Images of generalized of modified multivariable I-function and special functions pertaining to multiple Erdélyi-Kober operator of Weyl type

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ABSTRACT
The aim in this paper is to establish the images of the product of certain special functions and new generalized modified of multivariable I-function with $zt^h(t^\mu+c^\mu)^{-\rho}$ as an argument pertaining to the multiple Erdélyi-Saigo operator due to Galué et al. The results encompass several cases of interest for Riemann-Liouville operators, Erdélyi-Kober operator and Saigo operators et cetera involving the product of certain special functions of general arguments.

Keywords :Modified of multivariable I-function, modified multivariable H-function, multivariable I-function, multivariable H-function, multiple Erdélyi-Kober operator of Weyl type, general class of polynomials, multivariable I-function, Fox's H-function

2000 Mathematics Subject Classification. 33C99, 33C60, 44A20

1.Introduction and definitions.

The multiple Erdélyi-Kober operator of Weyl type introduced by Galué et al. [5] is defined as :

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r}f(x)=\int_1^\infty H^{r,0}_{r,r}\left[\frac{1}{y}\left|\begin{matrix}(\eta_w+\zeta_w+\frac{1}{\tau_w};\frac{1}{\tau_w})_{1,r}\\(\eta_w+\frac{1}{\lambda_w};\frac{1}{\lambda_w})_{1,r}\end{matrix}\right.\right]f(xy)\mathrm{d}y,\text{ if }\sum_{w=1}^r\zeta_w>0 \tag{1.1}$$

$$=f(x), if\zeta_w=0,\lambda_w=\tau_w,w=1,\cdots,r,\text{ else}$$

where $\sum_{w=1}^r\frac{1}{\lambda_w}\geqslant\sum_{w=1}^r\frac{1}{\tau_w}$ and $f(x)\in C^*_\beta$

The class C^*_β is defined in the form ([5],page.56) .

$$C^*_\beta=\left\{f(x)=x^q\bar{f};q<\beta^*,\bar{f}\in C(0,\infty),|\bar{f}(x)|<A_{\bar{f}}\right\}\text{ and }\beta^*\leqslant max(\lambda_w,\eta_w) \tag{1.2}$$

Galué et al ([5],p.56) have shown that :

Lemma

$$K^{(\eta_w),(\zeta_w)}_{(\tau_w),(\lambda_w),r}(x^\rho)=\prod_{w=1}^r\frac{\Gamma\left(\eta_w-\frac{\rho}{\lambda_w}\right)}{\Gamma\left(\eta_w+\zeta_w-\frac{\rho}{\lambda_w}\right)}x^\rho \tag{1.3}$$

Srivastava and Garg [12] introduced and defined a general class of multivariable polynomials as follows

$$S_n^{w_1,\cdots,w_s}[x_1,\cdots,x_s]=\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}(-n)_{w_1k_1++\cdots+w_sk_s}A(n;k_1,\cdots,k_s)\frac{x_1^{k_1}\cdots x_s^{k_s}}{k_1!\cdots k_s!} \tag{1.4}$$

$n,w_1,\cdots,w_s$ are integers and the coefficients $A(n;k_1,\cdots,k_s)$ are arbitrary constants real or complex.

For $s=1$ the polynomials (1.4) reduces to general class of polynomials due to Srivastava [11]

$$S_n^w(x)=\sum_{k=0}^{[n/w]}\frac{(-n)_{wk}}{k!}A_{n,k}x^k, n\in\mathbb{N} \tag{1.5}$$

where w is an arbitrary positive integer, the coefficients $A_{n,k}$ $(n,k\in\mathbb{N})$ are arbitrary constants real or complex.

a) Since $H_n(x)=\sum_{k=0}^{[n/2]}\frac{(-)^k n!}{k!(n-2k)!}(2x)^{n-2k}$ (1.6)

defines Hermite polynomials therefore in this case, if we take

$$w=2, A_{n,k}=(-)^k, S_n^2(x)\rightarrow x^{n/2}H_n(1/2\sqrt{x}) \tag{1.7}$$

b) On setting $w=1, A_{n,k}=\binom{\alpha+n}{n}\frac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k}$, S_n^1 reduces to the Jacobi polynomials $P_n^{(\alpha,\beta)}(1-2x)$

defined by Szegö ([16] ,p. 68, eq . (4.3.2))

$$P_n^{\alpha,\beta}(x)=\sum_{k=0}^{n}\binom{n+\alpha}{n-k}\binom{n+\beta}{k}\left(\frac{x-1}{2}\right)^k\left(\frac{x+1}{2}\right)^{n-k} \tag{1.8}$$

The following series representation of the H-function given in [4] will be required in the proof.

$$H_{R,S}^{K,L}(z)=H_{R,S}^{K,L}\left[\frac{1}{z}\Bigg|\begin{matrix}(\mathrm{e}_R,E_R)\\(\mathrm{f}_S,F_S)\end{matrix}\right]=\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\zeta)}{E_h}\left(\frac{1}{z}\right)^{\xi} \tag{1.9}$$

where $\zeta=\frac{e_h-1-v_1}{E_h}$, and $h=1,\cdots,L$ (1.10)

we note $\eta(\xi)=\frac{\prod_{j=1}^{K}\Gamma(f_j+\xi F_j)\prod_{j=1,j\neq h}^{L}\Gamma(1-e_j+E_j\xi)}{\prod_{j=K+1}^{S}\Gamma(1-f_j+F_j\xi)\prod_{j=L+1}^{R}\Gamma(e_j+E_j\xi)}$ (1.11)

which exits for $z\neq 0$ if $\mu<0$ and for $|z|>\beta$ if $\mu=0$

$\mu=\sum_{j=1}^{S}F_j-\sum_{j=1}^{R}E_j$ and $\beta=\prod_{j=1}^{R}(E_j)^{E_j}\prod_{j=1}^{S}(F_j)^{-F_j}$

Prasad and Singh [8] have defined the modified multivariable H-function. Later Prasad [7] have studied the multivariable I-function. These two functions are an extension of multivariable H-function defined by Srivastava and Panda [13,14]. Of the other hand, recently Prathima et al. [9] have defined the multivariable I-function, this function is an extension of multivariable H-function defined by Srivastava and Panda [13-14]. In this paper, we define a general function who unify the functions defined above. We note I this function. It's defined by the following multiple integrals contour of Mellin-Barnes. We have.

$$\mathrm{I}(z_1,\cdots,z_r)=I_{p_2,q_2,p_3,q_3;\cdots;p_r,q_r:|R:p',q';\cdots;p^{(r)},q^{(r)}}^{m_2,n_2;m_3,n_3;\cdots;m_r,n_r:|R':m',n';\cdots;m^{(r)},n^{(r)}}$$

$$\left(\begin{array}{c|l} z_1 & (a_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})_{1,p_2};(a_{3j};\alpha_{3j}^{(1)},\alpha_{3j}^{(2)},\alpha_{3j}^{(3)};A_{3j})_{1,p_3};\cdots; \\ \cdot & \\ \cdot & \\ \cdot & \\ z_r & (b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})_{1,q_2};(b_{3j};\beta_{3j}^{(1)},\beta_{3j}^{(2)},\beta_{3j}^{(3)},B_{3j})_{1,q_3};\cdots; \end{array}\right.$$

$$\left.\begin{array}{l} (a_{rj};\alpha'_{rj},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,p_r}:(e_j;u'_jg'_j,\cdots,u_j^{(r)}g_j^{(r)};E_j)_{1,R'}:(a_j^{(1)};\alpha_j^{(1)};A_j^{(1)};)_{1,p^{(1)}},(a_j^{(r)};\alpha_j^{(r)};A_j^{(r)})_{1,p^{(r)}} \\ (b_{rj};\beta'_{rj},\cdots,\beta_{rj}^{(r)};B_{rj})_{1,q_r}:(l_j;U'_jf'_j,\cdots,U_j^{(r)}f_j^{(r)};L_j)_{1,R}:(b_j^{(1)};\beta_j^{(1)};B_j^{(1)};)_{1,q^{(1)}},(b_j^{(r)};\beta_j^{(r)};B_j^{(r)})_{1,q^{(r)}} \end{array}\right)$$

$$=\frac{1}{(2\pi\omega)^r}\int_{L_1}\cdots\int_{L_r}\xi(s_1,\cdots,s_r)\prod_{i=1}^{r}\phi_i(s_i)z_i^{s_i}\mathrm{d}s_1\cdots\mathrm{d}s_r \tag{1.12}$$

where

$$\xi(s_1,\cdots,s_r)=\frac{\prod_{j=1}^{n_2}\Gamma^{A_{2j}}(1-a_{2j}+\sum_{i=1}^{2}\alpha_{2j}^{(i)}s_i)\prod_{j=1}^{n_3}\Gamma^{A_{3j}}(1-a_{3j}+\sum_{i=1}^{3}\alpha_{3j}^{(i)}s_i)\cdots}{\prod_{j=n_2+1}^{p_2}\Gamma^{A_{2j}}(a_{2j}-\sum_{i=1}^{2}\alpha_{2j}^{(i)}s_i)\prod_{j=n_3+1}^{p_3}\Gamma^{A_{3j}}(a_{3j}-\sum_{i=1}^{3}\alpha_{3j}^{(i)}s_i)\cdots}$$

$$\frac{\cdots\prod_{j=1}^{n_r}\Gamma^{A_{rj}}(1-a_{rj}+\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i)\prod_{j=1}^{m_2}\Gamma^{B_{2j}}(b_{2j}-\sum_{i=1}^{r}\beta_{2j}^{(i)}s_i)}{\cdots\prod_{j=n_r+1}^{p_r}\Gamma^{A_{rj}}(a_{rj}-\sum_{i=1}^{r}\alpha_{rj}^{(i)}s_i)\prod_{j=m_2+1}^{q_2}\Gamma^{B_{2j}}(1-b_{2j}+\sum_{i=1}^{2}\beta_{2j}^{(i)}s_i)}$$

$$\frac{\prod_{j=1}^{m_3}\Gamma^{B_{3j}}(b_{3j}-\sum_{i=1}^{3}\beta_{3j}^{(i)}s_i)\cdots\prod_{j=1}^{m_r}\Gamma^{B_{rj}}(b_{rj}-\sum_{i=1}^{r}\beta_{rj}^{(i)}s_i)}{\prod_{j=m_3+1}^{q_3}\Gamma^{B_{3j}}(1-b_{3j}+\sum_{i=1}^{3}\beta_{3j}^{(i)}s_i)\cdots\prod_{j=m_r+1}^{q_r}\Gamma^{B_{rj}}(1-b_{rj}+\sum_{i=1}^{r}\beta_{rj}^{(i)}s_i)}$$

$$\frac{\prod_{j=1}^{R'}\Gamma^{E_j}(e_j+\sum_{i=1}^{r}u_j^{(i)}g_j^{(i)}s_i)}{\prod_{j=1}^{R}\Gamma^{L_j}(l_j+\sum_{i=1}^{r}U_j^{(i)}f_j^{(i)}s_i)} \tag{1.13}$$

$$\phi(s_i;q)=\frac{\prod_{j=1}^{m^{(i)}}\Gamma^{B_j^{(i)}}(b_j^{(i)}-\beta_j^{(i)}s)\prod_{j=1}^{n^{(i)}}\Gamma^{A_j^{(i)}}(1-a_j^{(i)}+\alpha_j^{(i)}s_i)}{\prod_{j=1+m^{(i)}}^{q^{(i)}}\Gamma^{B_j^{(i)}}(1-b_j^{(i)}+\beta_j^{(i)}s_i)\prod_{j=n^{(i)}+1}^{p^{(i)}}\Gamma^{A_j^{(i)}}(a_j^{(i)}-\alpha_j^{(i)}s_i)} \tag{1.14}$$

where

$A_{ij}(i=2,\cdots,r;j=1,\cdots,p_i), B_{ij}(i=2,\cdots,r;j=1,\cdots,q_i)$, $A_j^{(i)}(i=1,\cdots,r;j=1,\cdots,p^{(i)})$,$B_j^{(i)}(i=1,\cdots,r;$ $j=1,\cdots,q^{(i)}), g_j^{(i)}(i=1,\cdots,r;j=1,\cdots,R), E_j(j=1,\cdots,R'), L_j(j=1,\cdots,R), f_j^{(i)}(i=1,\cdots,r,j=1,\cdots,q^{(i)})$, $\alpha_{kj}^{(i)}(i=1,\cdots,r;j=2,\cdots,r;k=1,\cdots,p_i)$, $\beta_{kj}^{(i)}(i=1,\cdots,r;j=2,\cdots,r;k=1,\cdots,q_i)$, $\alpha_j^{(i)}(i=1,\cdots,r;j=1,\cdots,q^{(i)})$, $\beta_j^{(i)}(i=1,\cdots,r;j=1,\cdots,q^{(i)})$ are positive real numbers.

$e_j(j=1,\cdots,R'), l_j(j=1,\cdots,R), a_j^{(i)}(i=1,\cdots,r;j=1,\cdots,p^{(i)}), b_j^{(i)}(i=1,\cdots,r,j=1,\cdots,q^{(i)})$, $a_{kj}(i=1,\cdots,r;$ $j=2,\cdots,p_i), b_{kj}(i=1,\cdots,r;j=2,\cdots,q_i)$ are complex numbers and here $m^{(i)},n^{(i)},p^{(i)},q^{(i)}(i=1,\cdots,r)$, $m_k,n_k,p_k,q_k(k=2,\cdots,r)$ are non-negative integers where $0\leqslant m_k\leqslant q_k$ $0\leqslant m^{(i)}\leqslant q^{(i)};0\leqslant n^{(i)}\leqslant p^{(i)}(i=1,\cdots,r)$ and $0\leqslant n_k\leqslant p_k$.

Here (i) denotes the numbers of dashes. The contour L_k is in the $s_k(k=1,\cdots,r)$- plane and run from $\sigma-i\infty$ to $\sigma+i\infty$ where σ if is a real number with loop, if necessary to ensure that the poles of $\Gamma^{A_{2j}}(1-a_{2j}+\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k)$,

$(j=1,\cdots,n_2)$, $\Gamma^{A_{3j}}(1-a_{3j}+\sum_{k=1}^{3}\alpha_{3j}^{(k)}s_k),(j=1,\cdots,n_3)$ $,\cdots,\Gamma^{A_{rj}}(1-a_{rj}+\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k),(j=1,\cdots,n_r)$ $\Gamma^{A_j^{(k)}}(1-a_j^{(k)}+_j^{(k)}s_k),(j=1,\cdots,n^{(k)}),(k=1,\cdots,r)$ to the left of the contour L_k and the poles of $\Gamma^{B_{2j}}(b_{2j}-\sum_{k=1}^{2}\alpha_{2j}^{(k)}s_k),(j=1,\cdots,m_2),\cdots,\Gamma^{B_{rj}}(b_{rj}-\sum_{k=1}^{r}\alpha_{rj}^{(k)}s_k),(j=1,\cdots,m_r),\Gamma^{B_j^{(k)}}(b_j^{(k)}-\beta_j^{(k)}s_k),(j=1,\cdots,m^{(k)})$, $(k=1,\cdots,r)$ lie to the right of the contour L_k. For further details and asymptotic expansion of the I-function one can refer by Prasad [7]. The poles of the integrand are assumed to be simple.. Also the pole of $\Gamma(e_j+\sum_{i=1}^{r}u_i^{(j)}g_j^{i)}s_i)$ lie to the left or right of it according $u_j^{(i)}$ is positive or negative. The various parameters being restricted so that these poles of the integrand are assumed to be simple. The point $z_i=0(i=1,\cdots,r)$ being tacitly excluded. The multiple contour integral (2.3) converges absolutely if

$|argz_i|<\frac{1}{2}\Omega_i\pi$, where

$$\Omega_i=\sum_{k=1}^{n^{(i)}}A_k^{(i)}\alpha_k^{(i)}-\sum_{k=n^{(i)}+1}^{p^{(i)}}A_k^{(i)}\alpha_k^{(i)}+\sum_{k=1}^{m^{(i)}}B_k^{(i)}\beta_k^{(i)}-\sum_{k=m^{(i)}+1}^{q^{(i)}}B_k^{(i)}\beta_k^{(i)}+\sum_{k=1}^{n_2}A_{2k}^{(i)}\alpha_{2k}^{(i)}-\sum_{k=n_2+1}^{p_2}A_{2k}^{(i)}\alpha_{2k}^{(i)}+\sum_{k=1}^{n_3}A_{3k}^{(i)}\alpha_{3k}^{(i)}$$

$$-\sum_{k=n_3+1}^{p_3}A_{3k}^{(i)}\alpha_{3k}^{(i)}+\cdots+\sum_{k=1}^{n_r}A_{rk}^{(i)}\alpha_{rk}^{(i)}-\sum_{k=n_r+1}^{p_r}A_{rk}^{(i)}\alpha_{rk}^{(i)}+\sum_{k=1}^{m_2}B_{2k}^{(i)}\beta_{2k}^{(i)}-\sum_{k=m_2+1}^{q_2}B_{2k}^{(i)}\beta_{2k}^{(i)}+\sum_{k=1}^{m_3}B_{3k}^{(i)}\beta_{3k}^{(i)}-$$

$$\sum_{k=m_3+1}^{q_3}B_{3k}^{(i)}\beta_{3k}^{(i)}+\cdots+\sum_{k=1}^{m_r}B_{rk}^{(i)}\beta_{rk}^{(i)}-\sum_{k=m_r+1}^{q_r}B_{rk}^{(i)}\beta_{rk}^{(i)}+\sum_{j=1}^{R'}G_j^{(i)}g_j^{(i)}-\sum_{j=1}^{R}F_j^{(i)}f_j^{(i)}>0(i=1,\cdots,r) \tag{1.15}$$

We shall use the following notations in this paper.

$$A=(a_{2j};\alpha_{2j}^{(1)},\alpha_{2j}^{(2)};A_{2j})_{1,p_2};\cdots;(a_{(r-1)j};\alpha_{(r-1)j}^{(1)},\cdots,\alpha_{(r-1)j}^{(r-1)};A_{(r-1)j})_{1,p_{r-1}} \tag{1.16}$$

$$B=(b_{2j};\beta_{2j}^{(1)},\beta_{2j}^{(2)};B_{2j})_{1,q_2};\cdots;(b_{(r-1)j};\beta_{(r-1)j}^{(1)},\cdots,\beta_{(r-1)j}^{(r-1)};B_{(r-1)j})_{1,q_{r-1}} \tag{1.17}$$

$$\mathbf{A}=(\mathrm{a}_{rj};\alpha_{rj}^{(1)},\cdots,\alpha_{rj}^{(r)};A_{rj})_{1,p_r};\mathrm{E}=(\mathrm{e}_j;u_j^{(1)}g_j^{(1)},\cdots,u_j^{(r)}g_j^{(r)};E_j)_{1,R'} \tag{1.18}$$

$$\mathfrak{A}=(a_j^{(1)},\alpha_j^{(1)};A_j^{(1)})_{1,p'};\cdots;(a_j^{(r)},\alpha_j^{(r)};A_j^{(r)})_{1,p^{(r)}} \tag{1.19}$$

$$\mathbf{B}=(\mathrm{b}_{rj};\beta_{rj}^{(1)},\cdots,\beta_{rj}^{(r)};B_{rj})_{1,q_r};\mathrm{L}=(\mathrm{l}_j;U_j^{(1)}f_j^{(1)},\cdots,U_j^{(r)}f_j^{(r)};L_j)_{1,R} \tag{1.20}$$

$$\mathfrak{B}=(\mathrm{b}_j^{(1)},\beta_j^{(1)};B_j^{(1)})_{1,q'};\cdots;(b_j^{(r)},\beta_j^{(r)};B_j^{(r)})_{1,q^{(r)}} \tag{1.21}$$

$$U=p_2,q_2;p_3,q_3;\cdots;p_{r-1},q_{r-1};V=0,n_2;0,n_3;\cdots;0,n_{r-1} \tag{1.22}$$

$$Y=(p',q');\cdots;(p^{(r)},q^{(r)});X=(m',n');\cdots;(m^{(r)},n^{(r)}) \tag{1.23}$$

2. Images under multiple Erdélyi-Kober operator.

We will consider the new generalized of modified multivariable I-function of t variables

Theorem.

If will be shown here that , if

$$f(x)=x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma}\,\mathrm{I}\left(\begin{array}{c} \mathrm{z}_1 x^{-h_1}(x^{\mu}+c^{\mu})^{-\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_t x^{-h_t}(x^{\mu}+c^{\mu})^{-\rho_t}\end{array}\right) S_n^{w_1,\cdots,w_s}\left(\begin{array}{c} \mathrm{x}^{P_1}(x^{\mu}+c^{\mu})^{-q_1}\\ \cdot\\ \cdot\\ \mathrm{x}^{P_s}(x^{\mu}+c^{\mu})^{-q_s}\end{array}\right)$$

$$H_{R,S}^{K,L}\left[zx^{-\ddot{t}}(x^{\mu}+c^{\mu})^{-\eta}\left|\begin{array}{c}(\mathrm{e}_R,E_R)\\ (\mathrm{f}_S,F_S)\end{array}\right.\right] \tag{2.1}$$

with $Re(-A+\min_{1\leqslant k\leqslant r}(\lambda_k\gamma_k))>0$, $\sum_{i=1}^{r}\frac{1}{\lambda_i}\geqslant\sum_{i=1}^{r}\frac{1}{\tau_i}$ and $\eta,\rho,\sigma,h_i,\rho_i(i=1,\cdots,t),p_i,q_i(i=1,\cdots,s)>0$ and

$|argz_i|<\frac{1}{2}\Omega_i\pi$, Ω_i is defined by (1.15), then there holds the following formula

$$K_{(\tau_w),(\lambda_w),r}^{(\eta_w),(\zeta_w)}[f(x)]=x^{\rho}c^{-\mu\sigma}\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}(-n)_{w_1k_1+\cdots+w_sk_s}A(n;k_1,\cdots,k_s)\frac{c^{-\mu\sum_{i=1}^{s}q_ik_i}}{k_1!\cdots k_s!}$$

$$\sum_{l=0}^{\infty}\frac{(-)^l x^{\mu l}}{l!c^{\mu l}}H_{R,S}^{K,L}(zx^{-\ddot{t}}c^{-\mu\eta})\mathrm{I}_{U;p_t+r+1,q_t+r+1:|R:Y}^{V;m_t+r+1,n_t:|R':X}\left(\begin{array}{c}\frac{z_1}{x^{h_1}c^{\mu\rho_1}}\\ \cdot\\ \cdot\\ \frac{z_t}{x^{h_t}c^{\mu\rho_t}}\end{array}\left|\begin{array}{c}\mathrm{A};\mathbf{A},[1\text{-}\Delta-l;\rho_1,\cdots,\rho_t],\\ \cdot\\ \cdot\\ \mathrm{B};\mathbf{B},[1\text{-}\Delta;\rho_1,\cdots,\rho_t],\end{array}\right.\right.$$

$$\left.\begin{array}{c}[1\text{-}\eta_j+E;\frac{h_1}{\lambda_j},\cdots,\frac{h_t}{\lambda_j}]_{1,r}:E:\mathfrak{A}\\ \cdot\\ \cdot\\ [1\text{-}\eta_j-\zeta_j+E;\frac{h_1}{\lambda_j},\cdots,\frac{h_t}{\lambda_j}]_{1,r}:L:\mathfrak{B}\end{array}\right) \tag{2.2}$$

where

$$E=\frac{\rho+\sum_{i=1}^{s}P_ik_i+\mu l-\ddot{t}\zeta}{\lambda_j},\Delta=\sigma+\sum_{i=1}^{s}q_ik_i-\eta\zeta \tag{2.3}$$

and the serie defined by the equation (2.2) is convergent.

Proof : Let $M\{.\}=\frac{1}{(2\pi\omega)^t}\int_{L_1}\cdots\int_{L_t}\xi(s_1,\cdots,s_t)\prod_{i=1}^{t}\phi_i(s_i)z_i^{\zeta_i}\{.\}$ (2.4)

and $f(x)=x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma}\,\mathrm{I}\left(\begin{array}{c} \mathrm{z}_1 x^{-h_1}(x^{\mu}+c^{\mu})^{-\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_t x^{-h_t}(x^{\mu}+c^{\mu})^{-\rho_t}\end{array}\right) S_n^{w_1,\cdots,w_s}\left(\begin{array}{c} \mathrm{x}^{P_1}(x^{\mu}+c^{\mu})^{-q_1}\\ \cdot\\ \cdot\\ \mathrm{x}^{P_r}(x^{\mu}+c^{\mu})^{-q_r}\end{array}\right)$

$$H_{R,S}^{K,L}\left[zx^{-\ddot{t}}(x^{\mu}+c^{\mu})^{-\eta}\left|\begin{array}{c}(\mathrm{e}_R,E_R)\\ (\mathrm{f}_S,F_S)\end{array}\right.\right] \tag{2.5}$$

First, we express the new generalized of modified multivariable I-function , general class of polynomials and H-function by using equations (1.12), (1.9) and (1.4) respectively.

We have : $f(x) = x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma}\, M\left\{\left[x^{-h_1}(x^{\mu}+c^{\mu})^{-\rho_1}\right]^{-\zeta_1}\cdots\left[x^{-h_t}(x^{\mu}+c^{\mu})^{-\rho_t}\right]^{-\zeta_t}\right]$

$$\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}(-n)_{w_1k_1+\cdots+w_sk_s}A(n;k_1,\cdots,k_s)\frac{\left[x^{P_1}(x^{\mu}+c^{\mu})^{-q_1}\right]^{k_1}}{k_1!}\cdots\frac{\left[x^{P_s}(x^{\mu}+c^{\mu})^{-q_s}\right]^{k_s}}{k_s!}$$

$$\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\zeta)}{E_h}\left(zx^{-\ddot{t}}(x^{\mu}+c^{\mu})^{-\eta}\right)^{\zeta}\Bigg\}\ \mathrm{d}s_1\cdots\mathrm{d}s_t \tag{2.6}$$

Now, change the order of integration ans summation (which is easily seen to be justified due to the absolute convergence of the integral and the summations involved in the process), we thus find that

$$f(x)=x^{\rho}\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}(-n)_{w_1k_1+\cdots+w_sk_s}\frac{A(n;k_1,\cdots,k_s)}{k_1!\cdots k_s!}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\zeta)}{E_h}z^{-\zeta}$$

$$M\left\{x^{\rho-\ddot{t}\zeta+\sum_{i=1}^{t}h_i\zeta_i+\sum_{i=1}^{s}P_ik_i}(x^{\mu}+c^{\mu})^{-\sigma+\eta\xi+\sum_{i=1}^{t}\rho_i\xi_i-\sum_{i=1}^{s}q_ik_i}\right\}\mathrm{d}s_1\cdots\mathrm{d}s_t \tag{2.7}$$

with algebraic manipulations, we obtain

$$f(x)=\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}(-n)_{w_1k_1+\cdots+w_sk_s}\frac{A(n;k_1,\cdots,k_s)}{k_1!\cdots k_s!}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\zeta)}{E_h}z^{-\zeta}$$

$$M\left\{x^{\rho-\ddot{t}\zeta+\sum_{i=1}^{t}h_i\zeta_i+\sum_{i=1}^{s}P_ik_i}\left(1+\frac{x^{\mu}}{c^{\mu}}\right)^{-\sigma+\eta\zeta+\sum_{i=1}^{t}\rho_i\zeta_i-\sum_{i=1}^{s}q_ik_i}c^{\mu(-\sigma+\eta\zeta+\sum_{i=1}^{t}\rho_i\zeta_i-\sum_{i=1}^{s}q_ik_i)}\right\}\mathrm{d}s_1\cdots\mathrm{d}s_t \tag{2.8}$$

Use the binomial formula, we get

$$f(x)=\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}(-n)_{w_1k_1+\cdots+w_sk_s}\frac{A(n;k_1,\cdots,k_s)}{k_1!\cdots k_s!}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\zeta)}{E_h}z^{-\zeta}$$

$$\sum_{l=0}^{\infty}\frac{(-)^{l}}{l!}\quad M\left\{\left(\sigma-\eta\zeta-\textstyle\sum_{i=1}^{t}\rho_i\zeta_i+\sum_{i=1}^{s}q_ik_i\right)_l\ c^{\mu(-\sigma-l+\eta\zeta+\sum_{i=1}^{t}\rho_i\zeta_i-\sum_{i=1}^{s}q_ik_i)}\right.$$

$$\left.x^{\rho+\mu l-\ddot{t}\zeta+\sum_{i=1}^{t}h_i\zeta_i+\sum_{i=1}^{s}P_ik_i}\right\}\mathrm{d}s_1\cdots\mathrm{d}s_t \tag{2.9}$$

Now to establish the images under multiple Erdélyi-Kober operator by using the lemma of the function defined by the equations (2.1), we get :

$$K_{(\tau_w),(\lambda_w),r}^{(\eta_w),(\zeta_w)}\left[f(x)\right]=\sum_{k_1,\cdots,k_s=0}^{w_1k_1+\cdots w_sk_s\leqslant n}\sum_{h=1}^{L}\sum_{v_1=0}^{\infty}(-n)_{w_1k_1+\cdots+w_sk_s}\frac{A(n;k_1,\cdots,k_s)}{k_1!\cdots k_s!}\frac{(-)^{v_1}}{v_1!}\frac{\eta(\zeta)}{E_h}z^{-\zeta}$$

$$\sum_{l=0}^{\infty}\frac{(-)^l}{l!}\ M\left\{\left(\sigma-\eta\zeta-\sum_{i=1}^{t}\rho_i\zeta_i+\sum_{i=1}^{s}q_ik_i\right)_l\ c^{\mu(-\sigma-l+\eta\zeta-\sum_{i=1}^{t}\rho_i\zeta_i-\sum_{i=1}^{s}q_ik_i)}\right.$$

$$\prod_{w=1}^{s}\frac{\Gamma\left[\eta_w-\frac{\rho+\mu l-\ddot{t}\zeta+\sum_{i=1}^{t}h_i\zeta_i+\sum_{i=1}^{s}P_ik_i}{\lambda_w}\right]}{\Gamma\left[\eta_w-\rho+\zeta_w-\frac{\mu l-\ddot{t}\zeta+\sum_{i=1}^{t}h_i\zeta_i+\sum_{i=1}^{s}P_ik_i}{\lambda_w}\right]}x^{\rho+\mu l-\ddot{t}\zeta+\sum_{i=1}^{t}h_i\zeta_i+\sum_{i=1}^{s}P_ik_i}\left.\right\}\mathrm{d}s_1\cdots\mathrm{d}s_t \tag{2.10}$$

Finally interpreting the result thus obtained with the multiple Mellin-barnes contour integrals with the help of (1.12) and use also the equation (1.9), we arrive at the desired result (2.2).

3. Applications.

Taking $s=1$ in the equation (2.2), the polynomial (1.4) will reduce to $S_n^w(x)$ which is defined by the equation (1.5) and consequently, we obtain the following result. On suitably specializing the coefficients $A_{N,K}$, $S_N^M(x)$ yields some known polynomials, these include the Jacobi polynomials, Laguerre polynomials, and others polynomials ([15],p. 158-161).

Corollary 1.

$$K_{(\tau_w),(\lambda_w),r}^{(\eta_w),(\zeta_w)}\left[f_1(x)\right]=x^{\rho}c^{-\mu\sigma}\sum_{k=0}^{[n/w]}\frac{A(n;k)x^{pk}}{k!}c^{-q\mu k}\sum_{l=0}^{\infty}\frac{(-)^l x^{\mu l}}{l!c^{\mu l}}\ H_{R,S}^{K,L}\left(zx^{-\ddot{t}}c^{-\mu\eta}\right)$$

$$\mathrm{I}_{U;p_t+r+1,q_t+r+1:|R:Y}^{V;m_t+r+1,n_t:|R':X}\left(\begin{array}{c|c}\frac{z_1}{x^{h_1}c^{\mu\rho_1}} & \mathrm{A};\mathbf{A},\ \left[1\text{-}\eta_j+E^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_t}{\lambda_j}\right]_{1,s},\left[1-\Delta^*-l;\rho_1,\cdots,\rho_t\right]:E:\mathfrak{A}\\ \cdot & \\ \cdot & \cdots\\ \cdot & \\ \frac{z_t}{x^{h_t}c^{\mu\rho_t}} & \mathrm{B};\mathbf{B};\ \left[1\text{-}\eta_j-\zeta_j+E^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_t}{\lambda_j}\right]_{1,s},\left[1-\Delta^*;\rho_1,\cdots,\rho_t\right]:L:\mathfrak{B}\end{array}\right) \tag{3.1}$$

where $,\ E^*=\dfrac{\rho+pk+\mu l-\ddot{t}\zeta}{\lambda_j},\Delta^*=\sigma+qk-\eta\zeta$ (3.2)

and $f_1(x)=x^{\rho}(x^{\mu}+c^{\mu})^{-\sigma}\,\mathrm{I}\left(\begin{array}{c}\mathrm{z}_1x^{-h_1}(x^{\mu}+c^{\mu})^{-\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_tx^{-h_t}(x^{\mu}+c^{\mu})^{-\rho_t}\end{array}\right)S_n^w(x^p(x^{\mu}+c^{\mu})^{-q})$

$$H_{R,S}^{K,L}\left[zx^{-\ddot{t}}(x^{\mu}+c^{\mu})^{-\eta}\left|\begin{array}{c}(\mathrm{e}_R,E_R)\\(\mathrm{f}_S,F_S)\end{array}\right.\right] \tag{3.3}$$

under the same validity conditions that (2.2)

Setting $s=1,w=2,A_{n,k}=(-)^k$ in (2.2), then by vertue of the result (1.7), we have the following result

Corollary 2.

$$K_{(\tau_w),(\lambda_w),r}^{(\eta_w),(\zeta_w)}\left[f_2(x)\right]=x^{\rho}c^{-\mu\sigma}\sum_{k=0}^{[n/2]}\ (-)^k(-n)_{2k}\frac{c^{-kq\mu}}{k!}x^{pk}\sum_{l=0}^{\infty}\frac{(-)^l x^{\mu l}}{l!c^{\mu l}}\ H_{R,S}^{K,L}\left(zx^{-\ddot{t}}c^{-\mu\eta}\right)$$

$$\mathrm{I}_{U;p_t+r+1,q_t+r+1:|R:Y}^{V;m_t+r+1,n_t:|R':X}\left(\begin{array}{c}\frac{z_1}{x^{h_1}c^{\mu\rho_1}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{z_t}{x^{h_t}c^{\mu\rho_t}}\end{array}\left|\begin{array}{c}\mathrm{A};\mathbf{A},\ [1\text{-}\eta_j+E^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_t}{\lambda_j}]_{1,s},[1-\Delta^*-l;\rho_1,\cdots,\rho_t]:E:\mathfrak{A}\\ \cdots\\ \mathrm{B};\mathbf{B};\ [1\text{-}\eta_j-\zeta_j+E'^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_t}{\lambda_j}]_{1,s},[1-\Delta^*;\rho_1,\cdots,\rho_t]:L:\mathfrak{B}\end{array}\right.\right) \tag{3.4}$$

and $f_2(x)=x^{\rho+\frac{np}{2}}(x^\mu+c^\mu)^{-\sigma-\frac{nq}{2}}\,\mathrm{I}\left(\begin{array}{c}\mathrm{z}_1x^{-h_1}(x^\mu+c^\mu)^{-\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_tx^{-h_t}(x^\mu+c^\mu)^{-\rho_t}\end{array}\right)H_n\left[\frac{(x^\mu+c^\mu)^{q/2}}{2x^{p/2}}\right]$

$$H_{R,S}^{K,L}\left[zx^{-\ddot{\imath}}(x^\mu+c^\mu)^{-\eta}\left|\begin{array}{c}(\mathrm{e}_R,E_R)\\ (\mathrm{f}_S,F_S)\end{array}\right.\right] \tag{3.5}$$

Next , if we set $s=1, w=1$ and $A_{n,k}=\binom{n+\alpha}{n}\frac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k}$ in (2.2), then by vertue of the result (1.8), we have the following result under the same validity conditions and notations that (3.1)

Corollary 3.

$$K_{(\tau_w),(\lambda_w),r}^{(\eta_w),(\zeta_w)}[f_3(x)]=x^\rho c^{-\mu\sigma}\sum_{k=0}^{n}\binom{n+\alpha}{n}\frac{(\alpha+\beta+n+1)_k}{(\alpha+1)_k}(-n)_k\frac{c^{-kq\mu}}{k!}x^{pk}\sum_{l=0}^{\infty}\frac{(-)^lx^{\mu l}}{l!c^{\mu l}}\ H_{R,S}^{K,L}(zx^{-\ddot{\imath}}c^{-\mu\eta})$$

$$\mathrm{I}_{U;p_t+r+1,q_t+r+1:|R:Y}^{V;m_t+r+1,n_t:|R':X}\left(\begin{array}{c}\frac{z_1}{x^{h_1}c^{\mu\rho_1}}\\ \cdot\\ \cdot\\ \cdot\\ \frac{z_t}{x^{h_t}c^{\mu\rho_t}}\end{array}\left|\begin{array}{c}\mathrm{A};\mathbf{A},\ [1\text{-}\eta_j+E^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_t}{\lambda_j}]_{1,s},[1-\Delta^*-l;\rho_1,\cdots,\rho_t]:E:\mathfrak{A}\\ \cdots\\ \mathrm{B};\mathbf{B};\ [1\text{-}\eta_j-\zeta_j+E'^*;\frac{h_1}{\lambda_j},\cdots,\frac{h_t}{\lambda_j}]_{1,s},[1-\Delta^*;\rho_1,\cdots,\rho_t]:L:\mathfrak{B}\end{array}\right.\right) \tag{3.6}$$

and $f_3(x)=x^\rho(x^\mu+c^\mu)^{-\sigma}\,\mathrm{I}\left(\begin{array}{c}\mathrm{z}_1x^{-h_1}(x^\mu+c^\mu)^{-\rho_1}\\ \cdot\\ \cdot\\ \mathrm{z}_tx^{-h_t}(x^\mu+c^\mu)^{-\rho_t}\end{array}\right)P_n^{(\alpha,\beta)}\left[1-2x^p(x^\mu+c^\mu)^{-q}\right]$

$$H_{R,S}^{K,L}\left[zx^{-\ddot{\imath}}(x^\mu+c^\mu)^{-\eta}\left|\begin{array}{c}(\mathrm{e}_R,E_R)\\ (\mathrm{f}_S,F_S)\end{array}\right.\right] \tag{3.7}$$

under the same validity conditions and notations that (3.1)

Remarks :
We obtain the same formula concerning the generalized of modified multivariable H-function defined by Prasad and Singh [8], the generalized of multivariable I-function defined by Prasad [7], the generalized of modified of multivariable I-function defined by Prathima et al. [9], the generalized of multivariable I-function defined by Prathima et al. [9], the multivariable Aleph-function defined by Ayant [1], the multivariable Gimel-function defined by Ayant [2], the multivariable A-function defined by Gautam et al. [6] and the generalized of the multivariable H-function defined by Srivastava and Panda [13,14]. Chaurasia and Dubey [3] have obtained these results concerning the multivariable H-function.

4. Conclusion.

In this paper we have evaluated the images of the product of certain special functions and new generalized of modified multivariable I-function , a class of polynomials and Fox's H-function of one variable pertaining to multiple Erdélyi-Kober operator .The images established in this paper is of very general nature as it contains this generalized of modified multivariable I-function, which is a general function of several variables studied so far. Thus, the integral established in this research work would serve as a key formula from which, upon specializing the parameters, as many as desired results involving the special functions of one and several variables can be obtained.

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