Jackknife And Bootstrap Techniques In The Estimation of regression Parameters

[1]Ijomah Maxwell Azubuike, [2]Chris-Chinedu, Joy Nonso
[1]Department of Mathematics & Statistics, [2]Department of Mathematics
[1]University of Port Harcourt, [2]Ignatius Ajuru University of Education
[1]Choba, Port Harcourt, Nigeria, [2]Rumuolumeni Port Harcourt, Nigeria

Abstract *- The classifications according to various criteria into successive levels for building a regression model by using jackknife and bootstrap resampling methods were the basis in this study. Bootstrap techniques on the basis ofthe observations and errors resampling re-computing the estimates, while jackknife methodis based on the delete-one and delete-d observations were considered. Wealso estimatedjackknife and bootstrap bias, theconfidence levels and standard errors of the regression coefficients, and to compare them with the concerned estimates of ordinary least squares(OLS). The regression coefficients of jackknife bias, the standard errors and confidence intervals are considerably larger than the bootstrap and the estimated related to OLS standard errors. The bootstrap percentile intervals are smaller than that of thejackknife percentile intervals of the regression coefficients.*

Keywords: *jackknife, bootstrap, resampling and regression*

I. INTRODUCTION

Regression analysis is one of the most widely used statistical techniques in which we study the effects of explanatory variables on a response variable. Two of the most important problems in applied statistics are the determination of an estimator for a particular parameter of interest and the evaluation of the accuracy of that estimator through estimates of the standard error of the estimator and the determination of confidence intervals for the parameter [1].

Consider the model

$$Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + ... + \beta_P X_P + \varepsilon \quad (1)$$

where β_0 = a constant $\beta_1, \beta_2, \ldots, \beta_p$ = regression parameters ε = random error . When we have n observations on Y and X_i's this equation can be represented as follows:

$$Y = X\beta + \varepsilon$$

where $Y = (y_1, y_2, ..., y_n)'$ is a n-vector of responses

$$X_{nx(p+1)} = \begin{bmatrix} 1 x_{11} x_{21} ...... x_{p1} \\ : \\ 1 x_{12} x_{22} ...... x_{p2} \\ : \\ 1 x_{1n} x_{2n} ...... x_{pn} \end{bmatrix} \quad (2)$$

and $\beta = (\beta_0, \beta_1, \ldots\ldots, \beta_p)'$ is a p+1-vector parameters and $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots\ldots\ldots, \varepsilon_n)'$ is a n-vector of error terms.An ordinary least square solution to (1) is one in which the Euclidean norm of the vector $(Y - X\beta)$ is minimized. That is ,

$$\min \|Y - X\beta\|_2$$

By setting the gradient of the square of this norm, $(Y - X\beta)^1 (Y - X\beta),$ to zero with respect to the vector β, the necessary condition for the solution vector $\hat{\beta}$ is that $\hat{\beta}$ must be a solution to

$$X^1 X\beta = X^1 Y \quad (3)$$

In other symbols, the solution is

$$\hat{\beta} = (X^1 X)^{-1} X^1 Y \quad (4)$$

Estimating $\hat{\beta}$ from the above equation (4) using ordinary least square (OLS) rely strongly on some major modeling assumptions. Though furnished with intuitive assumptions are based on asymptotical or frequently similar properties. The dependability of the statistical analysis therefore relies on the values of these suppositions and size of the sample. There are several useful approaches for identifying the nature and treating infringements of the regression assumptions. Robust estimation strategies and residual diagnostics have improved the usefulness of these techniques. Nevertheless, they may not be furnished by these assumptions by use of these methods. The data observed was considered to represent the picture of the entire population in resampling procedures. “Hence, the main idea is to constitute statistical inference on the basis of a crafty resample, which is pulled from the full sample” [7]. The common sampling methods use some of the assumptions similar to the form of the estimator distribution, while on the other hand resampling techniques need not these assumptions for the reason that the sample is thought as population. The jackknife and bootstrap are not involving an estimation of the parameters of a statistic and particular resampling methods that set forth of deriving estimates of confidence intervals of a population parameters like the average, median values, symmetrical, odds ratio, coefficient of correlation or regression calculations and standard errors without composing distributional suppositions when those suppositions are in uncertainty of condition, or the impossibility of parametric inference or the calculation of standard errors requires very intricate formulas [4]. And hence offers for more accurate inference if the data distribution is unknown or the sample size is small.

The Jackknife and Bootstrap are the most popular resampling methods that are becoming increasingly popular as statistical tools as they are generally very robust, their simplicity is compelling and their computational demands are largely no longer an issue to their widespread implementation which are designed to estimate standard errors, bias, confidence intervals, and prediction error. The basic objective of all statistical analyses is to extract information from the available data under consideration. The use of the jackknife and bootstrap to estimate the sampling distribution of the parameter estimates in linear regression model was first proposed by Efron [3] and further developed by Freedman([10], [27]). These resampling methods are essentially computer intensive techniques to extract as much information as possible from the data at hand. Bootstrap and Jackknife use the sample data to estimate the relevant properties of the population. They empirically construct the sampling distribution of a statistic by resampling.These resampling methods replace the rigorous theoretical derivations required in applying traditional methods (such as substitution and linearization) in statistical analysis by repeatedly resampling the original data and making inferences from the resamples. Thus these methods can routinely answer questions which are far too complicated for traditional statistical analysis. The resampling procedure is in parallel with sample withdrawal from the population. The application of bootstrap methods to regression models helps approximate the distribution of the coefficients [10]and the distribution of the prediction errors when the regressors are data [24] or random variables [17]. We are concerned here with the application of the bootstrap techniques to determine prediction intervals on econometric models when the regressors are known. Wolter [26] discusses use of resampling methods for variance estimation especially in the context of sample surveys. Shao and Tu [22] discuss the Jackknife and Bootstrap methods applied to diverse areas of statistics. Lahiri[15]discusses Bootstrap and related resampling methods for temporal and spatial data exhibiting various forms of dependence without requiring stringent structural assumptions. Good [11] also discusses various software for re-sampling purposes among other things. The bootstrap is a resampling method that draws a large collection of samples from the original data. It is used to select the observation randomly with replacement from the original data sample [6].

In this study, we focus on the accuracy of the jackknife and bootstrap resampling methods in estimating the unknown beta coefficients in Linear Regression. The true interest lies in determining whether the bootstrap or jackknife techniques for variance estimation provide better estimates. The contents of this article may be divided into five sections. In sections 2 we briefly review the jackknifing and bootstrapping regression model respectively. Section 3 deals with the material and methods while section 4 gives results for bootstrap resampling and Jackknife methods to estimates parameters of linear regression. Finally, the last section summarizes the conclusions of the study.

II. THE JACKKNIFE TECHNIQUE

The Jackknife method was proposed by Quenouille [20]as a method of bias reduction which eventually in later years became widely useful for variance estimation. Given a sample data, the Jackknife method uses all but some units at a time to form a group. Following Tukey [25]'s conjecture that the so–called "pseudovalues" obtained from these groups are IID random variables one can get a simple estimator of variance. The generalized Jackknife is called the delete–d–Jackknife.

Let $\hat{\theta}_n$ be an estimator of θ based on n i.i.d. random vectors $X_1,\ldots\ldots\ldots, X_n$, i.e., $\hat{\theta}_n = f_n(X_1,\ldots\ldots\ldots\ldots, X_n)$ for some function fn.

Let $\hat{\theta}_{n,-i} = f_{n-1}(X_1,\ldots\ldots, X_{i-1,} X_{i+1,}\ldots., X_n)$ be the corresponding recomputed statistic based on all but the i[th] observation. The jackknife estimator of bias $E(\hat{\theta}_n) - \theta$ is given by

$$bias_j = \frac{(n-1)}{n}\sum_{i=1}^{n}\left(\hat{\theta}_{n,-i} - \hat{\theta}_n\right)$$

Jackknife estimator θ_j of θ is given by

$$\theta_j = \hat{\theta}_n - biasj = \frac{1}{n}\sum_{i=1}^{n}\left(n\hat{\theta}_n - (n-1)\hat{\theta}_{n,-i}\right) \qquad (5)$$

Such a bias corrected estimator hopefully reduces the overall bias. The summands above $\theta_{n,i} = \theta_n - (n-1)\hat{\theta}_{n-1}, i = 1\ldots.., n$ are called pseudo-values.

The Jackknife bias, the variance, the confidence and the percentile interval.

The jackknife variance and the confidence intervals are calculated by using these equations from F$(\hat{\theta}_{n,-i})$ distribution [18].

The jackknife variance equals,

$$Var_j(\hat{\theta}_n) = \frac{(n-1)}{n}\sum_{i=1}^{n}(\hat{\theta}_{n,-i} - \bar{\theta}_n)(\hat{\theta}_{n,-i} - \bar{\theta}_n)^1 \ (6)$$

Where $\hat{\theta}_{n,-i}$ is the estimate obtained from the replicate with i[th] observation set or j[th] group deleted [7] and $\bar{\theta} = \frac{1}{n}\sum_{i=1}^{n}\hat{\theta}_{n,-i}$. For most statistics, jackknife estimator of variance is $Var_j(\hat{\theta}_n)/Var(\hat{\theta}_n) \to 1$ .Jackknife (1-α) 100 % confidence interval equals [6].

$$\hat{\theta}_{n,-i} - t_{n-p,\alpha/2} * S_e(\hat{\theta}_{n,-i}) < \theta < \hat{\theta}_{n,-i} + t_{n-p,\alpha/2} * S_e(\hat{\theta}_{n,-i}) \quad (7)$$

where $t_{n-p,\alpha/2}$ is the critical value of t with probability α/2 the right for n-p degrees of freedom; and $S_e(\hat{\theta}_{n,-i})$ is the standard error of the $\hat{\theta}_{n,-i}$ .The jackknife percentile Interval can be estimated from the quintile's of the jackknife sampling distribution of $\hat{\theta}_{n,-i}$. The (α/2)% and (1-α/2)% percentile interval is

$$\hat{\theta}_{n,-i}(lower) < \theta < \hat{\theta}_{n,-i}(upper) \quad (8)$$

III. BOOTSTRAP

The bootstrap is a method for estimating the distribution of an estimator or test statistic by resampling one's data or a model estimated from the data. Under conditions that hold in a wide variety of applications, the bootstrap provides approximations to distributions of statistics, coverage probabilities of confidence intervals, and rejection probabilities of tests that are at least as accurate as the approximations of first-order asymptotic distribution theory.The bootstrap makes use of the resampling of the error term at a fixed regressors, and resampling of observation sets at random regressor([22]-[23]).Often, the bootstrap provides approximations that are more accurate than those of first-order asymptotic theory. The importance of the bootstrap emerged during the 1980s when mathematical study demonstrated that it gives nearly optimal estimate of the distribution of many statistics under a wide range of circumstances. The greatest significant advantages of the bootstrap regression techniques are to require little sample than OLS procedure and its theoretical accomplishments is often much better yet this is not warranted [13].In several cases, the method yields better results than those obtained by the classical normal approximation theory. For this reason, it is a misconception to trust that bootstrap regression technique often gives trusted results. The trust rests on the data structure and distribution function.

The Bootstrap bias, the variance, the confidence and percentile interval.

The mean of sampling distribution of $\hat{\theta}$ often differs from θ, usually by an amount = c/n for large n . The bootstrap bias equals,

$$\hat{bias} = \hat{\theta}_b - \hat{\theta} \quad (9)$$

The variance of the bootstrap from the distribution of $F(\hat{\theta}_b)$ are estimated by [24]

$$Var(\hat{\theta}_b) = \sum_{b=1}^{B} \left[(\hat{\theta}_{br} - \hat{\theta}_b)(\hat{\theta}_{br} - \hat{\theta}_b)^1 \right] / (B-1), \quad r = 1,2,....,\theta \quad (10)$$

The bootstrap confidence interval by normal approach is obtained as

$$\hat{\theta}_b - t_{n-p,\alpha/2} * S_e(\hat{\theta}_b) < \theta < \hat{\theta}_b + t_{n-p,\alpha/2} * S_e(\hat{\theta}_b) \quad (11)$$

where $t_{n-p,\alpha/2}$ is the critical value of t with probability α/2 the right for n-p degrees of freedom; and $S_e(\hat{\theta}_b)$ is the standard error of the $\hat{\theta}_b$. The Z-distribution values were used rather than that of t in estimating the confidence intervals when the sample size is n =30 [2].

The percentile interval which is nonparametric confidence interval can be derived from the quartiles of the sampling distribution of bootstrap of $\hat{\theta}_{br}$. The (α/2)% and (1-α/2)% percentile interval is:

$$\hat{\theta}_{br}(lower) < \theta_i < \hat{\theta}_{br}(upper) \quad (12)$$

where $\hat{\theta}_{br}$ is the ordered bootstrap estimates of regression coefficient from Equation 2 or 5, lower = (α/2)B, and upper = (1-α/2)B.

IV. MATERIAL AND THE METHODS

Material.

The results of the samples obtained from the households listing of 15 alongside with the size of farmland owned (SFO) and the size cultivated (SFC) (in hectares) are ***y*** and ***x*** respectively, constitute a simple random sample from 162 households. The sample results are tabulated in Table 1.The R statistical packageand SPSS was used for the statistical analysis of the data.

Method.

The purpose of this work is to exemplify by means of figures, comparisons and examples the jackknife and bootstrap regression parameter estimation as the methodology in method.

We start with an n sized sample to depict the resampling methods $W_i = (Y_i, X_{ji})^1$ and assumed that W$_i$'s are pulled independently and indistinguishably from a distribution of **F**, where $Y_i = (y_1, y_2, ---, y_n)^1$ contains the responses, $X_{ji} = (x_{j1}, x_{j2}, ---, x_{jn})^1$ is a matrix in the dimension of nxk, where j=1,2,...k, i=1,2,3,...,n.

V. RESULTS

The OLS regression model was first fitted to data given inTable 1 displays the summary of the results of the ordinary least squares regression. The regression of size of farmland cultivated yielded aninsignificant result of variance analysis (P>0.05**). The overall model is significant (P<0.03**).

Table 1: The summary statistics of regression coefficient for OLS regression

Variables	$\hat{\theta}$	**S.E($\hat{\theta}$)**	**T**	**Sig.**	**95% C.I Upper, lower**
Constant (SFO)	0.543	0.222	-0.111	0.029	0.065,1.022
SFC	-0.312	0.237	2.453	0.913	-6.369,5.745
R^2 = 0.316, N = 15, F = 6.016, P-value = 0.02906					

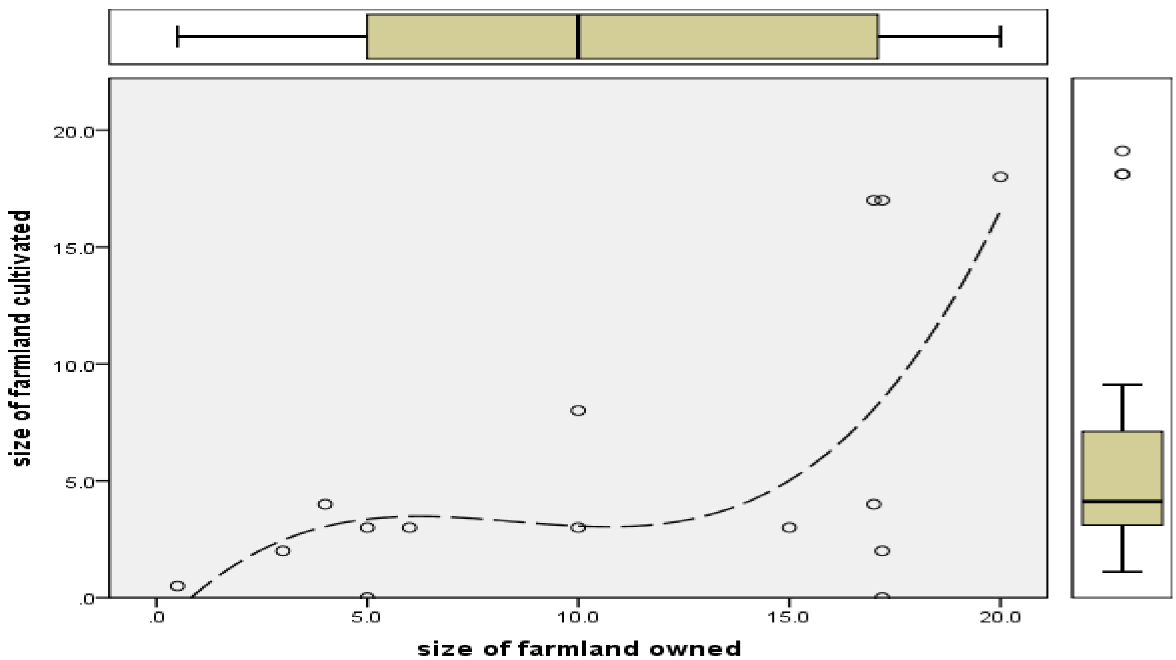

Figure 1. The fitted Ordinary Least Square regression line

The jackknifeillustration (jackknife samples, each of size n-1=162-1=161) and as well the bootstrap (B=10000 bootstrap samples, each of size n=162) regression processes, from the given data in Figure 1, estimating the jackknife and bootstrap estimates of the regression parameters for each sample are displayed in Table 2 and 3.

Table 2. The illustration of the jackknife (jackknife samples, each of size n-1=162-1=161) regression processes from the data of Table 1, estimating the jackknife estimates of the parameters of regression for each sample for the size of farmland model.

		Observations set					
$W_i^{(J)}$	Variables	1	2	---	15	$\hat{\theta}_{0J}$	$\hat{\theta}_{1J}$
1	SFO(y) SFC(x)	Omitted	18.0 20.0	--- ---	8.0 10.0	-1.461	0.851
2	SFO(y) SFC(x)	3.0 6.0	omitted	--- ---	8.0 10.0	-1.401	0.800
. . .	. . .	. . .	. . .	--- ---	. . .	. . .	. . .
15	SFO(y) SFC(x)	3.0 6.0	18.0 20.0	--- ---	Omitted	-1.431	0.831
$\theta_{0J} = \sum_{j=1}^{J} \hat{\theta}_{Ji} / 162$		-1.478	0.831				

Table 3: The bootstrap illustration (B=10000 bootstrap samples, n = 162 of each size) regression processes from the data of Table 1, estimating the bootstrap estimates of the parameters of regression for each sample of Size of farmland model

R	Variables	$W_1^{(b)}$	$W_2^{(b)}$	---	$W_{15}^{(b)}$	$\hat{\beta}_0^{(b)}$	$\hat{\beta}_1^{(b)}$
1	SFO(y) SFC(x)	3.0 6.0	8.0 10.0	---	2.00 3.0	-1.479	0.853
2	SFO(y) SFC(x)	17.0 17.0	3.0 15.0	---	17.0 17.0	-1.541	0.801
. . .	. . .	. . .	. . .	---	. . .	. . .	. . .
10000	SFO(y) SFC(x)	3.0 15.0	2.0 3.0	---	4.0 17.0	-1.432	0.831
$\hat{\theta}_b = \sum_{b=1}^{B} \hat{\theta}_{br} = \theta_{br}$		-1.479	0.832				

Table 4. Thisdisplaysthe summary statistics of the coefficients of regression for the jackknife and bootstrap regression (n=162, B=10000)

	Variables	Observed	Average	Std.Error	Bias	95% C.I Lower,Upper	5%,95% P.I Lower,Upper
Bootstrap	SFO SFC	-1.479 0.832	-1.432 0.831	0.222 0.237	0.002 -0.202	0.065,1.022 -6.369,5.745	0.064,1.021 -6.369,5.755
Jackknife	SFO SFC	-1.478 0.833	-1.477 0.832	0.212 0.231	0.0002 -0.210	0.062,1.021 -6.332,5.710	0.061,1.020 -6.322,5.720

B=10000 bootstrap samples are randomly generated to show the exact feature of the bootstrap processes and the distributions of regression parameter of bootstrap estimations ($\hat{\theta}_J$)which were displayed in Figure 2. For all regression parameters, the bootstrap estimateshistogram quite well concursto the limiting normal distribution. Nevertheless, where B is abundantly large (B=10000) the confidence intervals should thus be on the basis of that distribution. Also, the jackknife samples were gotten by the omission of each one of the n-observation sets andestimated the coefficients of regression as $\hat{\theta}_{Ji}$. To throw back on the main feature of the jackknife regression parameter estimations $\hat{\theta}_{Ji}$. The jackknife estimates histograms quite well concur to the untypical to the limiting normal distribution for all the coefficients of regression.

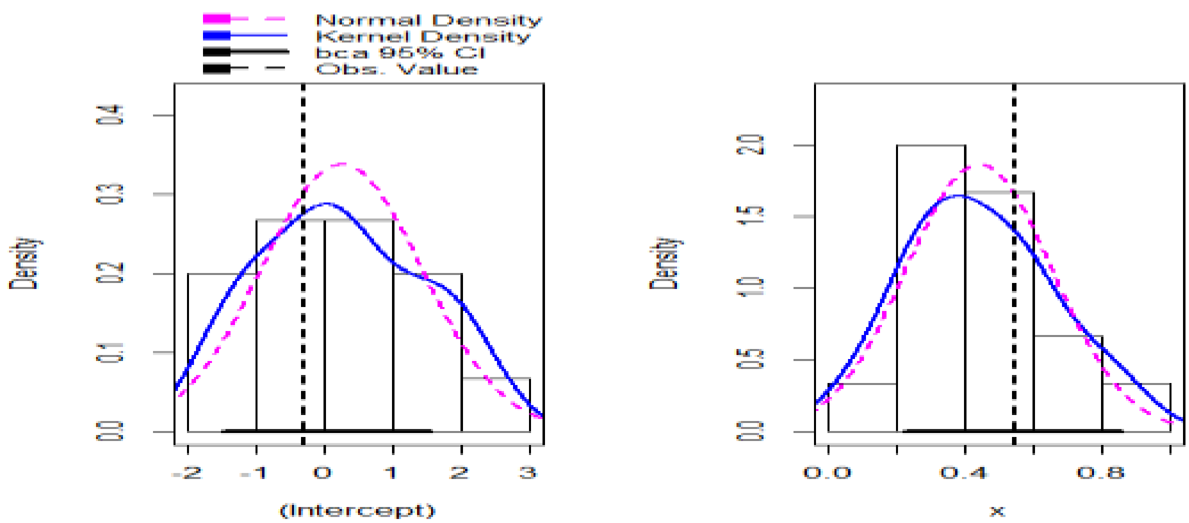

Figure 2: The Histograms of the bootstrap (B=10000) and the jackknife estimates of regression parameter.

The vertical dashed line marks the original point-estimate, and the thick horizontal line gives a confidence interval based on the bootstrap and Jackknife estimates. Whereas the two density estimates for the intercept are similar, the normal approximation is poor for the other coefficient, and confidence intervals are not close to symmetric about the original values. This suggests that inference from the bootstrap is diff erent from the asymptotic theory, and that the bootstrap is likely to be more accurate in this small sample.The standard errors of the bootstrap on the size of farmland owned(SFO) and the size of farmland cultivated(SFC) coefficients in a substantial manner larger than the calculated asymptotic Ordinary Least Square(OLS) standard errors, the reason is because of the inadequateness of the bootstrap in a smaller samples ([9], [14]). Based on the standard errors of the bootstrapping procedure, the confidence intervals are closely the same to that of the percentile intervals of the SFO and SFCcoefficients; thus, based on the bootstrap the Ordinary Least Square standard errors, the confidence intervals are entirelydifferent from the percentile and the confidence intervals based on the standard errors of bootstrap techniques.In comparison of the averages of the bootstrap coefficients $\bar{\hat{\theta}}_{0br}$ and $\bar{\hat{\theta}}_{1br}$ with the suitable Ordinary Least Square estimates(OLS): $\hat{\theta}_0$, and $\bar{\hat{\theta}}_1$ indicates that there is a small bias in thebootstrap coefficients.Here it can be seen that the estimates from both the methods compare well with each other but their standard errors differ significantly with the Bootstrap version higher than that obtained by the least squares procedure. Mong and Wang [19]have proposed a Jackknife based procedure for estimating non-linear regression model parameters which is bias-reducing without increasing the variance. A detailed account on Jackknife, Bootstrap and other resampling procedures in regression analysis can be found in [27].

VI. DISCUSSION AND CONCLUSIONS

The procedures of the bootstrap and jackknife estimate the modifications of a statistic from the varying of that statistic among sub samples, prior from the parametric suppositions that may generate likely results in most cases. Also, they additionally pave a way of reducing bias and getting the standard errors in cases where the standard techniques might be expectedly inappropriate. From the statistical theory of the bootstrap, it is noted that there exists a limited sum of nn possible bootstrap samples. The parameter estimates for each one this nn sample if computed, will get the real bootstrap estimates of parameters unless such radical computation is a waste and not necessary [24]. A suggestion was made on the bootstrap repetitions sufficient to be for the estimation of variance 50 <=B<=100, B≅1000 for the estimation of standard errors, peradventure it may not be enough for the confidenceintervals, ([5], [14], [16]). The number of bootstraprepetitions B rests on the application and the size of sample population and the available computer software.Heltshe and Forrester, [13]as well give report that not only the size of the sample population, unless with the importance in improvement of the jackknife estimators of the total number of individuals in the sample population. However, the bias, standard errors and confidence intervals of jackknife on the SFO and SFC coefficients on the basis of the distribution F $(\hat{\theta}_J)$ are essentially bigger than the bootstrap and the calculated asymptotic ordinary least square standard errors. The percentile intervals of jackknife are as well large compared to the bootstrap percentile intervals of the SFO and SFC coefficients. The application of bootstrap and jackknife techniques wholly depend on the advancement of computer technologies and would be more often used if statistical computer packages possess these analyses. Conclusively, bootstrap technique is more preferred in linear regression due to some of the practical properties such as having distributional assumptions on residuals and if the errors do not have normal distribution, it allows for inference.

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