Computation of Status Indices of Graphs

V.R.Kulli

Department of Mathematics. Gulbarga University, Gulbarga 585106, India

Abstract: *In this paper, we introduce the vertex status index, total status index, modified vertex status index, status inverse degree, status zeroth order index, F-status index, general vertex status index of a graph. Also we propose the total status polynomial vertex status polynomial, F-status polynomial of a graph. We compute exact formulas for certain standard graphs and friendship graphs.*

Keywords: *vertex status index, F-status index, status polynomial, graph.*

Mathematics Subject Classification: 05*C*05, 05*C*07, 05*C*12, 05*C*35.

I. Introduction

Let $G = (V(G), E(G))$ be a simple, finite, connected graph. The degree $d_G(v)$ of vertex v is the number of vertices adjacent to v. The distance $d(u, v)$ between any two vertices u and v is the length of shortest path containing u and v The status $\sigma(u)$ of a vertex u in a graph G is the sum of distances of all other vertices from u in G. For undefined term and notation, we refer [1].

A graph index or topological index is a numerical parameter mathematically derived from graph structure. In Mathematical Chemistry, graph indices have found some applications in chemical documentation, isomer discrimination *QSAR*/*QSPR* study [2, 3, 4]. Some different graph indices may be found in [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17].

In [18], Ramane et al. introduced the first and second status connectivity indices of a graph G defined as

$$S_1(G) = \sum_{uv\in E(G)} [\sigma(u)+\sigma(v)], \qquad S_2(G) = \sum_{uv\in E(G)} \sigma(u)\sigma(v).$$

We introduce the vertex status index of a graph G defined as

$$S_v(G) = \sum_{u\in V(G)} \sigma(u)^2.$$

We now propose the following status indices:

The total status index of a graph G is defined as

$$T_s(G) = \sum_{u\in V(G)} \sigma(u).$$

The modified vertex status index of a graph G is defined as

$${}^mS_v(G) = \sum_{u\in V(G)} \frac{1}{\sigma(u)^2}.$$

The status inverse degree of a graph G is defined as

$$SI(G) = \sum_{u\in V(G)} \frac{1}{\sigma(u)}.$$

The status zeroth order index of a graph G is defined as

$$SZ(G) = \sum_{u\in V(G)} \frac{1}{\sqrt{\sigma(u)}}.$$

The F-status index of a graph G is defined as

$$FS(G) = \sum_{u\in V(G)} \sigma(u)^3.$$

We continue this generalization and propose the general vertex status index of G, defined as

$$S_v^a(G) = \sum_{u\in V(G)} \sigma(u)^a.$$

where a is a real number.

Recently, some variants of status indices were studied in [19,20,21,22,23].

We also introduce the total status polynomial, vertex status polynomial and F-status polynomial of a graph G, defined as

$$T_s(G,x)=\sum_{u\in V(G)} x^{\sigma(u)}.$$

$$S_v(G,x)=\sum_{u\in V(G)} x^{\sigma(u)^2}.$$

$$FS(G,x)=\sum_{u\in V(G)} x^{\sigma(u)^3}.$$

Recently, some different polynomials were studied in [24,25,26,27,28,29,30,31,32,33,34].

In this paper, the vertex status index, total status index, modified vertex status index, F-status index, the general vertex status index of some standard graphs, wheel graphs, friendship graphs are determined. Also the total status polynomial, vertex status polynomial, F-status polynomial of some standard graphs are obtained.

II. Results for complete Graphs

Let K_n be a complete graph with n vertices.

Theorem 1. The general vertex status index of a complete graph K_n is

$$S_v^a(K_n)=n(n-1)^a. \tag{1}$$

Proof: If K_n is a complete graph with n vertices, then $d_{K_n}(u)=n-1$ and $\sigma(u)=n-1$ for any vertex u in K_n. Thus

$$S_v^a(K_n)=\sum_{u\in V(K_n)} \sigma(u)^a = n(n-1)^a.$$

We establish the following results by using Theorem 1.

Corollary 1.1. Let K_n be a complete graph with n vertices. Then

(i) $S_v(K_n)=n(n-1)^2$.

(ii) $T_s(K_n)=n(n-1)$.

(iii) ${}^mS_v(K_n)=\dfrac{n}{(n-1)^2}$.

(iv) $SI(K_n)=\dfrac{n}{n-1}$.

(v) $SZ(K_n)=\dfrac{n}{\sqrt{n-1}}$.

(vi) $FS(K_n)=n(n-1)^3$.

Proof: Put a = 2, 1, –2, –1, –½, 3 in equation (1), we get the desired results.

Theorem 2. The total status polynomial, vertex status polynomial and F-status polynomial of a complete graph K_n are given by

(i) $T_s(K_n,x)=nx^{n-1}$.

(ii) $S_v(K_n,x)=nx^{(n-1)^2}$.

(iii) $FS(K_n,x)=nx^{(n-1)^3}$.

Proof: Let K_n be a complete graph with n vertices. Then $\sigma(u) = n - 1$ for any vertex u in K_n.

(i) $$T_s(K_n,x)=\sum_{u\in V(K_n)} x^{\sigma(u)} = nx^{n-1}.$$

(ii) $$S_v(K_n,x)=\sum_{u\in V(K_n)} x^{\sigma(u)^2} = nx^{(n-1)^2}.$$

(iii) $$FS(K_n,x)=\sum_{u\in V(K_n)} x^{\sigma(u)^3} = nx^{(n-1)^3}.$$

III. Results for Cycles

Theorem 3. If C_n is a cycle with n vertices, then the general vertex status index of C_n is

$$S_v^a(C_n) = n\left(\frac{n^2}{4}\right)^a, \quad \text{if } n \text{ is even,} \qquad (2)$$

$$= n\left(\frac{n^2-1}{4}\right)^a, \quad \text{if } n \text{ is odd.} \qquad (3)$$

Proof: Case 1. Suppose n is even. If C_n is a cycle with n vertices then $\sigma(u)=\frac{n^2}{4}$ for every vertex u of C_n. Thus

$$S_v^a(C_n) = \sum_{u\in V(C_n)} \sigma(u)^a = n\left(\frac{n^2}{4}\right)^a.$$

Case 2. Suppose n is odd. If C_n is a cycle with n vertices, then $\sigma(u)=\frac{n^2-1}{4}$ for every vertex u of C_n. Thus

$$S_v^a(C_n) = \sum_{u\in V(C_n)} \sigma(u)^a = n\left(\frac{n^2-1}{4}\right)^a.$$

We obtain the following results by using Theorem 3.

Corollary 3.1. Let C_n be a cycle with n vertices. Then

(i) $S_v(C_n) = \frac{n^5}{16}$, if n is even,

$= \frac{n(n^2-1)^2}{16}$, if n is odd.

(ii) $T_s(C_n) = \frac{n^3}{4}$, if n is even,

$= \frac{n(n^2-1)}{4}$, if n is odd.

(iii) ${}^mS_v(C_n) = \frac{16}{n^3}$, n is even,

$= \frac{16n}{(n^2-1)^2}$, n is odd.

(iv) $SI(C_n) = \frac{16}{n}$, n is even,

$= \frac{16n}{n^2-1}$, n is odd.

(v) $SZ(C_n) = 2.$ if n is even,

$= \frac{2n}{\sqrt{n^2-1}}$, if n is odd.

(vi) $FS(C_n) = \frac{n^7}{64}$, if n is even,

$= \frac{n(n^2-1)^2}{16}$, if n is odd.

Proof: Put a = 2, 1, –2, –1, –½, 3 in equations (2), (3), we get the desired results.

Theorem 4. The total status polynomial, vertex status polynomial and F-status polynomial of a cycle C_n are given by

(i) $T_s(C_n, x) = nx^{\frac{n^2}{4}}$, if n is even,

$= nx^{\frac{n^2-1}{4}}$, if n is odd.

(ii) $S_v(C_n, x) = nx^{\frac{n^4}{16}}$, if n is even,

$= nx^{\frac{(n^2-1)^2}{16}}$, if n is odd.

(iii) $FS(C_n, x) = nx^{\frac{n^6}{64}}$, if n is even

$= nx^{\frac{(n^2-1)^3}{64}}$, if n is odd.

Proof: Let C_n be a cycle with n vertices.

Case 1. Suppose n is even. Then $\sigma(u) = \frac{n^2}{4}$ for every vertex u in C_n. Thus

(i) $T_s(C_n, x) = \sum_{u \in V(C_n)} x^{\sigma(u)} = nx^{\frac{n^2}{4}}$. (ii) $S_v(C_n, x) = \sum_{u \in V(C_n)} x^{\sigma(u)^2} = nx^{\frac{n^4}{16}}$.

(iii) $FS(C_n, x) = \sum_{u \in V(C_n)} x^{\sigma(u)^3} = nx^{\frac{n^6}{64}}$.

Case 2. Suppose n is odd. Then $\sigma(u) = \frac{n^2-1}{4}$ for every vertex u in C_n. Thus

(i) $T_s(C_n, x) = \sum_{u \in V(C_n)} x^{\sigma(u)} = nx^{\frac{n^2-1}{4}}$. (ii) $S_v(C_n, x) = \sum_{u \in V(C_n)} x^{\sigma(u)^2} = nx^{\frac{(n^2-1)^2}{16}}$.

(iii) $FS(C_n, x) = \sum_{u \in V(C_n)} x^{\sigma(u)^3} = nx^{\frac{(n^2-1)^3}{64}}$.

IV. Results for Complete Bipartite Graphs

Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices and pq edges. In $K_{p,q}$, there are two types of status vertices as given in Table 1.

$\sigma(u) \backslash u \in V(K_{p,q})$	$p + 2(q - 1)$	$q + 2(p - 1)$
Number of vertices	q	p

Table 1. Status vertex of $K_{p,q}$

Theorem 5. The general vertex status index of $K_{p,q}$ is

$$S_v^a(K_{p,q}) = q[p + 2(q-1)]^a + p[q + 2(p-1)]^a. \quad (4)$$

Proof: Let $K_{p,q}$ be a complete bipartite graph. By definition, we have

$$S_v^a(K_{p,q}) = \sum_{u \in V(K_{p,q})} \sigma(u)^a.$$

By using Table 1, we deduce

$$S_v^a(K_{p,q}) = q[p + 2(q-1)]^a + p[q + 2(p-1)]^a.$$

We obtain the following results by using Theorem 5.

Corollary 5.1. If $K_{p,q}$ is a complete bipartite graph, then

(i) $S_v(K_{p,q}) = q[p+2(q-1)]^2 + p[q+2(p-1)]^2$.

(ii) $T_s(K_{p,q}) = 2pq + 2(p^2+q^2) - 2(p+q)$.

(iii) $^mS_v(K_{p,q}) = \frac{q}{(p+2q-2)^2} + \frac{p}{(q+2p-2)^2}$.

(iv) $SI(K_{p,q}) = \frac{q}{p+2q-2} + \frac{p}{q+2p-2}$.

(v) $SZ(K_{p,q}) = \frac{q}{\sqrt{p+2q-2}} + \frac{p}{\sqrt{q+2p-2}}$.

(vi) $SI(K_{p,q}) = q(p+2q-2)^3 + p(q+2p-2)^3$.

Proof: Put a = 2, 1, –2, –1, –½, 3 in equation (4), we get the desired results.

Theorem 6. The total status polynomial, vertex status polynomial and F-status polynomial of $K_{p,q}$ are given by

$$T_s(K_{p,q}, x) = qx^{p+2q-2} + px^{q+2p-2}.$$

$$S_v(K_{p,q}, x) = qx^{(p+2q-2)^2} + px^{(q+2p-2)^2}.$$

$$FS(K_{p,q}, x) = qx^{(p+2q-2)^3} + px^{(q+2p-2)^3}.$$

Proof: Let $K_{p,q}$ be a complete bipartite graph with $p+q$ vertices. Then by using Table 1, we deduce

(i) $T_s(K_{p,q}, x) = \sum_{u \in V(K_{p,q})} x^{\sigma(x)} = qx^{p+2q-2} + px^{q+2p-2}$.

(ii) $S_v(K_{p,q}, x) = \sum_{u \in V(K_{p,q})} x^{\sigma(u)^2} = qx^{(p+2q-2)^2} + px^{(q+2p-2)^2}$.

(iii) $FS(K_{p,q}, x) = \sum_{u \in V(K_{p,q})} x^{\sigma(u)^3} = qx^{(p+2q-2)^3} + px^{(q+2p-2)^3}$.

V. Results for Wheel Graphs

A wheel graph W_n is a join of K_1 and C_n. Clearly W_n has n+1 vertices and 2n edges. A graph W_4 is shown in figure 1.

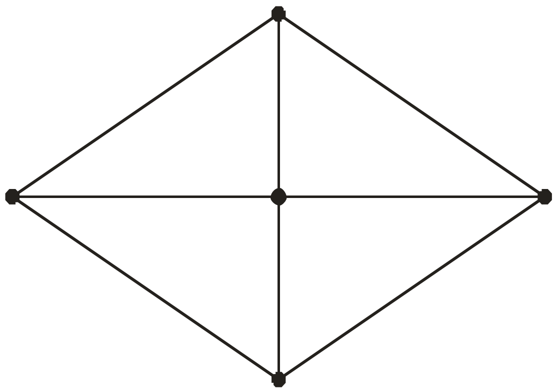

Figure 1. Wheel graph W_4

In W_n , there are two types of status vertices as given in Table 2.

$\sigma(u) \setminus u \in V(W_n)$	n	$2n-3$
Number of edges	1	n

Table 2. Status vertex partition of W_n

Theorem 7. The general vertex status index of a wheel graph W_n is

$$S_v^a(W_n) = n^a + n(2p-3)^a. \quad (5)$$

Proof: Let W_n be a wheel graph with n+1 vertices and 2n edges. By definition and by using Table 2, we derive

$$S_v^a(W_n) = \sum_{u \in V(W_n)} \sigma(u)^a = n^a + n(2p-3)^a.$$

We establish the following results by using Theorem 7.

Corollary 7.1. Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(i) $S_v(W_n) = 4n^3 + 11n^2 + 9n.$ (ii) $T_s(W_n) = 2n^2 - 2n.$

(iii) ${}^m S_v(W_n) = \frac{1}{n^2} + \frac{n}{(2n-3)^2}.$ (iv) $SI\ (W_n) = \frac{1}{n} + \frac{n}{2n-3}.$

(v) $SZ(W_n) = \frac{1}{\sqrt{n}} + \frac{n}{\sqrt{2n-3}}.$ (vi) $FS(W_n) = 8n^4 - 35n^3 + 54n^2 - 27n.$

Proof: Put a = 2, 1, –2, –1, –½, 3 in equation (5), we obtain the desired results.

Theorem 8. Let W_n be a wheel graph with n+1 vertices and 2n edges. Then

(i) $T_s(W_n, x) = x^n + nx^{2n-3}.$

(ii) $S_v(W_n, x) = x^{n^2} + nx^{(2n-3)^2}.$

(iii) $FS(W_n, x) = x^{n^3} + nx^{(2n-3)^3}.$

Proof: By using equations and Table 2, we derive

(i) $T_s(W_n, x) = \sum_{u \in V(W_n)} x^{\sigma(u)} = x^n + nx^{2n-3}.$

(ii) $S_v(W_n, x) = \sum_{u \in V(W_n)} x^{\sigma(u)^2} = x^{n^2} + nx^{(2n-3)^2}.$

(iii) $FS(W_n, x) = \sum x^{\sigma(u)^3} = x^{n^3} + nx^{(2n-3)^3}.$

VI. Results for Friendship Graphs

A friendship graph F_n is the graph obtained by taking $n \geq 2$ copies of C_3 with vertex in common. The graph of F_4 is shown in Figure 2.

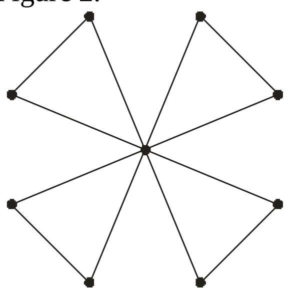

Figure 2. Friendship graph F_4

In F_n, there are two types of status vertices as follows:

$V_1 = \{u \in V(F_n) \mid \sigma(u) = 2n\}$, $\quad |V_1| = 1$.
$V_2 = \{u \in V(F_n) \mid \sigma(u) = 4n-2\}$, $\quad |V_2| = 2n$.

Theorem 9. The general vertex status index of a friendship graph F_n is

$$S_v^a(F_n) = (2n)^a + 2n(2n-2)^a. \tag{6}$$

Proof: Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

$$S_v^a(F_n) = \sum_{u \in V(F_n)} \sigma(u)^a = |V_1|(2n)^a + |V_2|(4n-2)^a$$

$$= (2n)^a + 2n(2n-2)^a.$$

We obtain the following results by using theorem 9.

Corollary 9.1. Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

(i) $S_v(F_n) = 32n^3 - 28n^2 + 8n.$

(ii) $T_s(F_n) = 8n^2 - 2n.$

(iii) ${}^m S_v(F_n) = \dfrac{1}{4n^2} + \dfrac{2n}{(4n-2)^2}.$

(iv) $SI(F_n) = \dfrac{1}{2n} + \dfrac{2n}{4n-2}.$

(v) $SZ(F_n) = \dfrac{1}{\sqrt{2n}} + \dfrac{2n}{\sqrt{4n-2}}$

(vi) $FS(F_n) = 128n^4 - 184n^3 + 96n^2 - 16n.$

Proof: Put $a = 2, 1, -2, -1, -½, 3$ in equation (6), we get the desired results.

Theorem 10. Let F_n be a friendship graph with $2n+1$ vertices and $3n$ edges. Then

(i) $T_s(F_n, x) = x^{2n} + 2nx^{4n-2}.$

(ii) $S_v(F_n, x) = x^{4n^2} + 2nx^{(2n-2)^2}.$

(iii) $FS(F_n, x) = x^{8n^3} + 2nx^{(4n-3)^3}.$

Proof: By using equations, we deduce

(i) $$T_s(F_n, x) = \sum_{u \in V(F_n)} x^{\sigma(u)} = |V_1| x^{2n} + |V_2| x^{4n-2}.$$
$$= x^{2n} + 2nx^{4n-2}.$$

(ii) $$S_v(F_n, x) = \sum_{u \in V(F_n)} x^{\sigma(u)^2} = |V_1| x^{(2n)^2} + |V_2| x^{(4n-2)^2}.$$
$$= x^{4n^2} + 2nx^{(2n-2)^2}.$$

(iii) $$FS(F_n, x) = \sum_{u \in V(F_n)} x^{\sigma(u)^3} = |V_1| x^{(2n)^3} + |V_2| x^{(4n-2)^3}.$$
$$= x^{8n^3} + 2nx^{(4n-3)^3}.$$

References

[1] V.R.Kulli, College Graph Theory, Vishwa International Publications, Gulbarga, India (2012).
[2] I. Gutman and O.E. Polansky, Mathematical Concepts in Organic Chemistry, Springer, Berlin (1986).
[3] V.R.Kulli, Multiplicative Connectivity Indices of Nanostructures, LAP LEMBERT Academic Publishing (2018).
[4] R. Todeschini and V. Consonni, Molecular Descriptors for Chemoinformatics, Wiley-VCH, Weinheim, (2009)
[5] B. Basavanagoud and P. Jakkannavar, Kulli-Basava indices of graphs, Inter. J. Appl. Eng. Research, 14(1) (2018) 325-342.
[6] S. Ediz, Maximal graphs of the first reverse Zagreb beta index, TWMS J. Appl. Eng. Math. 8(2018) 306-310.
[7] V.R.Kulli, The Gourava indices and coindices of graphs, Annals of Pure and Applied Mathematics, 14(1) (2017) 33-38.
[8] V.R.Kulli, On KV indices and their polynomials of two families of dendrimers, Interational Journal of Current Research in Life Sciences,7(9) (2018) 2739-2744.
[9] V.R.Kulli, Dakshayani indices, Annals of Pure and Applied Mathematics, 18(2) (2018) 139-146.
[10] V.R.Kulli, Neighborhood Dakshayani indices, International Journal of Mathematical Archive, 10(7) (2019) 23-31.
[11] V.R.Kulli, Leap Gourava indices of certain windmill graphs, International Journal of Mathematical Archive, 19(11) (2019) 7-14.
[12] V.R.Kulli, Computation of some temperature indices of HC5C7[p,q] nanotubes, Annals of Pure and Applied Mathematics, 20(2) (2019) 69-74.
[13] V.R.Kulli, The (a,b)-KA indices of polycyclic aromatic hydrocarbons and benzenoid systems, International Journal of Mathematics Trends and Technology, 65(11) (2019) 115-120.
[14] V.R.Kulli, Some new topological indices of graphs, International Journal of Mathematical Archive, 10(5) (2019) 62-70.

[15] V.R.Kulli, Some KV indices of certain dendrimers, Earthline Journal of Mathematical Sciences, 2(1) (2019) 69-86.

[16] V.R.Kulli, New connectivity topological indices, Annals of Pure and Applied Mathematics,20(1) (2019) 1-8.

[17] I. Gutman, V.R. Kulli, B. Chaluvaraju and H. S. Boregowda, On Banhatti and Zagreb indices, Journal of the International Mathematical Virtual Institute, 7 (2017) 53-67.

[18] H.S. Ramane and A.S. Yalnaik, Status connectivity indices graphs and its applications to the boiling point of benzenoid hydrocarbons, Journal of Applied Mathematics and Computing, 55 (2017) 607-627.

[19] H.S. Ramane, B. Basavanagoud and A.S. Yalnaik, Harmonic status index of graphs, Bulletin of Mathematical Sciences and Applications, 17(2016) 24-32.

[20] H.S.Ramane, A.S. Yalnaik and R. Sharafdini, Status connectivity indices and coindices of graphs and its computation to some distance balance graphs, AKCE International Journal of Graphs and Combinatorics, (2018) https://doi.org/10.101bj.akeej.2018.09.002.

[21] V.R.Kulli, Some new status indices of graphs, International Journal of Mathematics Trends and Technology, 10(10) (2019) 70-76.

[22] V.R.Kulli, Some new multiplicative status indices of graphs, International Journal of Recent Scientific Research, 10(10) (2019) 35568-35573.

[23] V.R.Kulli, Multiplicative status indices of graphs, submitted.

[24] V.R.Kulli, On augmented Revan index and its polynomial of certain families of benzenoid systems, International Journal of Mathematics and its Applications, 6(4) (2018) 43-50.

[25] V.R.Kulli, Reduced second hyper-Zagreb index and its polynomial of certain silicate networks, Journal of Mathematics and Informatics, 14 (2018) 11-16.

[26] V.R.Kulli, Leap hyper-Zagreb indices and their polynomials of certain graphs, International Journal of Current Research in Life Sciences, 7(10) (2018) 2783-2791.

[27] V.R.Kulli, Square reverse index and its polynomial of certain networks, International Journal of Mathematical Archive, 9(10) (2018) 27-33.

[28] V.R.Kulli, F-Revan index and F-Revan polynomial of some families of benzenoid systems, Journal of Global Research in mathematical Archives, 5(11) (2018) 1-6.

[29] V.R.Kulli, Minus leap and square leap indices and their polynomials of some special graphs, International Research Journal of Pure Algebra, 8(10) (2018) 54-60.

[30] V.R.Kulli, On F-leap indices and F-leap polynomials of some graphs, International Journal of Mathematical Archive, 9(12) (2018) 41-49.

[31] V.R.Kulli, Computing square Revan index and its polynomial of certain benzenoid systems, International Journal of Mathematics and its Applications, 6(4) (2018) 213-219.

[32] V.R.Kulli, On hyper KV and square KV indices and their polynomials of certain families of dendrimers. Journal of Computer and Mathematical Sciences, 10(2) (2019) 279-286.

[33] V.R.Kulli, Minus F and square F-indices and their polynomials of certain dendrimers, Earthline Journal of Mathematical Sciences, 1(2) (2019) 171-185.

[34] V.R.Kulli, On augmented leap index and its polynomial of some wheel type graphs, International Research Journal of Pure Algebra, 9(4) (2019) 1-7.