DYNAMICS OF CRIMINAL ACTIVITY

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Abstract- *The issue of Criminal activities in our societies remains a major challenge. It is very difficult to eradicate Criminal activities from the societies. In recent years researchers highlight applied mathematics as valuable theoretical resources that may help us better understand criminal activity. In this paper we review the Berestycki and Nadal's model of Dynamics of criminal activity and formation of crime hotspots. We also propose adding repeated victimization leaving aside the term 'risk associated with cost for criminal activities' in our model.*

Keywords*:* *crime hotspots, repeated victimization, acting out function*

I. Introduction

Criminal Activity depends upon the place and time. The Spatio-temporal behavior of the criminal activity can be studied in many categories. Criminal activity generally does not occur in day-light except metropolitan cities. However, in the remote places, night and evening time is preferred for stealing and other crimes. Highway robbery often takes place at lonely and non-congested road at midnight. The peak hour of marketing, vehicle or bike theft is much more common than in normal time at parking places. Criminal activity – such as burglary, larceny *etc*. are committed during day time in the absence of capable owners and during evening time with the help of darkness [15,16] . Criminal activity related to sex is committed in the lonely place at late night. Murder and sexual crimes take place at evening or at night in bars and restaurants of metropolitan cities. The departure time of trains from stations is the boon time for pick-pockets to commit crimes like snatching *etc*. When a mob becomes uncontrollable, criminal finds an opportunity to indulge themselves in loot and damage of the national and private goods.

Mathematical models using concepts from different subjects contribute to the field of criminology with their own methods. Many mathematical models have been developed in the criminology literature. As for examples Economic crime models were presented by G.S.Becker [1], and Isaac Ehrlich [2,3,4], social crime models were proposed by Glaeser *et al* [5,6,7,8]. There is a huge number of publications in the other literature of crime. Among them, we may mentioned the relation between education and crime [9], relation between crime and unemployment[10]. Cambel *et al* [11] treat criminality as an epidemics problem using differential equations. Another mathematical model of crime was introduced [12] using population dynamics.

We are interested to focus our attention on the dynamics of criminal activity was recently presented in [13]. The author derived a dynamics of criminal activity and formation of crime hotspots. In the present paper, dynamics of criminal activity is formed leaving the risk aversion term and adding the effect of repeated victimization [17].

II. Formation of Functional relation

There is a functional relation between the number of criminal activity and place (spatial) and time (temporal). If we denote the number of criminal activity by A and place by x and time by t, then A is a function of x and t. This relation can be written as $A = A(x,t)$.

Further, Let, T be the tendency(or propensity in [13]) of the Criminal to commit crime at the place x at time t. This relation can be written as a spatio-temporal field : $T = T(x,t)$

With the hypothesis, that every potential criminal have a monotonic increasing tendency to act crime [13]. It is remarkable that according to the routine activity theory of everyday life, it is known that the criminal activity is possible only for the potential criminal, in presence of suitable targets and in the weak strength of security [14],[15].

Now we consider a 'city' which is defined as an open set S in $\mathbb{R}$ or $\mathbb{R}^2$. We consider that the number of criminal activity $A(x,t)$ at place x and at time *t* is represented by the function of tendency of criminal activity $T(x,t)$. This relation can be expressed as

$$A(x,t) = F\left(T(x,t)\right) \Rightarrow A(x,t) = F(T) \qquad (1)$$

where $T = T(x,t)$ and F is the acting out function [3], which can be expressed as

$$F(T) = \begin{cases} 0, & T \le 0 \\ 1 - e^{-\gamma T}, & T > 0 \end{cases} \qquad (2)$$

Here $\gamma > 0$, is given constant and $F'(0) = \gamma$, which measures the strength of a positive tendency $T(x,t)$. In particular, γ can be taken as a parameter that varies in spatial location. The spatio-temporal field $T(x,t)$ represents such a condition that, when $T > 0$, criminal activity is successful and if $T < 0$, the criminal have to face high risk to commit the criminal activity, at x on time t. For the simplicity, we can consider $T \to \infty$, so that $(1 - e^{-\gamma T}) \to 1$, so that acting out function $F(T)$ can be considered as a step function-

$$F(T) = \begin{cases} 0, & T \le 0 \\ 1, & T > 0 \end{cases} \qquad (3)$$

Thus the acting out function has the binary nature of decision to act or not to act the criminal activity. In this case strength, γ of Tendency $T(x,t)$ of the criminal becomes very strong.

III. Properties of the acting out function F

(*i*) $F(T(x,t))=0$ for $T(x,t)\le 0$ and $F(T(x,t))>0$, otherwise (2a)

(*ii*) F is a increasing function of $T>0$ with $F\to 1$ as $T\to\infty$ (2b)

IV. Berestycki. and Nadal's Model

The dynamics of criminal activity can be written [13] as

$$\frac{\partial T}{\partial t}=-T(x,t)+P(x,t)-C(x,t)A_m(x,t) \tag{3a}$$

and

$$\mu_A\frac{\partial A_m}{\partial t}=A(x,t)-A_m(x,t) \tag{3b}$$

or

$$\mu_A\frac{\partial A_m}{\partial t}=F(T(x,t))-A_m(x,t) \tag{3c}$$

where A_m = moving average of past criminal activity or crime rate, $C(x,t)$ = cost of committing a crime or cost for punishment by guard, $P(x,t)$ = Local propensity to act of a crime, $C(x,t)A_m(x,t)$ = risk associated with criminal activity, μ_A = relative time scale for the criminal activity $A_m(x,t)$, and μ_A is greater than 1 (one).

We will make the simplifying assumption that the environment we are considering is uniform in the propensity to act as the local expected payoff (*i.e.* $(P(x,t)\approx P(x))$, then the equation (3a) expresses the relation of $T(x,t)$ towards $P(x)$, which is the average value of $P(x,t)$ over the spatial domain.

V. Time independent of P and C in spatial location

Let us assume that $P(x,t)$ and $C(x,t)$ are independent of time.

So that $P(x,t)=P(x)$ and $C(x,t)=C(x)$

Then the system (3) can be written in spatial location as

$$\frac{\partial T(x,t)}{\partial t}=-T(x,t)+P(x)-C(x)A_m(x,t) \tag{4a}$$

and

$$\mu_A\frac{\partial A_m(x,t)}{\partial t}=F(T(x,t))-A_m(x,t) \tag{4b}$$

If we dropped the spatial term x, then the two coupled equations (4a) and (4b) for the scalars $C > 0$ and P can be written as

$$\frac{\partial T(t)}{\partial t} = -T(t) + P - CA_m(t) \tag{5a}$$

and

$$\mu_A \frac{\partial A_m(t)}{\partial t} = F(T(t)) - A_m(t) \tag{5b}$$

VI. **Equilibrium points of the dynamics of criminal activities**

Let $\left(T^0, A_m{}^0\right)$ be the equilibrium point of the above system

The equilibrium points are obtained from (5a) and (5b) equating $\frac{\partial T(t)}{\partial t}$ and $\mu_A \frac{\partial A_m(t)}{\partial t}$ to 0 (zero), when $T(t) \to T^0$, and $A_m(t) \to A_m{}^0$

Thus, $0 = \frac{\partial T(t)}{\partial t} \Rightarrow 0 = -T^0 + P - CA_m^0$ and $0 = \mu_A \frac{\partial A_m}{\partial t} \Rightarrow 0 = F(T^0) - A_m^0$

$\therefore$ $\qquad T^0 = P - CA_m^0$ and $A_m^0 = F(T^0)$

$$\Rightarrow \qquad T^0 = P - CF(T^0) \tag{6}$$

and

$$A_m^0 = F(T^0) \tag{7}$$

From the property (2a) and (2b) of acting out function $F\left(T(x,t)\right)$, we have, $F(T) = 0$ for $T \le 0$, and $F(T) > 0$, for $T > 0$. F is a increasing function on $\left[0, +\infty\right]$ when $T > 0$ with $F \to 1$ as $T \to \infty$, satisfying $F(0^+) = 0$, when γ is defined by $\gamma = F'(0^+)$ and $F(T) = 1 - e^{-\gamma T}$ using these properties of F, we have seen that there exists a unique solution of the system (5a) and (5b) for all values of $\left\{P, C\right\}$.

When, $P < 0$, $(T < 0)$, we then, have $F(T^0) = 0$

So, (6) $\Rightarrow$ $T^0 = P$ (8)

(7) $\Rightarrow$ $A_m^0 = 0$ (9)

Hence the equilibrium point (T^0, A_m^0) is given by $(P,0)$

For $P>0$, $T^0>0$ is the unique intersection of the graph $\varphi(T)=T$ and $\varphi(T)=P-CF(T)$ for $T\geq 0$ as in Fig.2, this guarantee (T) become high for greater positive (P) and consequently the rate of the criminal activity A_m become very large number.

VII. Analysis of Stability

The stability is determined by linearization of equation (5a) and (5b) near the equilibrium point $(T^0, A_m^0)=(P,0)$ as in (8) and (9).

Now the systems (5a) and (5b) can be rewritten

as $$\frac{\partial}{\partial t}(T(t)) = -T(t)+P-CA_m(t) = f(T,A_m), \quad \text{say}$$

and $$\frac{\partial}{\partial t}\left(A_m(t)\right) = \frac{1}{\mu_A}F\left(T(t)\right)-\frac{1}{\mu_A}A_m(t) = g\left(T,A_m\right), \text{ say}$$

Jacobian matrix of the right side of the system at the equilibrium point $(T^\circ, A_m^\circ)=(P,0)$ is

$$J_{(T^0,A_m^0)} = \begin{bmatrix} \frac{\partial f}{\partial T} & \frac{\partial f}{\partial A_m} \\ \frac{\partial g}{\partial T} & \frac{\partial g}{\partial A_m} \end{bmatrix}_{(T^0,A_m^0)} = \begin{bmatrix} -1 & -C \\ \frac{1}{\mu_A}F'(T) & -\frac{1}{\mu_A} \end{bmatrix}_{(P,0)}$$

$$= \begin{bmatrix} -1 & -C \\ \frac{1}{\mu_A}F'(P) & -\frac{1}{\mu_A} \end{bmatrix} = \begin{bmatrix} -1 & -C \\ 0 & -\frac{1}{\mu_A} \end{bmatrix} \qquad (10)$$

Since near the equilibrium point, we have, when $P<0$, $F'(P)=F'(T^0)=0$

The Jacobian matrix (10) is a triangular matrix. Therefore, the eigen values of the matrix (10) are

$$\lambda_1 = -1 \text{ and } \lambda_2 = -\frac{1}{\mu_A} \qquad (11)$$

It is clear that two eigenvalues of the Jacobian matrix J are negative. Hence the whole system (5) is stable.

Again Jacobian matrix, near equilibrium poin (T^0, A_m^0) is $J_{(T^0,A_m^0)} = \begin{bmatrix} -1 & -C \\ \frac{1}{\mu_A}F'(T) & -\frac{1}{\mu_A} \end{bmatrix}_{(T^0,A_m^0)}$

When, $P>0$ then $J_{(T^0,A_m^0)} = \begin{bmatrix} -1 & -C \\ \frac{1}{\mu_A}F'(T^0) & -\frac{1}{\mu_A} \end{bmatrix}$ as $P>0,\ \gamma^0 = F'(T^0)$ [as $\gamma = F'(0^+)$

$\therefore\ J_{(T^0,A_m^0)} = \begin{bmatrix} -1 & -C \\ \frac{\gamma^0}{\mu_A} & -\frac{1}{\mu_A} \end{bmatrix}$ is 2×2 matrix, if λ be the eigenvalues then we have

The characteristics equation is given by $\det(J-\lambda I)=0$

$$\Rightarrow \quad \det\begin{bmatrix} -1-\lambda & -C \\ \frac{\gamma^0}{\mu_A} & -\frac{1}{\mu_A}-\lambda \end{bmatrix} = 0 \Rightarrow (-1-\lambda)(-\frac{1}{\mu_A}-\lambda)+\frac{\gamma^0 C}{\mu_A}=0$$

$$\Rightarrow \mu_A\lambda^2+(\mu_A+1)\lambda+(1+\gamma^0 C)=0 \Rightarrow \lambda=[-(\mu_A+1)\pm\left\{(\mu_A-1)^2-4\mu_A\gamma^0 C\right\}^{\frac{1}{2}}]/2\mu_A$$

For the real roots the discriminant $(\mu_A-1)^2-4\mu_A\gamma^0 C\ge 0$

$$(\mu_A-1)^2 \ge 4\mu_A\gamma^0 C \ \Rightarrow\ \frac{(\mu_A-1)^2}{4\mu_A}\ge\gamma^0 C \qquad (12)$$

Then we have found two real eigenvalues, as-

$$\lambda_{1,2} = \frac{-(\mu_A+1)\pm\left\{(\mu_A-1)^2-4\mu_A\gamma^0 C)\right\}^{\frac{1}{2}}}{2\mu_A} \qquad (13)$$

These two eigenvalues of the Jacobian matrix can be written as

or $$\lambda_1 = -\frac{(\mu_A+1)}{2\mu_A}+\frac{1}{2\mu_A}\left\{(\mu_A-1)^2-4\mu_A\gamma^0 C)\right\}^{\frac{1}{2}}$$

and $$\lambda_2 = -\frac{(\mu_A+1)}{2\mu_A}-\frac{1}{2\mu_A}\left\{(\mu_A-1)^2-4\mu_A\gamma^0 C)\right\}^{\frac{1}{2}}$$

From (11) and (13), it is clear that the real part of the eigenvalues of the Jacobian matrix at the equilibrium point for the systems (5a) and (5b) are always negative. So that the equilibrium point is always stable.

Again in (13) if $(\mu_A-1)^2-4\mu_A\gamma^0 C)<0 \Rightarrow (\mu_A-1)^2/4\mu_A<\gamma^0 C$ (14)

then, we have two complex eigenvalues from (13), say $\lambda_{3,4}$

$$\therefore \quad \lambda_{3,4} = -\frac{(\mu_A+1)}{2\mu_A}\pm\frac{1}{2\mu_A}\left\{(\mu_A-1)^2-4\mu_A\gamma^0 C\right\}^{\frac{1}{2}} \tag{15}$$

or $$\lambda_3 = -\frac{(\mu_A+1)}{2\mu_A}+\frac{1}{2\mu_A}\left\{(\mu_A-1)^2-4\mu_A\gamma^0 C\right\}^{\frac{1}{2}}$$

and $$\lambda_4 = -\frac{(\mu_A+1)}{2\mu_A}-\frac{1}{2\mu_A}\left\{(\mu_A-1)^2-4\mu_A\gamma^0 C\right\}^{\frac{1}{2}}$$

In (15), since $\frac{(\mu_A-1)^2}{4\mu_A}<\gamma^0 C$

Here the limit of γ^0 is taken into account as large enough. This means that for $P<C$, if the function $F(T)$ has a steep slope at $T>0$, as shown in Figure (2.5), and hence γ^0 is considered large enough. For this reason we have got complex Eigen values and convergence towards the equilibrium point occurs with damped oscillations.

VIII. The Modified Model of Dynamics of Criminal activity

In the previous model, Berestycki and Nadal proposed for time evolution of T, added a quantity local cost $C(x,t)$ of criminal activity. They subtracted the product term $C(x,t)A(x,t)$ (which represents the risk associated with criminal activity for a criminal due to deterrent factors or other environment) from the system.

In this present Model this risk term is neglected for a potential criminal in the location where the deterrent factors are very weak. Moreover, the repeated victimization effect $\rho(x,t)A_m(x,t)$ as in [13] is added, where ρ is the density of criminals.

In the absence of risk aversion and in the presence of repeated victimization effect [17], the following system is proposed as

$$\frac{\partial T}{\partial t} = -T(x,t) + P(x,t) + \rho(x,t)A_m(x,t) \tag{16}$$

$$\frac{\partial A_m}{\partial t} = \frac{1}{\mu_A}[F(T(x,t)) - A_m(x,t)] \tag{17}$$

We restrict the analysis to the simplest case, i.e. with time independent values of P and ρ and without risk of criminal act, the system can be rewritten (dropping spatial term) as

$$\frac{\partial T(t)}{\partial t} = -T(t) + P + \rho A_m(t) \tag{18a}$$

$$\frac{\partial A_m(t)}{\partial t} = \frac{1}{\mu_A}[F(T(t)) - A_m(t)] \tag{18b}$$

The equilibrium points of the above system are obtained as $T^0 = P, A_m^0 = 0$ and the jacobian is given by

$$J = J_{(P,0)} = \begin{bmatrix} -1 & \rho \\ \frac{1}{\mu_A}F'(T) & -\frac{1}{\mu_A} \end{bmatrix}_{(P,0)} = \begin{bmatrix} -1 & \rho \\ 0 & -\frac{1}{\mu_A} \end{bmatrix}$$

Hence the eigen values are -1 and $-\frac{1}{\mu_A}$ and hence the system (18) is stable. Obviously the dynamics are the same as for the simplest case discussed in [13].

IX. Analysis of Hotspots of Criminal Activity

We recall the acting out functions

$$F(T) = \begin{cases} 0\,, & T \le 0 \\ 1 - e^{-\gamma T}, & T > 0 \end{cases}$$

when $\gamma \to \infty$, F converges to the step function

$$F(T) = \begin{cases} 0\,, & T \le 0 \\ 1\,, & T > 0 \end{cases}$$

For the equilibrium point of (6) and (7) we estimate as below:

For $\gamma \to \infty$, when $P > C,\ T^0 : \ P - C$

and $A_m^0 : \; 1$ $\left[\begin{array}{l} \text{Q}\, T^0 = P - CA_m^0 \\ A_m^0 = F(T^0) : \; 1 \end{array}\right.$

i.e. hot spot, when $P > C$

Thus as $\gamma \to \infty$, we have $T^0 = P - C$ and $A_m^0 = 1$, for $P > C$. In this case there is a criminal

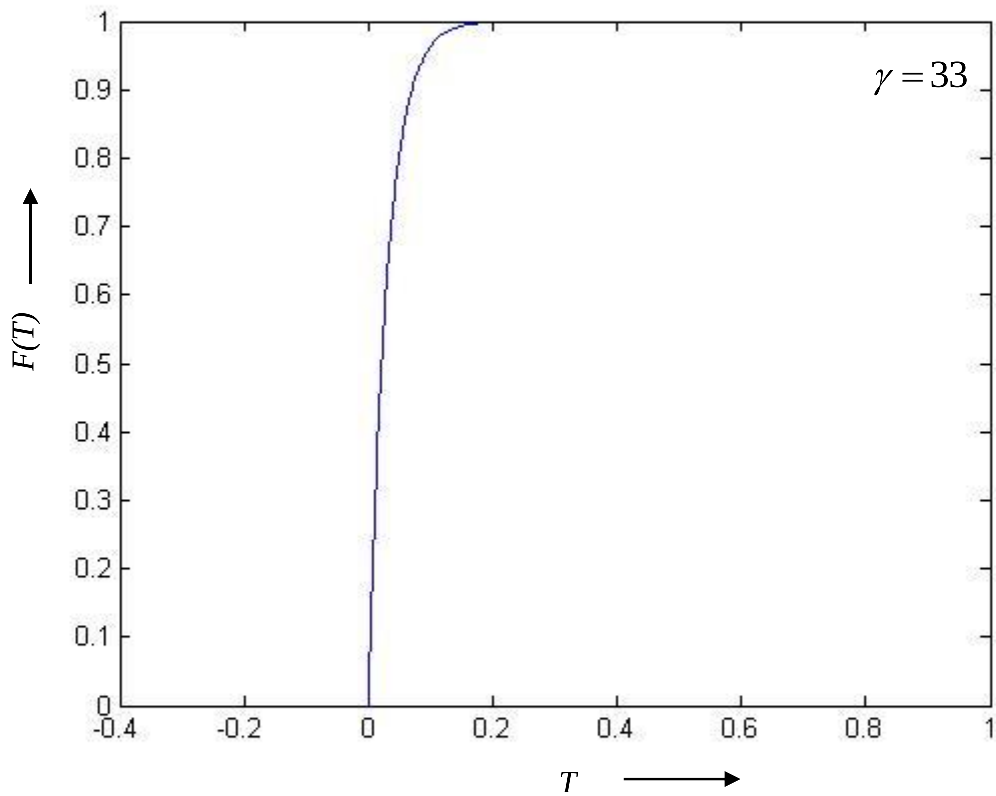

Fig. : 1. Graph of acting out function

The Fig.1 represents the graph of acting out function defined by

$$F(T) = \begin{cases} 0\,,\; T \le 0 \\ 1 - e^{-\gamma T},\; T > 0 \end{cases} \text{ Where } \gamma = 33$$

The Fig. 2 represents the graph of the dynamics of criminal Activity of the system

$$\frac{\partial T(t)}{\partial t} = -T(t) + P - CA_m(t)$$

$$\mu_A \frac{\partial A_m(t)}{\partial t} = F(T(t)) - A_m(t) \text{ With } \gamma = 33$$

Equilibrium point for a cost $C = 1.7$ and two particular values of P, say $P_1 = 0.9(P < C)$ and $P_2 = 3(P > C)$ are shown. Equilibrium point found as the point of intersection of $\phi(T) = P - CF(T)$ and $\phi(T) = T$. When $P < C$, the equilibrium point is at small value of T.

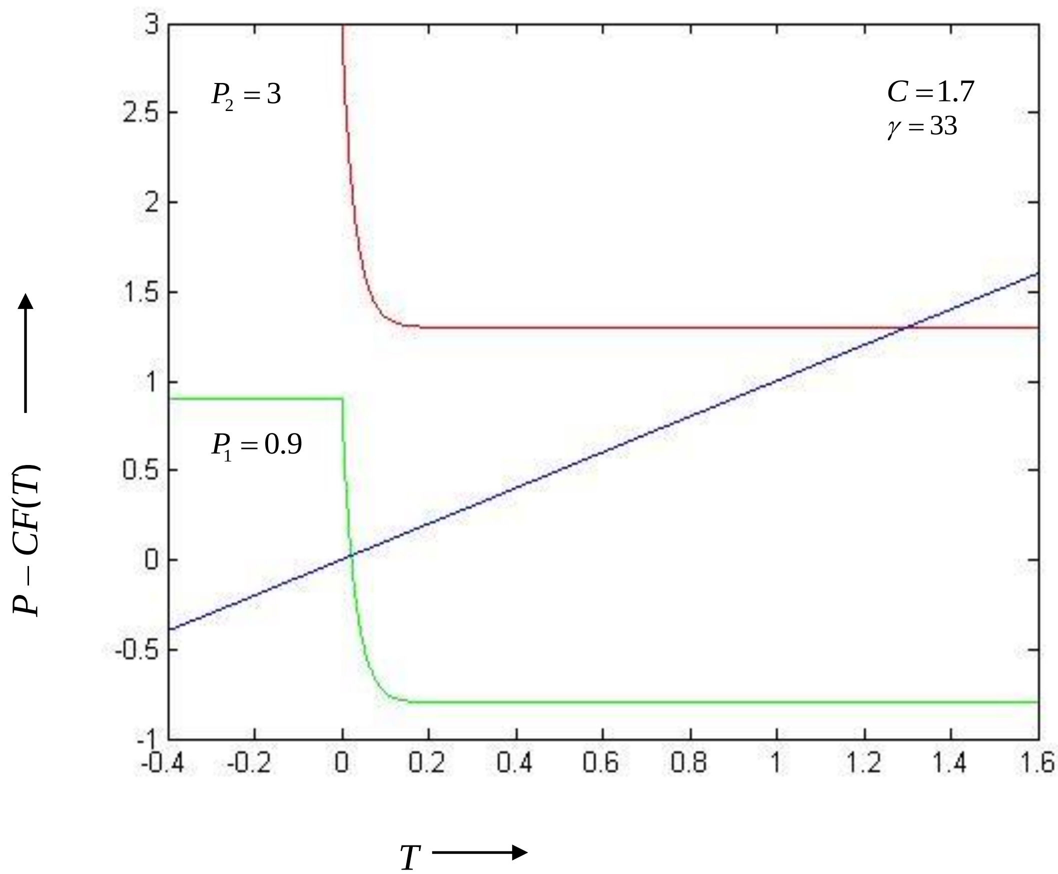

Fig. : 2 Graph of the Dynamics criminal activity

X. Conclusions

Acting out function of tendency to commit crime determines the criminal activity. The spatial locations are characterized by the local Propensity to act. When local propensity to act is greater than cost for punishment then tendency of criminal activity is given by the gap between cost and local propensity to act, and then the criminal activity is maximal, $A_m^0 : 1$. Then in this case there is a criminal hotspot. The dynamics of criminal activity is not affected by the removal of risk associated with criminal act.

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