Fuzzy Approach; Stair-case linear programming Problem with cost coefficient

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Abstract:- *In this paper, Modelling planning problems that extend over many time periods as linear programs leads to a special structure called a "staircase" or "dynamic" linear program. The simplex method may be adapted to better solve linear programs that have a staged or staircase structure . We focus on the solution procedure of The linear Stair-case programming Problem is a special type of vector minimum problem in which cost coefficients of the objectives function has been expressed as interval values by decision makers and constraints are all equality type or less than equal to type or greater than equal to type, where the per unit cost matrix contains the elements along the main diagonal and a band below it only, This problem has been transformed into a classical linear programming problem where to minimize the interval objective function, Finally the equivalent transformed problem has been solved by fuzzy programming technique.*

Keyword :- *Stair-case linear programming, fuzzy membership function*

I. INTRODUCTION

The linear Staircase – Structured is studied about as long as linear programming itself. Application of The linear Staircase programs in production scheduling, inventory, transportation, control, and design of multistage structures Dantzig observed [2,3,4] the general mathematical problem is concerned with maximization of a linear form of nonnegative variables subject to a system of linear equalities, in linear programming case one finds by observing the staircase system that grand matrix of coefficients is composed mostly of blocks of zeros except for sub matrices along and just of the diagonal.

The linear Stair-case Problem is a special type of vector minimum problem in which constraints are all equality type or less than equal to type or greater than equal to type, where the per unit cost matrix contains the elements along the main diagonal and a band below it only. All existing methods generate a set of non-dominated solutions or construct a single compromise solution.

Hoffman and Veinott[6] presented the stair-case transportation problem with super-additive rewards and cumulative capacities.

Das, Goswami and Alam [5] have proposed a method to solve the multiobjective transportation problem in which the co-efficients of the objective functions as well as the source and destination parameters are in the form of interval.

Verma, Biswal [7] used the fuzzy programming Approach To Multiobjective Stair-case Transportation Problem.

In this paper, we focus on the solution procedure of The linear Stair-case Problem is a special type of vector minimum problem in which cost coefficients of the objectives function has been expressed as interval values by decision makers and constraints are all equality type or less than equal to type or greater than equal to type, where the per unit cost matrix contains the elements along the main diagonal and a band below it only, This problem has been transformed into a classical linear programming where to minimize the interval objective function, Finally the equivalent transformed problem has been solved by fuzzy programming technique.

II. Mathematical model for Square Stair-Case linear programming Problem with cost coefficient

Stair case programs has square partitioned structure consisting of t x t sub matrices' all of whose elements are zero except possibly the minimize element of the submartrices on and just below the main diagonal

We may have more than one objective in a staircase linear programming problem.

$$\text{Minimize } Z(x) = \sum_{t=1}^{T} [c_{Lt}, c_{Rt}] X_t \quad (1)$$

Subject to

$$A_{11}X_1 = b_1, \quad (2)$$

$$A_{21}X_1 + A_{22}X_2 = b_2 \quad (3)$$

$$A_{32}X_2 + A_{33}X_3 = b_3 \quad (4)$$

$$A_{43}X_3 + A_{44}X_4 = b_4 \quad (5)$$

$$X_t \geq 0 \quad \text{for all } t = 1,2 \ldots.N \quad (6)$$

$$\text{where } \sum_{i=1}^{t} m_i = \sum_{j=1}^{t} n_j \quad \text{(Balanced condition)}$$

Such a matrix A, with all of its nonzero elements found in blocks centered roughly on and just below the main diagonal, is called a staircase matrix because of its resemblance to a set of steps. The sub matrices At, t = 1, ... , T, are called diagonal blocks and are of dimensions m_t x n_t where $\sum_{t=1} m_t = m$ and $\sum_{t=1} n_t = n$ For any nonzero column of an off diagonal block, the associated column of A is called a linking column, with the corresponding linking variable being the appropriate component of the vector x. On the other hand, an all zero column in B_t is associated with a variable that is said to be local to period t (since it has no effect on period t + 1 through the matrix B_t). With $[c_{Lt}, c_{Rt}]$ is an interval representing the uncertain cost for the transportation problem. the problem may be restated as

$$\text{Minimize} \left\{ Z = \sum_{t=1} c_t x_t \;\middle|\; \begin{array}{l} \sum_{t=1} m_t = m \\ \sum_{t=1} n_t = n \\ t \geq 0 \\ \sum_{t=1} m_t = \sum_{t=1} n_t \text{ for all } t \varepsilon M \text{ and } t \varepsilon N \end{array} \right\} \quad (7)$$

where $Z \in R$ and $c_{ij} = [c_{Lt}, c_{Rt}]$ are left bound and right bound of c_t. The set of all feasible solutions of the problem will be denoted by S

III. FORMULATION OF THE CRISP OBJECTIVE FUNCTION

In this section, we show that the formulation of the original interval objective function as a crisp one. $x^0 \in S$ is an optimal solution of problem (1-6) iff there is no other solution $x \in S$ which satisfies $Z(x) <_{RC} Z(x^0)$ .The right limit $Z_R(x)$ of the interval objective function $Z_R(x)$ in the problem (1-6) may be calculated from, Bodkhe[1] fuzzy programming approach to interval transportation problem

$$Z_R(x) = \sum_{t=1}^{T} C_{ct}X_t + \sum_{t=1}^{T} C_{wt}X_t \tag{8}$$

where C_{Ct} is the center and C_{Wt} is the half width of the co-efficient C_t of Z(x). In the case when $X_t \geq 0$, t= 1,2,…,N, $Z_R(x)$ can be modified as

$$Z_R(x) = \sum_{t=1}^{T} C_{ct}X_t + \sum_{t=1}^{T} C_{wt}X_t \tag{9}$$

The center of the objective function Z_c can be elicited as

$$Z_C(x) = \sum_{t=1}^{T} C_{ct}X_t \tag{10}$$

The solution set of equation (1-6) defined by definition (7) can be obtained as the Pareto optimal solution of the following interval objective problem.

Min $\{Z_R, Z_C\}$

Subject to constraints

(2) to (6)

where Z_R and Z_C are stated in equations (9) and (10).

IV. FUZZY PROGRAMMING TECHNIQUE FOR THE SOLUTION OF THE PROBLEM

The first step to solve the problem is to assign, for each objective, two values U_p and L_p as upper and lower bounds respectively, for the Z_R and Z_C objectives, where lower bound indicates aspiration level of achievement and upper bound indicates highest acceptable level of achievement for the Z_R and Z_C objectives. Let $d_p = (U_p - L_p)$ is the degradation allowance for the Z_R and Z_C objectives.

Once the aspiration levels and degradation allowance for each objective have been specified, we have formed the fuzzy model. Our next step is to transform the fuzzy model into a "Crisp" model. The steps of the fuzzy programming technique are as follows:

Step 1: Solve the interval Stair-case linear problem as a single objective linear problem using, each time, only one objective and ignoring all others.

Step 2: From the results of step 1, determine the corresponding values for every objective at each solution derived.

Step 3: From Step 2, we may find, each objective, the worst (U_p) and the best (L_p) values corresponding to the set of solutions. The initial fuzzy model can then be stated in terms the aspiration levels of each objective, as follows.

$$\text{Find } \{ x_t, \ t = 1, 2, \ldots, N; \} \tag{11}$$

so as to satisfy $Z \lessapprox L_,$ and constraints

(2) to (6)

$\lessapprox$ (fuzzification symbol)) indicates nearly less than equal to

Step 4: Defining membership functions

Linear membership function is defined as follows:

$$\mu_p(x) = \begin{cases} 1, & \text{if } \sum_{t=1}^{T} [Z_R, Z_C] x_t \le L_p \\ \dfrac{U_p - \sum_{t=1}^{T} [Z_R, Z_C] x_t}{L_p - U_p}, & \text{if } L_p < \sum_{t=1}^{T} [Z_R, Z_C] x_t < U_p \\ 0, & \text{if } \sum_{t=1}^{T} [Z_R, Z_C] x_t \ge U_p \end{cases} \tag{12}$$

Step 5: Find an equivalent crisp model by using a linear membership function for the initial fuzzy model.

Step 6: From case (i), solve the crisp model by an appropriate mathematical programming algorithm. The solution obtained in step 6 will be the optimal compromise solution of the problem. If we will use the linear membership function as defined in (12) then an equivalent crisp model for the fuzzy model can be formulated as:

then the crisp model can be simplified as:

$$\text{Maximize } \lambda \tag{15}$$

subject to

$$\sum_{t=1}^{T} [(Z_R, Z_C)] x_t + \lambda (U_p - L_p) \le U_{p,} \tag{16}$$

$$(2) \text{ to } (6) \tag{17}$$

V. NUMERICAL EXAMPLE

Minimize

$$Z(x) = [4,10]X_{11} + [8,1]X_{21} + [2,8]X_{22} + [1,5]X_{32} + [5,12]X_{33} + [6,2]X_{43} + [3,11]X_{44} \tag{18}$$

subject to

$X_{11} = 8$

$X_{21}+X_{22}= 10$

$X_{32}=X_{33}=16$

$X_{43}+X_{44}=12$ (19)

$X_t \geq 0$, t = 1,2,3,4

Solution:-

The equivalent deterministic problem becomes:

$$Z_R(x)= 10X_{11}+1X_{21}+8X_{22}+5X_{32}+12X_{33}+2X_{43}+11X_{44} \quad (20)$$

$$Z_c(x)= 7X_{11}+4.5X_{21}+5X_{22}+3X_{32}+8.5X_{33}+4X_{43}+7X_{44} \quad (21)$$

subject to

(19)

Optimal solution which minimizes Z_R subject to constraints (19) is as follows:

X_{21} = 10., X_{32} = 16., X_{43} = 12.

with $Z_R(X_1)$ =114, $Z_R(X_2)$ = 141

Optimal solution which minimizes Z_C subject to constraints (19) is as follows:

X_{11} = 8., X_{21}=10, X_{32} = 16., X_{43} = 12

$Z_C(X_1)$ =194, $Z_C(X_2)$ = 197

If we use the linear membership function, The membership function $\mu_1(x)$, $\mu_2(x)$, for the objectives Z_R and Z_C ,

The problem was solved by the Linear Interactive and Discrete Optimization (LINDO) Software The optimal solution is presented as follows:

X_{11} = 8., X_{21} = 10., X_{32} = 16., X43 = 12.

$Z_C(X_1)$ = 194, $Z_C(X_2)$ = 194 and λ= 0.50

VI. CONCLUSION

The present paper proposes a solution procedure of the interval Stair –case linear programming problem, where the co-efficient of the objective functions have been considered as interval. Initially, the problem has been converted into a classical linear programming problem where the objectives, which are right limit and center of the interval objective functions, are minimized. These objective functions can be considered as the minimization of the worst case and the average case., the fuzzy linear programming techniques has been used for pareto optimum solution . we observe that for a balanced stair case linear programming problem number of decision variables is equal to 2n-1 and number of constraints is equal to 2n. therefore the problem has exactly 2n-1 independent equations and 2n-1 decision variables only one solution is possible to the system so we find the ideal solution is equal to optimal compromise

VII. REFERENCES

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