A Common Fixed Point Theorem in Intuitionistic Fuzzy Metric Space

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Abstract:

The aim of the present paper is to establish a common fixed point theorem for faintly compatible pair of self maps in intuitionistic fuzzy metric space. Our results improve the results of [4, 6, 18].

Keywords:

Intuitionistic fuzzy metric space, Property (E.A.), sub sequentially continuity, faintly compatible maps.

1. INTRODUCTION

The concept of fuzzy sets was introduced initially by Zadeh [19] in 1965. After this, fuzzy set theory was further developed and a series of research were done by several mathematicians. In the sequel the concept of fuzzy metric space was introduced by Kramosil and Michalek [7] in 1975. Moreover, A. George and P. Veeramani [5] modified the notion of fuzzy metric spaces of [7] with the help of t-norm in 1994.

As a generalization of fuzzy set, Atanassov [2] introduced the idea of intuitionistic fuzzy set. Park [10] introduced and discussed a notion of intuitionistic fuzzy metric spaces, which is based both on the idea of intuitionistic fuzzy sets and the concept of a fuzzy metric space given by George and Veeramani [5]. After this several research has been done in the theory of fuzzy metric space and intuitionistic fuzzy metric spaces.

On the other hand, Aamri and El. Moutawakil [1] generalized the concepts of non-compatibility by defining the notion of (E.A) property and proved common fixed point theorems under strict contractive condition in metric space. Pant and Pant [12] introduced the concept of property (E.A.) in fuzzy metric space. Pant and Bisht [11], introduced the concept of conditional compatible maps. Faintly compatible maps introduced by Bisht and Shahzad [3] as an improvement of conditionally compatible maps. Some recent result related to faintly compatible maps can be seen in [15, 16]. Pant [9] introduced the notion of reciprocal continuity of mappings in metric spaces. Recently, Wadhwa et.al.[14], introduced the notion of sub sequentially continuous mappings in fuzzy metric space which is weaker than reciprocal continuous mappings. Recently, [4, 18] proved some fixed point theorems for occasionally weakly compatible mappings in fuzzy metric space. Gupta and Gupta [6] proved their result in intuitionistic fuzzy metric space by using occasionally weakly compatible maps in rational form.

We prove some common fixed point theorems using for faintly compatible pair and our results improve the result of [4, 6, 18].

II. PRELIMINARIES

In this section, we recall some definitions and useful results which are already in the literature.

Definition 2.1 [13]: A binary operation $*$: $[0, 1] \times [0, 1] \to [0, 1]$ is continuous t- norm if $*$ satisfies the following conditions:

i. is commutative and associative;
ii. is continuous;
iii. $a * 1 = a \ \forall a \in [0. 1]$;
iv. $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d \ \forall a, b, c, d \in [0,1]$.

Example of continuous t-norm: $a * b = \min\{a, b\}$, minimum t-norm.

George and Veeramani [5] modified the notion of fuzzy metric space of Kramosil and Michalek [7] as follows:

Definition 2.2 [5]: The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2 \times (0,\infty)$ satisfying the following conditions: $\forall x, y, z \in X, t, s > 0$;

(GV - 1) $M(x, y, t) > 0$;
(GV - 2) $M(x, y, t) = 1$ iff $x = y$;
(GV - 3) $M(x, y, t) = M(y, x, t)$;
(GV - 4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
(GV - 5) $M(x, y, \cdot): [0,\infty) \to [0, 1]$ is continuous.

Definition 2.3 [6]: A 5-tuple $(X,M,N, *, \Diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\Diamond$ is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times [0,1)$ satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;
(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;
(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(vi) for all $x, y \in X$, $M(x, y, .) : [0,1) \to [0, 1]$ is left continuous;
(vii) $\lim_{t\to\infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;
(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;
(ix) $N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;
(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;
(xi) $N(x, y, t) \Diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y, z \in X$ and $s, t > 0$;
(xii) for all $x, y \in X$, $N(x, y, .) : [0,1) \to [0, 1]$ is right continuous;
(xiii) $\lim_{t\to\infty} N(x, y, t) = 0$ for all $x, y \in X$.

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of nonnearness between x and y with respect to t, respectively.

Remark 2.4: Every fuzzy metric space $(X,M, *)$ is an intuitionistic fuzzy metric space of the form $(X,M, 1 - M,*, \Diamond)$ such that t-norm $*$ and t-conorm $\Diamond$ are associated as

$$x \Diamond y = 1 - ((1 - x) * (1 - y)) \text{ for all } x, y \in X.$$

Remark 2.5: In intuitionistic fuzzy metric space X, $M(x, y, .)$ is non-decreasing and $N(x, y, .)$ is non-increasing for all $x, y \in X$.

Definition 2.6 [17]: Let $(X, M, N,*,\Diamond)$ be a IFMS. A pair of self-maps (A, S) on X is said to be conditionally compatible iff whenever the set of sequences x_n satisfying

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n$$

is non-empty, there exists a sequence z_n in X such that

$$\lim_{n\to\infty} Az_n = \lim_{n\to\infty} Sz_n = t, \text{ for some } t \in X,$$

and

$$\lim_{n\to\infty} M(ASz_n, SAz_n, t) = 1 \text{ and } \lim_{n\to\infty} N(ASz_n, SAz_n, t) = 0, \forall\, t > 0.$$

Definition 2.7 [17]: Let $(X, M, N, *, \Diamond)$ be a IFMS. A pair of self-maps (A, S) on X is said to be faintly compatible iff (A, S) is conditionally compatible and they commute on a non-empty subset of the set of coincidence points, whenever the set of coincidence points is nonempty.

Definition 2.8 [12]: Let A and S be two self mappings of a FMS $(X, M, *)$. We say that A and S satisfy the property (E.A.) if there exists a sequence x_n such that

$$lim_{n\to\infty} Ax_n = lim_{n\to\infty} Sx_n = t \text{, for some } t \in X.$$

Definition 2.9 [17]: Let $(X, M, N, *, \Diamond)$ be a IFMS. Self maps A and S on X are said to be subsequentially continuous iff there exists a sequence x_n in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x,\ x \in X$$

and satisfy

$$\lim_{n\to\infty} ASx_n = Ax,\quad \lim_{n\to\infty} SAx_n = Sx.$$

Lemma 2.10 [17]: Let $(X, M, N, *, \Diamond)$ be an IFMS if there exists $k \in (0,1)$ such that

$$M(x, y, kt) \geq M(x, y, t),$$

$$N(x, y, kt) \leq N(x, y, t),$$

for all $x, y \in X$ then $x = y$.

III. MAIN RESULTS

Theorem 3.1: Let $(X, M, N, *, \Diamond)$ be a fuzzy metric space. Let A, B, S and T be mappings from X into itself such that

(3.1.1) for all x, y∈X, t > 0 and for some k∈(0, 1),

$$M(Ax, By, kt) \leq \min\left\{\begin{matrix} M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t), \\ \frac{aM(Ax,Ty,t)+bM(By,Sx,t)+cM(Sx,Ty,t)}{a+b+c} . \frac{1+M(Ax,Sx,t)}{2} \end{matrix}\right\};$$

and

$$N(Ax, By, kt) \geq \max\left\{\begin{matrix} N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Sx, By, t), N(Ty, Ax, t), \\ \frac{aN(Ax,Ty,t)+bN(By,Sx,t)+cN(Sx,Ty,t)}{a+b+c} . \frac{1+N(Ax,Sx,t)}{2} \end{matrix}\right\};$$

where a,b,c≥0 with a, b and c cannot be simultaneously 0;

(3.1.2) If pair (A, S) and (B, T) satisfies E.A. property with sub sequentially continuous, faintly compatible maps, then A, B, S and T have a common fixed point in X.

Proof: E.A. Property of (A, S) and (B, T) implies that there exist sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = t_1, \text{ for some } t_1 \in X,$$

and

$$\lim_{n\to\infty} Bx_n = \lim_{n\to\infty} Tx_n = t_2, \text{ for some } t_2 \in X,$$

Since pairs (A, S) and (B, T) are faintly compatible therefore conditionally compatibility of (A, S) and (B, T) implies that there exist sequences $\{z_n\}$ and $\{z_n'\}$ in X satisfying

$$\lim_{n\to\infty} Az_n = \lim_{n\to\infty} Sz_n = u \text{ for some } u\in X,$$

such that

$$\lim_{n\to\infty} M(ASz_n, SAz_n, t)=1 \text{ and } \lim_{n\to\infty} N(ASz_n, SAz_n, t)=0;$$

and

$$\lim_{n\to\infty} Bz_n' = \lim_{n\to\infty} Tz_n' = v \text{ for some } v\in X,$$

such that

$$\lim_{n\to\infty} M(BTz_n', TBz_n', t)=1 \text{and } \lim_{n\to\infty} N(BTz_n', TBz_n', t)=0.$$

As the pairs (A, S) and (B, T) are sub sequentially continuous, we get

$$\lim_{n\to\infty} ASz_n = Au,\ \lim_{n\to\infty} SAz_n = Su$$

so Au = Su; and

$$\lim_{n\to\infty} BTz_n' = Bv,\ \lim_{n\to\infty} TBz_n' = Tv$$

so Bv = Tv.

Since pairs (A, S) and (B, T) are faintly compatible, we get

$$ASu = SAu \text{ then } AAu=ASu=SAu=SSu;$$

and

$$BTv=TBv \text{ then } BBv=BTv=TBv=TTv.$$

Now we show that Au=Bv.

Let Au≠Bv, condition (3.1.1) with x=u and y=v implies that

$$M(Au, Bv, kt) \le \min\left\{\begin{matrix} M(Su, Tv, t), M(Su, Au, t), M(Tv, Bv, t), M(Su, Bv, t), M(Tv, Au, t), \\ \frac{aM(Au,Tv,t)+bM(Bv,Su,t)+cM(Su,Tv,t)}{a+b+c} \cdot \frac{1+M(Au,Su,t)}{2} \end{matrix}\right\};$$

$$M(Au, Bv, kt) \le \min\left\{\begin{matrix} M(Au, Bv, t), M(Au, Au, t), M(Bv, Bv, t), M(Au, Bv, t), M(Bv, Au, t), \\ \frac{aM(Au,Bv,t)+bM(Bv,Au,t)+cM(Au,Bv,t)}{a+b+c} \cdot \frac{1+M(Au,Au,t)}{2} \end{matrix}\right\};$$

$M(Au, Bv, kt) \le \min\{M(Au, Bv, t), 1, 1, M(Au, Bv, t), M(Bv, Au, t), M(Au, Bv, t)\};$

$M(Au, Bv, kt) \le M(Au, Bv, t),$ (1)

and

$$N(Au, Bv, kt) \ge \max\left\{\begin{matrix} N(Su, Tv, t), N(Su, Au, t), N(Tv, Bv, t), N(Su, Bv, t), N(Tv, Au, t), \\ \frac{aN(Au,Tv,t)+bN(Bv,Su,t)+cN(Su,Tv,t)}{a+b+c} \cdot \frac{1+N(Au,Su,t)}{2} \end{matrix}\right\};$$

$$N(Au, Bv, kt) \ge \max\left\{\begin{matrix} N(Au, Bv, t), N(Au, Au, t), N(Bv, Bv, t), N(Au, Bv, t), N(Bv, Au, t), \\ \frac{aN(Au,Bv,t)+bN(Bv,Au,t)+cN(Au,Bv,t)}{a+b+c} \cdot \frac{1+N(Au,Au,t)}{2} \end{matrix}\right\};$$

$N(Au, Bv, kt) \ge \max\{N(Au, Bv, t), 0, 0, N(Au, Bv, t), N(Bv, Au, t), N(Au, Bv, t)\};$

$N(Au, Bv, kt) \ge N(Au, Bv, t),$ (2)

Condition (1) and (2) implies a contradiction of Lemma 2.10. Therefore we have Au=Bv.

Now we show that AAu=Au.

Let AAu≠Au, condition (3.1.1) with x=Au and y=v implies that

$$M(AAu, Bv, kt) \le \min\left\{\begin{matrix} M(SAu, Tv, t), M(SAu, AAu, t), \\ M(Tv, Bv, t), M(SAu, Bv, t), M(Tv, AAu, t), \\ \frac{aM(AAu,Tv,t)+bM(Bv,SAu,t)+cM(SAu,Tv,t)}{a+b+c} \cdot \frac{1+M(AAu,SAu,t)}{2} \end{matrix}\right\};$$

$$M(AAu, Au, kt) \le \min\left\{\begin{matrix} M(AAu, Au, t), M(AAu, AAu, t), \\ M(Au, Au, t), M(AAu, Au, t), M(Au, AAu, t), \\ \frac{aM(AAu,Au,t)+bM(Au,AAu,t)+cM(AAu,Au,t)}{a+b+c} \cdot \frac{1+M(AAu,AAu,t)}{2} \end{matrix}\right\};$$

$M(AAu, Au, kt) \le \min\{M(AAu, Au, t), 1, 1, M(AAu, Au, t), M(Au, AAu, t), M(AAu, Au, t)\};$

$$M(AAu, Au, kt) \le M(AAu, Au, t); \qquad (3)$$

and

$$N(AAu, Bv, kt) \ge \max\left\{\begin{matrix} N(SAu, Tv, t), N(SAu, AAu, t), \\ N(Tv, Bv, t), N(SAu, Bv, t), N(Tv, AAu, t), \\ \frac{aN(AAu,Tv,t)+bN(Bv,SAu,t)+cN(SAu,Tv,t)}{a+b+c} \cdot \frac{1+N(AAu,SAu,t)}{2} \end{matrix}\right\};$$

$$N(AAu, Au, kt) \ge \max\left\{\begin{matrix} N(AAu, Au, t), N(AAu, AAu, t), \\ N(Au, Au, t), N(AAu, Au, t), N(Au, AAu, t), \\ \frac{aN(AAu,Au,t)+bN(Au,AAu,t)+cN(AAu,Au,t)}{a+b+c} \cdot \frac{1+N(AAu,AAu,t)}{2} \end{matrix}\right\};$$

$$N(AAu, Au, kt) \ge \max\{N(AAu, Au, t), 0, 0, N(AAu, Au, t), N(Au, AAu, t), N(AAu, Au, t)\};$$

$$N(AAu, Au, kt) \ge N(AAu, Au, t); \qquad (4)$$

Condition (3) and (4) implies a contradiction of Lemma 2.10. Therefore, we have AAu=Au.

Let Bv≠BBv, now taking x=u and y=Bv in (3.1.1) we have

$$M(Au, BBv, kt) \le \min\left\{\begin{matrix} M(Su, TBv, t), M(Su, Au, t), \\ M(TBv, BBv, t), M(Su, BBv, t), M(TBv, Au, t), \\ \frac{aM(Au,TBv,t)+bM(BBv,Su,t)+cM(Su,TBv,t)}{a+b+c} \cdot \frac{1+M(Au,Su,t)}{2} \end{matrix}\right\};$$

$$M(Bv, BBv, kt) \le \min\left\{\begin{matrix} M(Bv, BBv, t), M(Bv, Bv, t), \\ M(BBv, BBv, t), M(Bv, BBv, t), M(BBv, Bv, t), \\ \frac{aM(Bv,BBv,t)+bM(BBv,Bv,t)+cM(Bv,BBv,t)}{a+b+c} \cdot \frac{1+M(Bv,Bv,t)}{2} \end{matrix}\right\};$$

$$M(Bv, BBv, kt) \le \min\left\{\begin{matrix} M(Bv, BBv, t), 1, 1, M(Bv, BBv, t), M(BBv, Bv, t), \\ M(Bv, BBv, t) \end{matrix}\right\};$$

$$M(Bv, BBv, kt) \le M(Bv, BBv, t), \qquad (5)$$

and

$$N(Au, BBv, kt) \ge \max\left\{\begin{matrix} N(Su, TBv, t), N(Su, Au, t), \\ N(TBv, BBv, t), N(Su, BBv, t), N(TBv, Au, t), \\ \frac{aN(Au,TBv,t)+bN(BBv,Su,t)+cN(Su,TBv,t)}{a+b+c} \cdot \frac{1+N(Au,Su,t)}{2} \end{matrix}\right\};$$

$$N(Bv, BBv, kt) \ge \max\left\{\begin{matrix} N(Bv, BBv, t), N(Bv, Bv, t), \\ N(BBv, BBv, t), N(Bv, BBv, t), N(BBv, Bv, t), \\ \frac{aM(Bv,BBv,t)+bN(BBv,Bv,t)+cN(Bv,BBv,t)}{a+b+c} \cdot \frac{1+N(Bv,Bv,t)}{2} \end{matrix}\right\};$$

$$N(Bv, BBv, kt) \ge \max\left\{\begin{matrix} N(Bv, BBv, t), 0, 0, N(Bv, BBv, t), N(BBv, Bv, t), \\ N(Bv, BBv, t) \end{matrix}\right\};$$

$$N(Bv, BBv, kt) \ge N(Bv, BBv, t), \qquad (6)$$

Condition (5) and (6) implies a contradiction of Lemma 2.10. Therefore we have Bv=Au=BBv.

If we say Au=p then we have

Ap=Sp=p= BBv=Bp

and

p= BBv=TBv=Tp, since Bv=Au=p.

Hence

Ap =Sp =Bp =Tp = p.

Therefore p is a common fixed point of A, B, S and T.

To prove uniqueness: Suppose that q (≠ p) be another fixed point of A, B, S and T, i.e.

Aq =Sq =Bq =Tq = q.

Let Au≠Bv, condition (3.1.1) with x=p and y=q implies that

$$M(Ap, Bq, kt) \leq \min\left\{\begin{matrix} M(Sp, Tq, t), M(Sp, Ap, t), M(Tq, Bq, t), M(Sp, Bq, t), M(Tq, Ap, t), \\ \frac{aM(Ap, Tq, t)+bM(Bq, Sp, t)+cM(Sp, Tq, t)}{a+b+c} \cdot \frac{1+M(Ap, Sp, t)}{2} \end{matrix}\right\};$$

$$M(p, q, kt) \leq \min\left\{\begin{matrix} M(p, q, t), M(p, p, t), M(q, q, t), M(p, q, t), M(q, p, t), \\ \frac{aM(p, q, t)+bM(q, p, t)+cM(p, q, t)}{a+b+c} \cdot \frac{1+M(p, p, t)}{2} \end{matrix}\right\};$$

$M(p, q, kt) \leq \min\{M(p, q, t), 1, 1, M(p, q, t), M(q, p, t), M(p, q, t)\};$

$M(p, q, kt) \leq M(p, q, t),$ (7)

and

$$N(Ap, Bq, kt) \leq \max\left\{\begin{matrix} N(Sp, Tq, t), N(Sp, Ap, t), N(Tq, Bq, t), N(Sp, Bq, t), N(Tq, Ap, t), \\ \frac{aN(Ap, Tq, t)+bN(Bq, Sp, t)+cN(Sp, Tq, t)}{a+b+c} \cdot \frac{1+N(Ap, Sp, t)}{2} \end{matrix}\right\};$$

$$N(p, q, kt) \leq \max\left\{\begin{matrix} N(p, q, t), N(p, p, t), N(q, q, t), N(p, q, t), N(q, p, t), \\ \frac{aN(p, q, t)+bN(q, p, t)+cN(p, q, t)}{a+b+c} \cdot \frac{1+N(p, p, t)}{2} \end{matrix}\right\};$$

$N(p, q, kt) \leq \max\{N(p, q, t), 0, 0, N(p, q, t), N(q, p, t), N(p, q, t)\};$

$N(p, q, kt) \leq N(p, q, t),$ (8)

Condition (7) and (8) implies a contradiction of Lemma 2.10. Therefore we have p=q.
This completes the proof of the theorem.

Remark 3.2: Theorem 3.1 improves the result of [4, 18] for faintly compatible mappings in intuitionistic fuzzy metric space.

Remark 3.3: Theorem 3.1 is still true, If we replace (3.1.1) by the following inequalities
(3.1.1)* for all $x, y \in X$, $t > 0$ and for some $k \in (0, 1)$,

$$M(Ax, By, kt) \leq \min\left\{\begin{matrix} M(Sx, Ty, t), M(Sx, Ax, t), M(Ty, By, t), M(Sx, By, t), M(Ty, Ax, t), \\ \frac{aM(Ax, Ty, t)+bM(By, Sx, t)+cM(Sx, Ty, t)}{a+b+c} \cdot \frac{M(By, Ty, t)+M(Ax, Sx, t)}{2} \end{matrix}\right\};$$

and

$$N(Ax, By, kt) \geq \max\left\{\begin{matrix} N(Sx, Ty, t), N(Sx, Ax, t), N(Ty, By, t), N(Sx, By, t), N(Ty, Ax, t), \\ \frac{aN(Ax, Ty, t)+bN(By, Sx, t)+cN(Sx, Ty, t)}{a+b+c} \cdot \frac{N(By, Ty, t)+N(Ax, Sx, t)}{2} \end{matrix}\right\};$$

where a,b,c≥0 with a, b and c cannot be simultaneously 0.

In this case our result is an improvement of Gupta and Gupta [6] for faintly compatible mappings.

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