

On Common Fixed Point Theorems in Intuitionistic Fuzzy Metric Spaces

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Abstract: *In this paper, we prove some common fixed point theorems for weakly compatible maps in Intuitionistic Fuzzy metric.*

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1. Introduction

It proved a turning point in the development of fuzzy mathematics when the notion of fuzzy set was introduced by L.A.Zadeh [15]. K.Atanassov [14] introduced and studied the concept of intuitionistic fuzzy sets. D.Coker [4] introduced the concept of intuitionistic fuzzy topological spaces. Jungck [13] common fixed point theorem in the setting of intuitionistic fuzzy metric space. Turkoglu et al. [5] further formulated the notions of weakly commuting and R weakly commuting mappings in intuitionistic fuzzy metric spaces and proved the intuitionistic fuzzy version of Pant's theorem [16]. Gregori et al. [19], Saadati and Park [21] studied the concept of intuitionistic fuzzy metric space and its applications. No wonder that intuitionistic fuzzy fixed point theory has become an area of interest for specialists in fixed point theory as intuitionistic fuzzy mathematics has covered new possibilities for fixed point theorists. Recently, many authors have also studied the fixed point theory in fuzzy and intuitionistic fuzzy metric spaces Dimri et.al.[6], Grabiec [9], Imdad et. al.[11], J.S. Park, Y.C. Kwan, and J.H. Park[12]. H.Dubey and R.Jain [9] studied the concept on common fixed point theorems in intuitionistic fuzzy metric spaces.

2. Preliminaries

Definition 2.1[3]. A binary operation $*$: $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t-norm if it satisfies the following conditions:

- (1) $*$ is associative and commutative,
- (2) $*$ is continuous,
- (3) $a * 1 = a$ for all $a \in [0, 1]$,
- (4) $a * b \leq c * d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$,

Two typical examples of continuous t-norm are $a * b = ab$ and $a * b = \min(a, b)$.

Definition 2.2[3]. A binary operation \diamond : $[0, 1] \times [0, 1] \rightarrow [0, 1]$ is a continuous t-co-norm if it satisfies the following conditions:

- (1) \diamond is associative and commutative,

(2) \diamond is continuous,

(3) $a \diamond 1 = a$ for all $a \in [0, 1]$,

(4) $a \diamond b \leq c \diamond d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$,

Two typical examples of continuous t-co norm are $a \diamond b = ab$ and $a \diamond b = \min(a, b)$.

Definition 2.3[4]. A 5-tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, \diamond is a continuous t-co-norm and M, N are fuzzy sets on $X^2 \times [0, \infty)$ satisfying the following conditions:

(i) $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$;

(ii) $M(x, y, 0) = 0$ for all $x, y \in X$;

(iii) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(iv) $M(x, y, t) = M(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(v) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$ for all $x, y \in X$ and $s, t > 0$;

(vi) for all $x, y \in X, M(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$ is left continuous;

(vii) $\lim_{t \rightarrow \infty} M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$;

(viii) $N(x, y, 0) = 1$ for all $x, y \in X$;

(ix) $M(x, y, t) = 1$ for all $x, y \in X$ and $t > 0$ if and only if $x = y$;

(x) $N(x, y, t) = N(y, x, t)$ for all $x, y \in X$ and $t > 0$;

(xi) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$ for all $x, y \in X$ and $s, t > 0$;

(xii) for all $x, y \in X, N(x, y, \cdot): [0, \infty) \rightarrow [0, 1]$ is right continuous;

(xiii) $\lim_{t \rightarrow \infty} N(x, y, t) = 0$ for all $x, y \in X$ and $t > 0$.

Then (M, N) is called an intuitionistic fuzzy metric space on X . The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y w.r.t. t respectively.

Remark 2.1[4]. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1 - M, *, \diamond)$ such that t-norm $*$ and t-co norm \diamond defined by $a * a \geq a, a \in [0, 1]$ & $(1 - a) \diamond (1 - a) \leq (1 - a)$ for all $x, y \in X$, In intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$, $M(x, y, \cdot)$ is non-decreasing and $N(x, y, \cdot)$ is non-increasing.

Remark 2.2[17]. Let (X, d) be a metric space. Define t-norm $a * b = \min(a, b)$ and t-co norm $a \diamond b = \max(a, b)$, for all $x, y \in X$ & $t > 0$.

$M_d(x, y, t) = \frac{t}{t+d(x,y)}$, $N_d(x, y, t) = \frac{d(x,y)}{t+d(x,y)}$ Then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric

space induced by the metric. It is obvious that $N(x, y, t) = 1 - M(x, y, t)$

Alaca, Turkoglu and Yildiz [4] introduced the following notions:

Definition 2.4. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then

(a) a sequence $\{x_n\}$ in X is called Cauchy-sequence if, for all $t > 0$ and $P > 0$,

$$\lim_{n \rightarrow \infty} M(x_{n+p}, x_n, t) = 1 \quad \text{and} \quad \lim_{n \rightarrow \infty} N(x_{n+p}, x_n, t) = 0,$$

(b) a sequence $\{x_n\}$ in X is said to be convergent to a point $x \in X$ if, for all $t > 0$,

$$\lim_{n \rightarrow \infty} M(x_n, x, t) = 1 \quad \text{and} \quad \lim_{n \rightarrow \infty} N(x_n, x, t) = 0.$$

Definition 2.5. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and $\{y_n\}$ be a sequence in X if there exists a number $k \in (0, 1)$ such that:

$$1. M(y_{n+2}, y_{n+1}, kt) \geq M(y_{n+1}, y_n, t),$$

$$2. N(y_{n+2}, y_{n+1}, kt) \leq N(y_{n+1}, y_n, t)$$

for all $t > 0$ and $n = 1, 2, 3, \dots$ then $\{y_n\}$ is a Cauchy sequence in X

Definition 2.6. A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space

$(X, M, N, *, \diamond)$ is said to be compatible if $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) = 1$ & $\lim_{n \rightarrow \infty} N(fgx_n, gfx_n, t) = 0$ for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z$, for some. $z \in X$.

Definition 2.7. A pair of self-mappings (f, g) of an intuitionistic fuzzy metric space

$(X, M, N, *, \diamond)$ is said to be non-compatible if $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) \neq 1$ & $\lim_{n \rightarrow \infty} N(fgx_n, gfx_n, t) \neq 0$ for every $t > 0$, whenever $\{x_n\}$ is a sequence in X such that $\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z$, for some. $z \in X$.

Definition 2.8. An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent.

Definition 2.9[7]. A pair of self mappings (f, g) of a metric space is said to be weakly compatible if they commute at the coincidence points i.e. $fu = gu$ for some $u \in X$, then $fgu = gfu$.

Definition 2.10[7]. A pair of self mappings (f, g) intuitionistic fuzzy metric space is said to be weakly compatible if they commute at the coincidence points i.e. $fu = gu$ for some $u \in X$, then $fgu = gfu$.

Definition 2.11[8]. A pair of self mappings (f, g) intuitionistic fuzzy metric space is said to be occasionally weakly compatible iff there is a point x in X which is coincidence point of they commute at the coincidence points f and g at which f and g commute.

Lemma 2.1[7]. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. f and g be self maps on X and f and g have a unique point of coincidence, $w = fx = gx$, then w is the unique common fixed point of f and g .

Lemma 2.2[2]. Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space and for all $x, y \in X, t > 0$ and if for a number $k \in (0, 1)$ such that $M(x, y, kt) \geq M(y, x, t)$ and $N(x, y, kt) \leq N(y, x, t)$ then $x = y$.

3. Main results

Theorem 3.1. Let A, B, S and T be self maps of intuitionistic fuzzy metric spaces

$(X, M, N, *, \diamond)$ with continuous t -norm $*$ and continuous t -co norm \diamond defined by $t * t \geq t$ and $(1 - t) \diamond (1 - t) \leq (1 - t)$ for all $t \in [0, 1]$ satisfying the following condition:

(3.1.1) $A(X) \subseteq S(X)$ and $B(X) \subseteq T(X)$,

(3.1.2) If one of the A, B, S and T is a complete subspace of X then $\{A, T\}$ and $\{B, S\}$ have a coincidence point,

(3.1.3) The pairs (A, T) and (B, S) are weakly compatible,

(3.1.4)

$$M(Ax, By, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(Tx, Sy, t) * M(Tx, Ax, t) * M(Ax, Sy, t) * \\ M(Sy, Tx, t) * M(Bx, Ty, t) * M(Bx, Sx, t) * \\ M(Tx, Bx, t) * M(Tx, By, t) \end{array} \right) \right\}$$

and

$$N(Ax, By, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(Tx, Sy, t) \diamond N(Tx, Ax, t) \diamond N(Ax, Sy, t) \diamond \\ N(Sy, Tx, t) \diamond N(Bx, Ty, t) \diamond N(Bx, Sx, t) \diamond \\ N(Tx, Bx, t) \diamond N(Tx, By, t) \end{array} \right) \right\}$$

$\forall x, y \in X$ & $t > 0$, where $\emptyset, \varphi: [0, 1] \rightarrow [0, 1]$ is a continuous function such that $\emptyset(t) > t$ & $\varphi(t) < t$ for each $0 < t < 1$ and $\emptyset(1) = 1$ and $\varphi(0) = 0$ with $M(x, y, t) > 0$. Then A, B, S and T have a unique common fixed point in X .

Proof : Since $A(X) \subseteq S(X)$, therefore for any $x_0 \in X$, there exists a point $x_1 \in X$ such that $Ax_0 = Sx_1$ and for the point x_1 , we can choose a point $x_2 \in X$ such that $Bx_1 = Tx_2$ as

$B(X) \subseteq S(X)$. Inductively, we get Sequence $\{y_n\}$ in x as follows $y_{2n+1} = Bx_{2n+1} = Tx_{2n+2}$ and $y_{2n} = Ax_{2n} = Sx_{2n+1}$ for $n = 0, 1, 2, \dots$. Putting $x = x_{2n}$, $y = x_{2n+1}$ in (3.1.4) we have,

$$M(Ax_{2n}, Bx_{2n+1}, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(Tx_{2n}, Sx_{2n+1}, t) * M(Tx_{2n}, Ax_{2n}, t) * \\ M(Ax_{2n}, Sx_{2n+1}, t) * M(Sx_{2n+1}, Tx_{2n}, t) * \\ M(Bx_{2n}, Tx_{2n+1}, t) * M(Bx_{2n}, Sx_{2n}, t) * \\ M(Tx_{2n}, Bx_{2n}, t) * M(Tx_{2n}, Bx_{2n+1}, t) \end{array} \right) \right\}$$

$$M(y_{2n}, y_{2n+1}, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(y_{2n-1}, y_{2n}, t) * M(y_{2n-1}, y_{2n}, t) * \\ M(y_{2n}, y_{2n}, t) * M(y_{2n}, y_{2n-1}, t) * \\ M(y_{2n-1}, y_{2n}, t) * M(y_{2n-1}, y_{2n}, t) * \\ M(y_{2n-1}, y_{2n-1}, t) * M(y_{2n-1}, y_{2n}, t) \end{array} \right) \right\}$$

$$M(y_{2n}, y_{2n+1}, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(y_{2n-1}, y_{2n}, t) * M(y_{2n-1}, y_{2n}, t) * \\ 1 * M(y_{2n}, y_{2n-1}, t) * \\ M(y_{2n-1}, y_{2n}, t) * M(y_{2n-1}, y_{2n}, t) * \\ 1 * M(y_{2n-1}, y_{2n}, t) \end{array} \right) \right\}$$

i.e $M(y_{2n}, y_{2n+1}, t) \geq \emptyset\{M(y_{2n-1}, y_{2n}, t)\} > M(y_{2n-1}, y_{2n}, t)$ as $\emptyset(t) > t$ for each $0 < t < 1$ and

$$N(Ax_{2n}, Bx_{2n+1}, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(Tx_{2n}, Sx_{2n+1}, t) \diamond N(Tx_{2n}, Ax_{2n}, t) \diamond \\ N(Ax_{2n}, Sx_{2n+1}, t) \diamond N(Sx_{2n+1}, Tx_{2n}, t) \diamond \\ N(Bx_{2n}, Tx_{2n+1}, t) \diamond N(Bx_{2n}, Sx_{2n}, t) \diamond \\ N(Tx_{2n}, Bx_{2n}, t) \diamond N(Tx_{2n}, Bx_{2n+1}, t) \end{array} \right) \right\}$$

$$N(y_{2n}, y_{2n+1}, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(y_{2n-1}, y_{2n}, t) \diamond N(y_{2n-1}, y_{2n}, t) \diamond \\ N(y_{2n}, y_{2n}, t) \diamond N(y_{2n}, y_{2n-1}, t) \diamond \\ N(y_{2n-1}, y_{2n}, t) \diamond N(y_{2n-1}, y_{2n}, t) \diamond \\ N(y_{2n-1}, y_{2n-1}, t) \diamond N(y_{2n-1}, y_{2n}, t) \end{array} \right) \right\}$$

$$N(y_{2n}, y_{2n+1}, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(y_{2n-1}, y_{2n}, t) \diamond N(y_{2n-1}, y_{2n}, t) \diamond \\ 1 \diamond N(y_{2n}, y_{2n-1}, t) \diamond \\ N(y_{2n-1}, y_{2n}, t) \diamond N(y_{2n-1}, y_{2n}, t) \diamond \\ 1 \diamond N(y_{2n-1}, y_{2n}, t) \end{array} \right) \right\}$$

i.e $N(y_{2n}, y_{2n+1}, t) \leq \varphi\{N(y_{2n-1}, y_{2n}, t)\} < N(y_{2n-1}, y_{2n}, t)$ as $\varphi(t) < t$ for each $0 < t < 1$. Thus $\{M(y_{2n}, y_{2n+1}, t), n \geq 0\}$ is an increasing sequence of positive real numbers in $[0,1]$ which tends to a limit $l \leq 1$, also $\{N(y_{2n+1}, y_{2n+2}, t), n \geq 0\}$ is an decreasing sequence of positive real numbers $[0,1]$ which tends to a limit $k = 0$.

Therefore for every $n \in I^+$ $M(y_n, y_{n+1}, t) > M(y_{n-1}, y_n, t)$ & $\lim_{n \rightarrow \infty} M(y_n, y_{n+1}, t) = 1$,

$N(y_n, y_{n+1}, t) < N(y_{n-1}, y_n, t)$ & $\lim_{n \rightarrow \infty} N(y_n, y_{n+1}, t) = 0$. Now any positive integer p , we obtain $\lim_{n \rightarrow \infty} M(y_n, y_{n+p}, t) = 1$ and $\lim_{n \rightarrow \infty} N(y_n, y_{n+p}, t) = 0$. Which shows that $\{y_n\}$ is a

Cauchy sequence in X . Let $w \in S^{-1}u$ then $Sw = u$. we shall use the fact that subsequence $\{y_{2n+1}\}$ also converges to u . Now by putting $x = x_{2n}$, $y = w$ in (3.4) and taking $n \rightarrow \infty$

$$M(Ax_{2n}, Bw, t) \geq \phi \left\{ \min \left(\begin{array}{l} M(Tx_{2n}, Sw, t) * M(Tx_{2n}, Ax_{2n}, t) * \\ M(Ax_{2n}, Sw, t) * M(Sw, Tx_{2n}, t) * \\ M(Bx_{2n}, Tw, t) * M(Bx_{2n}, Sx_{2n}, t) * \\ M(Tx_{2n}, Bx_{2n}, t) * M(Tx_{2n}, Bw, t) \end{array} \right) \right\}$$

$$M(u, Bw, t) \geq \phi \left\{ \min \left(\begin{array}{l} M(u, u, t) * M(u, u, t) * \\ M(u, u, t) * M(u, u, t) * \\ M(u, u, t) * M(u, u, t) * \\ M(u, u, t) * M(u, u, t) \end{array} \right) \right\}$$

i.e $M(u, Bw, t) \geq \phi\{M(u, u, t)\} \geq \phi(1) = 1$ (*)

also

$$N(Ax_{2n}, Bw, t) \leq \phi \left\{ \max \left(\begin{array}{l} N(Tx_{2n}, Sw, t) \diamond N(Tx_{2n}, Ax_{2n}, t) \diamond \\ N(Ax_{2n}, Sw, t) \diamond N(Sw, Tx_{2n}, t) \diamond \\ N(Bx_{2n}, Tw, t) \diamond N(Bx_{2n}, Sx_{2n}, t) \diamond \\ N(Tx_{2n}, Bx_{2n}, t) \diamond N(Tx_{2n}, Bw, t) \end{array} \right) \right\}$$

$$N(u, Bw, t) \leq \phi \left\{ \max \left(\begin{array}{l} N(u, u, t) \diamond N(u, u, t) \diamond \\ N(u, u, t) \diamond N(u, u, t) \diamond \\ N(u, u, t) \diamond N(u, u, t) \diamond \\ N(u, u, t) \diamond N(u, u, t) \end{array} \right) \right\}$$

i.e $N(u, Bw, t) \leq \phi\{N(u, u, t)\} \leq \phi(0) = 0$ (**)

From (*) and (**), Let $u = Bw$. Since $Sw = u$ we have $Sw = Bw = u$ i.e. w is the coincidence point of B and S . As $B(X) \subseteq T(X)$, $= Bw \rightarrow u \in T(X)$. Let $v \in T^{-1}u$ then $Tv = u$. Now by putting $x = v$, $y = x_{2n+1}$ in (3.1.4)

$$M(Av, Bx_{2n+1}, t) \geq \phi \left\{ \min \left(\begin{array}{l} M(Tv, Sx_{2n+1}, t) * M(Tv, Av, t) * \\ M(Av, Sx_{2n+1}, t) * M(Sx_{2n+1}, Tv, t) * \\ M(Bv, Tx_{2n+1}, t) * M(Bv, Sv, t) * \\ M(Tv, Bv, t) * M(Tv, Bx_{2n+1}, t) \end{array} \right) \right\}$$

taking $n \rightarrow \infty$

$$M(Av, u, t) \geq \phi \left\{ \min \left(\begin{array}{l} M(u, u, t) * M(u, Av, t) * \\ M(Av, u, t) * M(u, u, t) * \\ M(u, u, t) * M(u, u, t) * \\ M(u, u, t) * M(u, u, t) \end{array} \right) \right\}$$

$$M(Av, u, t) \geq \phi\{\min(1 * M(u, Av, t) * M(Av, u, t) * 1 * 1 * 1 * 1 * 1)\}$$

i.e $M(Av, u, t) \geq \phi\{M(u, Av, t)\} > M(u, Av, t)$ and

$$N(Av, Bx_{2n+1}, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(Tv, Sx_{2n+1}, t) \diamond N(Tv, Av, t) \diamond \\ N(Av, Sx_{2n+1}, t) \diamond N(Sx_{2n+1}, Tv, t) \diamond \\ N(Bv, Tx_{2n+1}, t) \diamond N(Bv, Sv, t) \diamond \\ N(Tv, Bv, t) \diamond N(Tv, Bx_{2n+1}, t) \end{array} \right) \right\}$$

taking $n \rightarrow \infty$

$$N(Av, u, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(u, u, t) \diamond N(u, Av, t) \diamond \\ N(Av, u, t) \diamond N(u, u, t) \diamond \\ N(u, u, t) \diamond N(u, u, t) \diamond \\ N(u, u, t) \diamond N(u, u, t) \end{array} \right) \right\}$$

$$N(Av, u, t) \leq \varphi \{ \max(1 \diamond N(u, Av, t) \diamond N(Av, u, t) \diamond 1 \diamond 1 \diamond 1 \diamond 1 \diamond 1) \}$$

$$\text{i.e } N(Av, u, t) \leq \varphi \{ N(u, Av, t) \} < N(u, Av, t)$$

Therefore, we get $Av = u$. we have $Tv = Av = u$.

Thus v is a coincidence point of a and T .

Since the pairs $\{A, T\}$ and $\{B, S\}$ are weakly compatible i.e. $B(Sw) = S(Bw) \rightarrow Bu = Su$ and $A(Tv) = T(Av) \rightarrow Au = Tu$. Now by putting $x = u, y = x_{2n+1}$ in (3.1.4)

$$M(Au, Bx_{2n+1}, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(Tu, Sx_{2n+1}, t) * M(Tu, Au, t) * \\ M(Au, Sx_{2n+1}, t) * M(Sx_{2n+1}, Tu, t) * \\ M(Bu, Tx_{2n+1}, t) * M(Bu, Su, t) * \\ M(Tu, Bu, t) * M(Tu, Bx_{2n+1}, t) \end{array} \right) \right\}$$

taking $n \rightarrow \infty$

$$M(Au, u, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(Au, u, t) * M(Au, Au, t) * \\ M(Au, u, t) * M(u, Au, t) * \\ M(u, u, t) * M(u, u, t) * \\ M(Au, u, t) * M(Au, u, t) \end{array} \right) \right\}$$

$$M(Au, u, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(Au, u, t) * 1 * M(Au, u, t) * M(u, Au, t) * \\ 1 * 1 * M(Au, u, t) * M(Au, u, t) \end{array} \right) \right\}$$

i.e $M(Au, u, t) \geq \emptyset \{ M(Au, u, t) \} > M(Au, u, t)$ and

$$N(Au, Bx_{2n+1}, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(Tu, Sx_{2n+1}, t) \diamond N(Tu, Au, t) \diamond \\ N(Au, Sx_{2n+1}, t) \diamond N(Sx_{2n+1}, Tu, t) \diamond \\ N(Bu, Tx_{2n+1}, t) \diamond N(Bu, Su, t) \diamond \\ N(Tu, Bu, t) \diamond N(Tu, Bx_{2n+1}, t) \end{array} \right) \right\}$$

taking $n \rightarrow \infty$

$$N(Au, u, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(Au, u, t) \diamond N(Au, Au, t) \diamond \\ N(Au, u, t) \diamond N(u, Au, t) \diamond \\ N(u, u, t) \diamond N(u, u, t) \diamond \\ N(Au, u, t) \diamond N(Au, u, t) \end{array} \right) \right\}$$

$$N(Au, u, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(Au, u, t) \diamond 1 \diamond N(Au, u, t) \diamond N(u, Au, t) \diamond \\ 1 \diamond 1 \diamond N(Au, u, t) \diamond N(Au, u, t) \end{array} \right) \right\}$$

i.e $N(Au, u, t) \leq \varphi\{N(Au, u, t)\} < N(Au, u, t)$

Therefore, we get $Au = u$. So we have $Au = Tu = u$. similarly by putting $x = x_{2n}$, $y = u$ in (3.1.4) as $n \rightarrow \infty$ $u = Bu = Su$. Thus $Au = Bu = Su = Tu = u$ i.e. u is a common fixed point of A, B, S and T .

Uniqueness: Let $w (w \neq u)$ be another common fixed point of A, B, S and T . then by putting $x = u, y = w$ in (3.1.4)

$$M(Au, Bw, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(Tu, Sw, t) * M(Tu, Au, t) * M(Au, Sw, t) * \\ M(Sw, Tu, t) * M(Bu, Tw, t) * M(Bu, Su, t) * \\ M(Tu, Bu, t) * M(Tu, Bw, t) \end{array} \right) \right\}$$

$$M(u, w, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(u, w, t) * M(u, u, t) * M(u, w, t) * \\ M(w, u, t) * M(u, w, t) * M(u, u, t) * \\ M(u, u, t) * M(u, w, t) \end{array} \right) \right\}$$

$$M(u, w, t) \geq \emptyset \left\{ \min \left(\begin{array}{l} M(u, w, t) * 1 * M(u, w, t) * M(w, u, t) * \\ M(u, w, t) * 1 * 1 * M(u, w, t) \end{array} \right) \right\}$$

i.e $M(u, w, t) \geq \emptyset\{M(u, w, t)\} > M(u, w, t)$

and

$$N(Au, Bw, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(Tu, Sw, t) \diamond N(Tu, Au, t) \diamond N(Au, Sw, t) \diamond \\ N(Sw, Tu, t) \diamond N(Bu, Tw, t) \diamond N(Bu, Su, t) \diamond \\ N(Tu, Bu, t) \diamond N(Tu, Bw, t) \end{array} \right) \right\}$$

$$N(u, w, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(u, w, t) \diamond N(u, u, t) \diamond N(u, w, t) \diamond \\ N(w, u, t) \diamond N(u, w, t) \diamond N(u, u, t) \diamond \\ N(u, u, t) \diamond N(u, w, t) \end{array} \right) \right\}$$

$$N(u, w, t) \leq \varphi \left\{ \max \left(\begin{array}{l} N(u, w, t) \diamond 1 \diamond N(u, w, t) \diamond N(w, u, t) \diamond \\ N(u, w, t) \diamond 1 \diamond 1 \diamond N(u, w, t) \end{array} \right) \right\}$$

i.e $N(u, w, t) \leq \emptyset\{N(u, w, t)\} < N(u, w, t)$. Hence $u = w$ for all $x, y \in X$ and $t > 0$.

Therefore u is the unique common fixed point of a, B, S and T .

This completes the proof.

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