Intuitionistic Monoid Structure Spaces

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Abstract

The purpose of this paper is to introduce the concepts of intuitionistic monoids, intuitionistic monoid structure spaces, intuitionistic monoid connected spaces, intuitionistic monoid S-connected spaces and intuitionistic monoid totally semi-continuous functions. Also the concepts of intuitionistic $\mathfrak{M}$-Noetherian spaces, intuitionistic $\mathfrak{M}_{\mathcal{S}}$-Noetherian spaces and intuitionistic $\mathcal{S}$-irresolute functions are introduced and some interesting properties are discussed.

Keywords

intuitionistic monoids, intuitionistic monoid structure spaces, intuitionistic monoid connected spaces, intuitionistic monoid S-connected spaces, intuitionistic monoid totally semi-continuous functions, intuitionistic $\mathfrak{M}$-Noetherian spaces, intuitionistic $\mathfrak{M}_{\mathcal{S}}$-Noetherian spaces and intuitionistic $\mathcal{S}$-irresolute functions.

2000 AMS Classification: 54A40,54E55.

1 Introduction

Lallement and Rosenfield introduced the concept of monoid in [4,8]. In this chapter, some new concepts like intuitionistic monoids, intuitionistic monoid structure spaces, intuitionistic monoid connected spaces, intuitionistic monoid S-connected spaces, intuitionistic monoid totally semi-continuous functions, intuitionistic $\mathfrak{M}$-Noetherian spaces and intuitionistic $\mathfrak{M}_{\mathcal{S}}$-Noetherian spaces are introduced and studied. Some interesting properties are established.

2 Preliminaries

Definition 2.1. [**1,2**] Let X be a nonempty fixed set.An *intuitionistic set*(IS for short) A is an object having the form $A = \langle x, A^1, A^2 \rangle$ for all $x \in X$, where A^1 and A^2 are subsets of X satisfying $A^1 \cap A^2 = \phi$. The set A^1 is called the set of members of A, while A^2 is called the set of nonmembers of A.

Every crisp set A on a non-empty set X is obviously an intuitionistic set having the form $\langle x, A, A^c \rangle$, and one can define several relations and operations between intuitionistic sets as follows:

Definition 2.2. [**1,2**] Let X be a nonempty set, $A = \langle x, A^1, A^2 \rangle$ for all $x \in X$, $B = \langle x, B^1, B^2 \rangle$ for all $x \in X$ be intuitionistic sets on X, and let $\{A_i : i \in J\}$ be an arbitrary family of intuitionistic sets in X, where $A_i = \langle x, A_i^1, A_i^2 \rangle$ for all $x \in X$.

(i) $A \subseteq B$ if and only if $A^1 \subseteq B^1$ and $B^2 \subseteq A^2$

(ii) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$

(iii) $\overline{A} = \langle x, A^2, A^1 \rangle$

(iv) $\cup A_i = \langle x, \cup A_i^1, \cap A_i^2 \rangle$

(v) $\cap A_i = \langle x, \cap A_i^1, \cup A_i^2 \rangle$

(vi) $A - B = A \cap \overline{B}$

(vii) $\phi_\sim = \langle x, \phi, X \rangle$ and $X_\sim = \langle x, X, \phi \rangle$.

Definition 2.3. [**1,2**] Let X and Y be two nonempty sets and $f : X \to Y$ a function

(i) If $B = \langle x, B^1, B^2 \rangle$ for all $x \in X$ is an intuitionistic set in Y,then the *preimage* of B under f,denoted by $f^{-1}(B)$,is an intuitionistic set in X defined by $f^{-1}(B) = \langle x, f^{-1}(B^1), f^{-1}(B^2) \rangle$.

(ii) If $A = \langle x, A^1, A^2 \rangle$ for all $x \in X$ is an intuitionistic set in X,then the *image* of A under f,denoted by $f(A)$,is the intuitionistic set in Y defined by $f(A) = \langle y, f(A^1), f_-(A^2) \rangle$ where $f_-(A^2) = (f(A^2)^c)^c$.

Definition 2.4. [**1**] An *intuitionistic topology* (IT for short) on a nonempty set X is a family τ of intuitionistic sets in X satisfying the following axioms:

(i) $\phi_\sim$ and $X_\sim \in \tau$,

(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$;

(iii) $\cup G_i \in \tau$ for any arbitrary family $\{G_i \mid i \in J\} \subseteq \tau$.

In this case the ordered pair (X, τ) is called an *intuitionistic topological space* (ITS for short) and any intuitionistic set in τ is known as an *intuitionistic open set* (IOS for short) in X. The complement $\overline{A}$ of an intuitionistic open set A is called an *intuitionistic closed set* (ICS for short) in X.

Definition 2.5. [**1**] Let (X, τ) be an intuitionistic topological space and $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic set in X.Then the intuitionistic interior and intuitionistic closure of A are defined by
$cl(A) = \cap\{K : K$ is an intuitionistic closed set in X and $A \subseteq K\}$,
$int(A) = \cup\{G : G$ is an intuitionistic open set in X and $G \subseteq A\}$.
It can be also shown that $cl(A)$ is an intuitionistic closed set and $int(A)$ is an intuitionistic open set in X and A is an intuitionistic closed set in X if $cl(A) = A$ and A is an intuitionistic open set in X if $int(A) = A$.

Definition 2.6. Let (X, T) be a topological space. A subset A in X is said to be

(i) semi-open [10] if $A \subseteq cl(int(A))$.

(ii) semi-closed [5] if $int(cl(A)) \subseteq A$.

Definition 2.7. [**2**] Let (X, T) be an intuitionistic topological space. An intuitionistic set $A = \langle X, A^1, A^2 \rangle$ of X is said to be intuitionistic clopen if it is both intuitionistic open and intuitionistic closed.

Definition 2.8. [**4**] A non-empty set S is said to be monoid if there is defined a binary operation, denoted by "." such that

(i) $a, b \in S$ implies that $a.b \in S$.

(ii) $a, b, c \in S$ implies that $(a.b).c = a.(b.c)$.

(iii) There exists an element $e \in G$ such that $a.e = e.a = a$ for all $a \in G$ (the existence of an identity element in G).

Definition 2.9. [**6**] Let (X, T) and (Y, S) be any two topological spaces. A function $f : (X, T) \to (Y, S)$ is said to be semi-continuous if $f^{-1}(A)$ is semi-open in X, for every open set A of Y.

Definition 2.10. [7] Let (X, T) and (Y, S) be any two topological spaces. A function $f : (X, T) \rightarrow (Y, S)$ is said to be totally continuous if $f^{-1}(A)$ is clopen in X, for every open set A of Y.

Definition 2.11. [11] Let (X, T) and (Y, S) be any two topological spaces. A function $f : (X, T) \rightarrow (Y, S)$ is said to be totally semi-continuous if $f^{-1}(A)$ is semi-clopen in X, for every open set A of Y.

Definition 2.12. [9] A topological space (X, T) is called Noetherian if every subset in (X, T) is compact.

3 Connectedness in Intuitionistic Monoid Structure Spaces

In this section, the concepts of intuitionistic monoids and intuitionistic monoid structure spaces are introduced. Also the concepts of intuitionistic monoid connected spaces, intuitionistic monoid S-connected spaces and intuitionistic monoid totally semi-continuous functions are introduced and studied.

Definition 3.1. Let M be a monoid. An intuitionistic set $A = \langle x, A^1, A^2 \rangle$ in M is called an intuitionistic monoid on M if it satisfies the following conditions:

(i) $a.b \in A$ for all $a, b \in A$

(ii) $(a.b).c = a.(b.c)$ for all $a, b, c \in A$

(iii) $a.e = e.a = a$ for identity $e \in A$.

Definition 3.2. Let M be a monoid. A family $\mathfrak{M}$ of intuitionistic monoids in M is said to be intuitionistic monoid structure on M if it satisfies the following axioms:

(i) $\phi_\sim$ and $M_\sim \in \mathfrak{M}$,

(ii) $M_1 \cap M_2 \in \mathfrak{M}$ for any $M_1, M_2 \in \mathfrak{M}$;

(iii) $\cup M_i \in \mathfrak{M}$ for any arbitrary family $\{M_i \mid i \in J\} \subseteq \mathfrak{M}$.

Then the ordered pair $(M, \mathfrak{M})$ is called an intuitionistic monoid structure space. Every member of $\mathfrak{M}$ is called an intuitionistic open monoid in $(M, \mathfrak{M})$. The complement $\overline{A}$ of an intuitionistic open monoid $A = \langle x, A^1, A^2 \rangle$ in $(M, \mathfrak{M})$ is an intuitionistic closed monoid in $(M, \mathfrak{M})$.

Example 3.1. Let $M = \{0, 1, 2\}$ be a set of integers modulo 3 with the binary operation as follows:

.	0	1	2
0	0	0	0
1	0	1	2
2	0	2	1

Then $(M, .)$ is a monoid. Define $\mathfrak{M} = \{\phi_\sim, M_\sim, A\}$ the family of intuitionistic monoids in M, where $A = \langle x, \{0, 1\}, \{2\}\rangle$. Clearly the family $\mathfrak{M}$ is an intuitionistic monoid structure and the ordered pair $(M, \mathfrak{M})$ is an intuitionistic monoid structure space.

Notation 3.1. Let $(M, \mathfrak{M})$ be any intuitionistic monoid structure space. Then

(i) $IMO(M)$ denotes the family of all intuitionistic open monoids in $(M, \mathfrak{M})$.

(ii) $IMC(M)$ denotes the family of all intuitionistic closed monoids in $(M, \mathfrak{M})$.

Definition 3.3. Let $(M, \mathfrak{M})$ be any intuitionistic monoid structure space. Let $A = \langle x, A^1, A^2\rangle$ be an intuitionistic monoid in M. Then

(i) the intuitionistic monoid interior of A is defined and denoted as
$I\mathfrak{M}int(A) = \cup\{B = \langle x, B^1, B^2\rangle \mid B \in IMO(M) \text{ and } B \subseteq A\}$.

(ii) the intuitionistic monoid closure of A is defined and denoted as
$I\mathfrak{M}cl(A) = \cap\{B = \langle x, B^1, B^2\rangle \mid B \in IMC(M) \text{ and } A \subseteq B\}$.

Remark 3.1. Let $(M, \mathfrak{M})$ be any intuitionistic monoid structure space. Let $A = \langle x, A^1, A^2\rangle$ be an intuitionistic monoid in M. Then the following statements hold:

(i) $I\mathfrak{M}cl(A) = A$ if and only if A is an intuitionistic closed monoid.

(ii) $I\mathfrak{M}int(A) = A$ if and only if A is an intuitionistic open monoid.

(iii) $I\mathfrak{M}int(A) \subseteq A \subseteq I\mathfrak{M}cl(A)$.

(iv) $I\mathfrak{M}int(M_\sim) = M_\sim$ and $I\mathfrak{M}int(\phi_\sim) = \phi_\sim$.

(v) $I\mathfrak{M}cl(M_\sim) = M_\sim$ and $I\mathfrak{M}cl(\phi_\sim) = \phi_\sim$.

(vi) $I\mathfrak{M}cl(\overline{A}) = \overline{I\mathfrak{M}int(A)}$ and $I\mathfrak{M}int(\overline{A}) = \overline{I\mathfrak{M}cl(A)}$.

Definition 3.4. An intuitionistic monoid structure space $(M, \mathfrak{M})$ is said to be an intuitionistic monoid connected space if $M_\sim$ cannot be expressed as the union of two intuitionistic open monoids $A = \langle x, A^1, A^2\rangle$ and $B = \langle x, B^1, B^2\rangle$ in M with $A \neq \phi_\sim$ and $B \neq \phi_\sim$. If $(M, \mathfrak{M})$ is not an intuitionistic monoid connected space then it is called as an intuitionistic monoid disconnected space.

Remark 3.2. If $(M, \mathfrak{M})$ is an intuitionistic monoid connected space then there exists no intuitionistic monoid $A \neq \phi_\sim$ which is both intuitionistic open monoid and intuitionistic closed monoid.

Example 3.2. Let $M = \{a, b, c\}$. Define $\mathfrak{M} = \{M_\sim, \phi_\sim, P, Q, R\}$ the family of intuitionistic monoids in M, where $P = \langle x, \{a\}, \{b, c\}\rangle, Q = \langle x, \{c\}, \{a, b\}\rangle$ and $R = \langle x, \{a, c\}, \{b\}\rangle$. Thus, $(M, \mathfrak{M})$ is an intuitionistic monoid structure space. Clearly, $M_\sim$ cannot be expressed as the union of two intuitionistic open monoids in M. Hence, $(M, \mathfrak{M})$ is an intuitionistic monoid connected space.

Definition 3.5. Let $(M_1, \mathfrak{M}_1)$ and $(M_2, \mathfrak{M}_2)$ be any two intuitionistic monoid structure spaces. A function $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is said to be an intuitionistic monoid continuous function if $f^{-1}(A)$ is an intuitionistic open monoid of $(M_1, \mathfrak{M}_1)$ for each intuitionistic open monoid A in $(M_2, \mathfrak{M}_2)$.

Proposition 3.1. An intuitionistic monoid continuous image of an intuitionistic monoid connected space is an intuitionistic monoid connected space.

Definition 3.6. Let $(M, \mathfrak{M})$ be an intuitionistic monoid structure space. An intuitionistic monoid $A = \langle x, A^1, A^2\rangle$ of M is said to be an intuitionistic semi-open monoid if $A \subseteq I\mathfrak{M}cl(I\mathfrak{M}int(A))$. The complement of an intuitionistic semi-open monoid is called an intuitionistic semi-closed monoid.

Definition 3.7. Let $(M, \mathfrak{M})$ be an intuitionistic monoid structure space. An intuitionistic monoid $A = \langle x, A^1, A^2\rangle$ of M is said to be intuitionistic clopen monoid if A is both intuitionistic open monoid and intuitionistic closed monoid in M.

Definition 3.8. Let $(M, \mathfrak{M})$ be an intuitionistic monoid structure space. An intuitionistic monoid $A = \langle x, A^1, A^2\rangle$ of M is said to be intuitionistic semi-clopen monoid if A is both intuitionistic semi-open monoid and intuitionistic semi-closed monoid in M.

Definition 3.9. An intuitionistic monoid structure space $(M, \mathfrak{M})$ is said to be an intuitionistic monoid S-connected space if $M_\sim$ cannot be expressed as the union of two intuitionistic semi-open monoids A and B in M with $A \neq \phi_\sim$ and $B \neq \phi_\sim$. If $(M, \mathfrak{M})$ is not an intuitionistic monoid S-connected space then it is called as an intuitionistic monoid S-disconnected space.

Definition 3.10. Let $(M_1, \mathfrak{M}_1)$ and $(M_2, \mathfrak{M}_2)$ be any two intuitionistic monoid structure spaces. A function $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is said to be an intuitionistic monoid semi-continuous function if $f^{-1}(A)$ is intuitionistic semi-open monoid in M_1, for every intuitionistic open monoid A of M_2.

Definition 3.11. Let $(M_1, \mathfrak{M}_1)$ and $(M_2, \mathfrak{M}_2)$ be any two intuitionistic monoid structure spaces. A function $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is said to be an intuitionistic monoid totally continuous function if $f^{-1}(A)$ is intuitionistic clopen monoid in M_1, for every intuitionistic open monoid A of M_2.

Definition 3.12. Let $(M_1, \mathfrak{M}_1)$ and $(M_2, \mathfrak{M}_2)$ be any two intuitionistic monoid structure spaces. A function $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is said to be an intuitionistic monoid totally semi-continuous function if $f^{-1}(A)$ is intuitionistic semi-clopen monoid in M_1, for every intuitionistic open monoid A of M_2.

Proposition 3.2. If $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is intuitionistic monoid totally semi-continuous, surjective function and $(M_1, \mathfrak{M}_1)$ is an intuitionistic monoid connected space then $(M_2, \mathfrak{M}_2)$ is an intuitionistic monoid S-connected space.

Definition 3.13. An intuitionistic monoid structure space $(M, \mathfrak{M})$ is said to be an indiscrete space if the only intuitionistic open monoids of M are $M_\sim$ and $\phi_\sim$.

Proposition 3.3. If $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is an intuitionistic monoid totally semi-continuous, surjective function from an intuitionistic monoid S-connected space $(M_1, \mathfrak{M}_1)$ to another space $(M_2, \mathfrak{M}_2)$, then $(M_2, \mathfrak{M}_2)$ is an indiscrete space.

Definition 3.14. An intuitionistic monoid $A = \langle x, A^1, A^2 \rangle$ of an intuitionistic monoid structure space $(M, \mathfrak{M})$ is said to be an intuitionistic connected monoid if and only if the only intuitionistic monoids which are both intuitionistic open monoids and intuitionistic closed monoids in $(M, \mathfrak{M})$ are $M_\sim$ and $\phi_\sim$.

Proposition 3.4. The intuitionistic monoid closure of an intuitionistic connected monoid is an intuitionistic connected monoid.

4 On Intuitionistic $\mathfrak{M}$-Noetherian Spaces

In this section, the concepts of intuitionistic $\mathfrak{M}$-Noetherian spaces, intuitionistic $\mathfrak{M}_\mathcal{S}$-Noetherian spaces and intuitionistic $\mathcal{S}$-irresolute functions are introduced and studied. In this connection, some interesting properties are established.

Definition 4.1. Let $(M, \mathfrak{M})$ be an intuitionistic monoid structure space. Let $A = \langle x, A^1, A^2 \rangle$ be an intuitionistic monoid in $\mathfrak{M}$. Let J be an indexed set. If a family $\{\langle x, G^1_\alpha, G^2_\alpha \rangle : \alpha \in J\}$ of intuitionistic open monoids in $(M, \mathfrak{M})$ satisfies the condition $A \subseteq \cup\{\langle x, G^1_\alpha, G^2_\alpha \rangle : \alpha \in J\}$, then it is called an intuitionistic open monoid cover of A. A finite subfamily of an intuitionistic open monoid cover $\{\langle x, G^1_\alpha, G^2_\alpha \rangle : \alpha \in I\}$ of A, where I is a finite subset of J, which is also an intuitionistic open monoid cover of A is called a finite intuitionistic open monoid subcover of $\{\langle x, G^1_\alpha, G^2_\alpha \rangle : \alpha \in J\}$.

Definition 4.2. An intuitionistic monoid $A = \langle x, A^1, A^2 \rangle$ in an intuitionistic monoid structure space $(M, \mathfrak{M})$ is said to be an intuitionistic compact monoid if every intuitionistic open monoid cover has a finite intuitionistic open monoid subcover.

Definition 4.3. An intuitionistic monoid structure space $(M, \mathfrak{M})$ is said to be an intuitionistic $\mathfrak{M}$-Noetherian space if every intuitionistic open monoid in $(M, \mathfrak{M})$ is an intuitionistic compact monoid.

Proposition 4.1. If $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is intuitionistic monoid continuous, bijective function and $(M_1, \mathfrak{M}_1)$ is an intuitionistic $\mathfrak{M}$-Noetherian space then $(M_2, \mathfrak{M}_2)$ is an intuitionistic $\mathfrak{M}$-Noetherian space.

Definition 4.4. Let $(M_1, \mathfrak{M}_1)$ and $(M_2, \mathfrak{M}_2)$ be any two intuitionistic monoid structure spaces. A function $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is said to be an intuitionistic open monoid function if the image of every intuitionistic open monoid is intuitionistic open monoid.

Proposition 4.2. If $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is an intuitionistic open monoid function, bijective function and $(M_2, \mathfrak{M}_2)$ is an intuitionistic $\mathfrak{M}$-Noetherian space then $(M_1, \mathfrak{M}_1)$ is an intuitionistic $\mathfrak{M}$-Noetherian space.

Definition 4.5. Let $(M, \mathfrak{M})$ be an intuitionistic monoid structure space. Let J be an indexed set. If a family $\{\langle x, G_i^1, G_i^2\rangle : i \in J\}$ of intuitionistic semi-open monoids in $(M, \mathfrak{M})$ satisfies the condition $A \subseteq \cup\{\langle x, G_i^1, G_i^2\rangle : i \in J\}$, then it is called an intuitionistic semi-open monoid cover of A. A finite subfamily of an intuitionistic semi-open monoid cover $\{\langle x, G_i^1, G_i^2\rangle : i \in J\}$ of A, which is also an intuitionistic semi-open monoid cover of A is called a finite intuitionistic semi-open monoid subcover of $\{\langle x, G_i^1, G_i^2\rangle : i \in J\}$.

Definition 4.6. An intuitionistic monoid $A = \langle x, A^1, A^2\rangle$ in an intuitionistic monoid structure space $(M, \mathfrak{M})$ is said to be an intuitionistic $\mathcal{S}$-compact monoid if every intuitionistic semi-open monoid cover has a finite intuitionistic semi-open monoid subcover.

Definition 4.7. An intuitionistic monoid structure space $(M, \mathfrak{M})$ is said to be an intuitionistic $\mathfrak{M}_{\mathcal{S}}$-Noetherian space if every intuitionistic semi-open monoid in $(M, \mathfrak{M})$ is an intuitionistic $\mathcal{S}$-compact monoid.

Proposition 4.3. If $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is an intuitionistic monoid semi-continuous, bijective function and $(M_1, \mathfrak{M}_1)$ is intuitionistic $\mathfrak{M}_{\mathcal{S}}$-Noetherian space then $(M_2, \mathfrak{M}_2)$ is an intuitionistic $\mathfrak{M}$-Noetherian space.

Definition 4.8. Let $(M_1, \mathfrak{M}_1)$ and $(M_2, \mathfrak{M}_2)$ be any two intuitionistic monoid structure spaces. A function $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is said to be an intuitionistic monoid $\mathcal{S}$-irresolute function if $f^{-1}(A)$ is intuitionistic semi-open monoid in M_1, for every intuitionistic semi-open monoid A of M_1.

Proposition 4.4. If $f : (M_1, \mathfrak{M}_1) \to (M_2, \mathfrak{M}_2)$ is an intuitionistic monoid $\mathcal{S}$-irresolute, bijective function and $(M_1, \mathfrak{M}_1)$ is intuitionistic $\mathfrak{M}_{\mathcal{S}}$-Noetherian space then $(M_2, \mathfrak{M}_2)$ is an intuitionistic $\mathfrak{M}_{\mathcal{S}}$-Noetherian space.

Definition 4.9. Let (X, T) be an intuitionistic topological space and $\mathcal{G} = \langle x, \mathcal{G}^1, \mathcal{G}^2\rangle$ be an intuitionistic set of X. Then $T_{\mathcal{G}} = \{A \cap G \mid A = \langle x, A^1, A^2\rangle \in T\}$ is an intuitionistic topology on G and is called the induced intuitionistic topology. The pair $(\mathcal{G}, T_{\mathcal{G}})$ is called an intuitionistic subspace of (X, T)

Proposition 4.5. Every intuitionistic subspace N of an intuitionistic $\mathfrak{M}$-Noetherian space $(M, \mathfrak{M})$ with the induced intuitionistic topology is an intuitionistic $\mathfrak{M}$-Noetherian space.

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