Hydromagnetic Instability of Rivlin-Ericksen Dusty Fluid In Porous Medium : Effect of Inertia

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Abstract:- *Hydromagnetic instability of Rivlin-Ericksen dusty fluid in porous medium : effect of inertia statically unstable system is found.and also analysed sufficient condition for stability of stable system.*

I. INTRODUCTION

The problem finds its usefulness in petroleum engineering, paper and pulp technology and several geophysical situations. Sharma and Sharma [1] and Sharma and Rani [2] have made a successful attempt to find the effect of suspended dust particles on thermosolutal convection in porous medium. They have shown the effect of suspended dust particles to be destabilizing. Sharma and Kango [3] studied the thermal convection in Rivlin-Ericksen elastico-viscous fluid in porous medium in the presence of uniform magnetic field.

In a recent study, Prakash and Kumar [4] studied the thermal instability in Rivlin-Ericksen elastico-viscous fluid in the presence of Larmor radius and variable gravity in porous medium. Sharma et al. [5] have discussed by stability of stratified Rivlin-Eicksen fluid particle mixture in hydromagnetic in porous medium. Sharma and Kumar [6] have discussed the effect of suspended particles on thermal instability in Rivlin-Ericksen elastico-viscous medium. Kumar [7] has discussed the Rayleigh-Taylor instability of Rivlin-Ericksen elastico-viscous fluids in presence of suspended particles through porous medium. The stability of the plane interface separating two superposed visco-elastic fluids through porous medium has been investigated in a uniform two-dimensional horizontal magnetic field has been discussed by Khan and Bhatia [8].

The problem has been extensively investigated under various physical situations (such as for an electrically conducting fluid in the presence of a magnetic field, thermally conducting fluid with temperature variation and instability problem through porous medium, etc.). Sharma and Rana [9] have discussed the thermosolutal instability of Rivlin-Ericksen rotating fluid in the presence of magnetic field and variable gravity field in porous medium and found that stable solute gradient has a stabilizing effect on the system while the magnetic field and have stabilizing effect under certain condition. Gupta and Sharma [10] have studied Rivlin-Ericksen elastico-viscous fluid heated and soluted from below in the presence of compressibility rotation and Hall currents.

Kumar and Singh [11] studied the stability of the plane interface separating two visco-elastic superposed fluid in the presence of suspended particles. The stability analysis has been carried out, for mathematical simplicity, for highly visco-elastic fluids of equal kinematic viscosities and equal kinematic visco-elasticities. The system is found to be stable configuration and unstable for unstable configuration. Kumar et al. [12] studied by the thermal instability of Walters B' viscoelastic fluid permeated with suspended particles in hydromagnetics in porous medium. Kumar et al. [13] the thermal instability of a rotating Rivlin-Ericksen visco-elastic fluid in the presence of uniform vertical magnetic field has both the stabilizing and destabilizing effects.

In this paper, therefore, we have made an attempt to critically examine the effect of inertia on the hydromagnetic instability of Rivlin-Ericksen dusty fluid in porous medium. It can be looked upon as an extension of shear flow instability of gas in a porous medium effect of weak applied magnetic field discussed by Jaimala [14].

II. EQUATIONS OF MOTION

In the equations of motion for the gas, the presence of particles adds an extra force term proportional to the velocity difference between particles and gas. Assuming that the usual viscous dissipation along with the dissipation due to Darcy resistance is present, the governing equations for the gas can be written as

$$\nabla.\mathbf{u} = 0 \tag{1}$$

$$\frac{\rho}{\phi}\frac{\partial \mathbf{u}}{\partial t} + \frac{\rho}{\phi^2}(\mathbf{u}.\nabla)\mathbf{u} = -\nabla p - \frac{\mu \mathbf{u}}{K} + \frac{1}{\mu_0}(\nabla\times\mathbf{H})\times\mathbf{H} + \left(\mu + \mu'\frac{\partial}{\partial t}\right)\nabla^2\mathbf{u} + \frac{1}{\phi}K_1 N(\mathbf{v}-\mathbf{u}) - g\rho\boldsymbol{\lambda}, \tag{2}$$

$$\frac{\partial \rho}{\partial t} + \frac{1}{\phi}\mathbf{u}.\nabla\rho = 0, \tag{3}$$

$$\frac{D\mathbf{H}}{Dt} = \frac{1}{\phi}(\mathbf{H}.\nabla)\mathbf{u} - \frac{1}{\phi}(\nabla.\mathbf{u})\mathbf{H} \tag{4}$$

and $$\nabla.\mathbf{H} = 0 \tag{5}$$

where $\mathbf{u}$ and $\mathbf{v}$ define the velocity fields for fluid and particle passes respectively, ϕ and K respectively the constant porosity and constant permeability of the porous medium, N is the number density of fluid particles and $K_1 = 6\pi n\mu$, where n is the constant particle radius. Remaining physical quantities have their usual meaning.

If m is the mass of each particle, the equation of continuity and equation of motion under the above assumption, are

$$\nabla.\mathbf{v}=0 \tag{6}$$

and $$mN\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}.\nabla)\mathbf{v}=K_1N(\mathbf{u}-\mathbf{v})\right]. \tag{7}$$

All the equation from in Cartesian form are given by

$$\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}+\frac{\partial u_3}{\partial z}=0 \tag{.8}$$

$$\frac{\rho}{\in}\frac{\partial u_1}{\partial t}+\frac{1}{\phi^2}\left(u_1\frac{\partial u_1}{\partial x}+u_2\frac{\partial u_1}{\partial y}+u_3\frac{\partial u_1}{\partial z}\right)=-\frac{\partial p}{\partial x}-\frac{\rho\nu u_1}{K}+\frac{1}{\mu_0}\left[H_z\left(\frac{\partial H_z}{\partial x}-\frac{\partial H_x}{\partial z}\right)\right]$$

$$-H_y\left[\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}\right]+\rho\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 u_1+\frac{1}{\phi}K_1N(\mathrm{v}_1-u_1) \tag{9}$$

$$\frac{\rho}{\in}\frac{\partial u_2}{\partial t}+\frac{1}{\phi^2}\left(u_1\frac{\partial u_2}{\partial x}+u_2\frac{\partial u_2}{\partial y}+u_3\frac{\partial u_2}{\partial z}\right)=-\frac{\partial p}{\partial x}-\frac{\rho\nu u_2}{K}+\frac{1}{\mu_0}\left[H_x\left(\frac{\partial H_y}{\partial x}-\frac{\partial H_x}{\partial y}\right)\right]$$

$$-H_z\left[\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}\right]+\rho\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 u_2+\frac{1}{\phi}K_1N(\mathrm{v}_2-u_2) \tag{10}$$

$$\frac{\rho}{\in}\frac{\partial u_3}{\partial t}+\frac{1}{\phi^2}\left(u_1\frac{\partial u_3}{\partial x}+u_2\frac{\partial u_3}{\partial y}+u_3\frac{\partial u_3}{\partial z}\right)=-\frac{\partial p}{\partial z}-\frac{\rho\nu u_3}{K}-g\rho+\frac{1}{\mu_0}\left[H_y\left(\frac{\partial H_z}{\partial y}-\frac{\partial H_y}{\partial z}\right)\right]$$

$$-H_x\left[\frac{\partial H_x}{\partial z}-\frac{\partial H_z}{\partial x}\right]+\rho\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 u_3+\frac{1}{\phi}K_1N(\mathrm{v}_3-u_3) \tag{11}$$

$$\frac{\partial \rho}{\partial t}+\frac{1}{\phi}\left(u_1\frac{\partial \rho}{\partial x}+u_2\frac{\partial \rho}{\partial y}+u_3\frac{\partial \rho}{\partial z}\right)=0 \tag{12}$$

$$\frac{\partial H_x}{\partial t}+\frac{1}{\phi}\left(u_1\frac{\partial H_x}{\partial x}+u_2\frac{\partial H_x}{\partial y}+u_3\frac{\partial H_x}{\partial z}\right)=\frac{1}{\phi}\left(H_x\frac{\partial u_1}{\partial x}+H_y\frac{\partial u_1}{\partial y}+H_z\frac{\partial u_1}{\partial z}\right)$$

$$-\frac{1}{\phi}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}+\frac{\partial u_3}{\partial z}\right)H_x \tag{13}$$

$$\frac{\partial H_y}{\partial t}+\frac{1}{\phi}\left(u_1\frac{\partial H_y}{\partial x}+u_2\frac{\partial H_y}{\partial y}+u_3\frac{\partial H_y}{\partial z}\right)=\frac{1}{\phi}\left(H_x\frac{\partial u_2}{\partial x}+H_y\frac{\partial u_2}{\partial y}+H_z\frac{\partial u_2}{\partial z}\right)$$

$$-\frac{1}{\phi}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}+\frac{\partial u_3}{\partial z}\right)H_y \tag{14}$$

$$\frac{\partial H_z}{\partial t}+\frac{1}{\phi}\left(u_1\frac{\partial H_z}{\partial x}+u_2\frac{\partial H_z}{\partial y}+u_3\frac{\partial H_z}{\partial z}\right)=\frac{1}{\phi}\left(H_x\frac{\partial u_3}{\partial x}+H_y\frac{\partial u_3}{\partial y}+H_z\frac{\partial u_3}{\partial z}\right)$$

$$-\frac{1}{\phi}\left(\frac{\partial u_1}{\partial x}+\frac{\partial u_2}{\partial y}+\frac{\partial u_3}{\partial z}\right)H_z \tag{15}$$

$$\frac{\partial H_x}{\partial x}+\frac{\partial H_y}{\partial y}+\frac{\partial H_z}{\partial z}=0 \tag{16}$$

$$\frac{\partial \mathrm{v}_1}{\partial x}+\frac{\partial \mathrm{v}_2}{\partial y}+\frac{\partial \mathrm{v}_3}{\partial z}=0 \tag{17}$$

$$mN\left[\frac{\partial \mathrm{v}_1}{\partial t}+\left(\mathrm{v}_1\frac{\partial}{\partial x}+\mathrm{v}_2\frac{\partial}{\partial y}+\mathrm{v}_3\frac{\partial}{\partial z}\right)\mathrm{v}_1\right]=K_1N(u_1-\mathrm{v}_1) \tag{18}$$

$$mN\left[\frac{\partial \mathrm{v}_2}{\partial t}+\left(\mathrm{v}_1\frac{\partial}{\partial x}+\mathrm{v}_2\frac{\partial}{\partial y}+\mathrm{v}_3\frac{\partial}{\partial z}\right)v_2\right]=K_1N(u_2-\mathrm{v}_2) \tag{19}$$

and

$$mN\left[\frac{\partial \mathrm{v}_3}{\partial t}+\left(\mathrm{v}_1\frac{\partial}{\partial x}+\mathrm{v}_2\frac{\partial}{\partial y}+\mathrm{v}_3\frac{\partial}{\partial z}\right)\mathrm{v}_3\right]=K_1N(u_3-\mathrm{v}_3) \tag{20}$$

III. BASIC STATE AND PERTURBATION EQUATION

Basic flow occupies a porous medium bounded by two rigid parallel plates situated at $z=0$ and $z=d$. Fine dust particles are uniformly distributed in the fluid and it is subjected to a weak applied uniform magnetic field in the flow direction. The basic state is characterized by

$$\left.\begin{aligned}&\mathbf{u}=[U(z),0,0],\\&\mathbf{v}=[U(z),0,0],\\&\mathbf{H}=[H_0(uniform),0,0],\\&\rho=\rho(z),\\ \text{and}\quad &p=p(z)\end{aligned}\right\}. \tag{21}$$

The governing equations (.8) to (20) require that pressure distribution is governed by the equation

$$\frac{\partial p}{\partial z}+g\rho=0 \tag{22}$$

while the basic velocity profile is given by

$$U = pK\left[\frac{\left(1-e^{-d/\sqrt{K}}(e^{z/\sqrt{K}}+e^{d/\sqrt{K}}.e^{-z/\sqrt{K}}\right)}{\left(e^{d/\sqrt{K}}-e^{-d/\sqrt{K}}\right)}-1\right] \tag{23}$$

where p is the constant pressure gradient in x-direction.

Now, let the basic state described by (5.3.1)-(5.3.3) be slightly perturbed, so that after perturbation are introduced, the velocity, magnetic field, density and pressure become

$$\left.\begin{aligned} \mathbf{u} &= [U(z)+u_1^{'},\ u_2^{'}, u_3^{'}], \\ \mathbf{v} &= [U(z)+\mathrm{v}_1^{'}, \mathrm{v}_2^{'}, \mathrm{v}_3^{'}], \\ \mathbf{H} &= [H_0+h_x, h_y, h_z], \\ \rho &= \rho(z)+\delta\rho' \\ \text{and}\quad p &= p(z)+\delta p' \end{aligned}\right\}. \tag{24}$$

The following linearized equations are obtained in perturbations :

$$\frac{\partial u_1^{'}}{\partial x}+\frac{\partial u_2^{'}}{\partial y}+\frac{\partial u_3^{'}}{\partial z}=0 \tag{25}$$

$$\frac{\rho}{\phi}\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial x}\right]u_1^{'}+\frac{\rho}{\phi^2}\frac{\partial U}{\partial z}u_3^{'}=-\frac{\partial\delta p'}{\partial x}-\frac{\nu\rho}{K}u_1^{'}+\rho\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 u_1^{'}$$
$$+\frac{1}{\phi}K_1N(\mathrm{v}_1^{'}-u_1^{'}), \tag{26}$$

$$\frac{\rho}{\phi}\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial y}\right]u_2^{'}=-\frac{\partial\delta p'}{\partial x}-\frac{\nu\rho u_2^{'}}{K}+\rho\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 u_2^{'}$$
$$+\frac{H_0}{\mu_0}\left(\frac{\partial h_y^{'}}{\partial x}-\frac{\partial h_x^{'}}{\partial y}\right)+\frac{1}{\phi}K_1N(\mathrm{v}_2^{'}-u_2^{'}), \tag{27}$$

$$\frac{\rho}{\phi}\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial z}\right]u_3^{'}=-\frac{\partial\delta p'}{\partial z}-\frac{\nu\rho u_3^{'}}{K}+\rho\left(\nu+\nu'\frac{\partial}{\partial t}\right)\nabla^2 u_3^{'}$$
$$+\frac{H_0}{\mu_0}\left(\frac{\partial h_z^{'}}{\partial x}-\frac{\partial h_x^{'}}{\partial z}\right)+\frac{1}{\phi}K_1N(\mathrm{v}_3^{'}-u_3^{'})-g\delta\rho' \ , \tag{28}$$

$$\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial x}\right]\delta\rho'+u_3^{'}\frac{d\rho'}{dz}=0, \tag{29}$$

$$\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial x}\right]h_x^{'}=\frac{1}{\phi}h_z\left(\frac{dU}{dz}\right)-\frac{1}{\phi}H_0\frac{d}{dz}u_3^{'}, \tag{30}$$

$$\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial x}\right]h_y^{'}=-\frac{1}{\phi}H_0\frac{du_2^{'}}{dx}, \tag{31}$$

$$\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial x}\right]h_z^{'}=-\frac{1}{\phi}H_0\frac{du_3^{'}}{dx}, \tag{32}$$

$$\frac{\partial h_x^{'}}{\partial x}+\frac{\partial h_y^{'}}{\partial y}+\frac{\partial h_z^{'}}{\partial z}=0, \tag{33}$$

$$\frac{\partial \mathrm{v}_1^{'}}{\partial x}+\frac{\partial \mathrm{v}_2^{'}}{\partial y}+\frac{\partial \mathrm{v}_3^{'}}{\partial z}=0, \tag{34}$$

$$\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial x}\right]\mathrm{v}_1^{'}+\frac{1}{\phi}\frac{dU}{dz}\mathrm{v}_3^{'}=\frac{u_1^{'}-\mathrm{v}_1^{'}}{T} \tag{35}$$

$$\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial x}\right]\mathrm{v}_2^{'}=\frac{u_2^{'}-\mathrm{v}_2^{'}}{T} \tag{36}$$

$$\left[\frac{\partial}{\partial t}+\frac{U}{\phi}\frac{\partial}{\partial x}\right]\mathrm{v}_3^{'}=\frac{u_3^{'}-\mathrm{v}_3^{'}}{T} \tag{37}$$

where, $T=\dfrac{m}{K_1}$.

For the present problem also, it is easy to confirm that three dimensional disturbance problem equivalent to a two-dimensional disturbance problem. Decomposing the disturbances into normal modes, we assume the dependence of any perturbation quantity $f\,'(x, z, t)$ on x, z and t in the form.

$$f(z)\exp\left[ik\left(x-\frac{c}{\phi}t\right)\right], \tag{38}$$

where k is the real wave number and c, in general, is complex. With the above mentioned space time dependence of perturbation quantities, the equations (25) to (27) reduce to

$$iku_1+Du_3=0, \tag{39}$$

$$\frac{i\rho k}{\phi^2}(U-c)u_1+\frac{\rho}{\phi^2}(DU)u_3=-ik\delta p-\frac{\nu\rho u_1}{K}+\rho\left(\nu+i\nu'\frac{k}{t}c\right)(D^2-k^2)u$$
$$+\frac{1}{\phi}K_1N(\mathrm{v}_1-u_1),$$

or
$$\frac{i\rho k}{\phi^2}(U-c)u_1+\frac{\rho}{\phi^2}(DU)u_3=-ik\delta p-\frac{\nu\rho u_1}{K}+\rho\left(\nu-\frac{ikc\nu'}{\phi}\right)(D^2-k^2)u+\frac{1}{\phi}K_1N(\mathrm{v}_1-u_1) \qquad (40)$$

$$\frac{i\rho k}{\phi^2}(U-c)u_1+\frac{\rho}{\phi^2}(DU)u_3=-ik\delta p-\frac{\nu\rho u_1}{K}+\rho\left(\nu-\frac{ikc\nu'}{\phi}\right)(D^2-k^2)u+\frac{1}{\phi}K_1N(\mathrm{v}_1-u_1) \qquad (41)$$

$$\frac{i\rho k}{\phi^2}(U-c)u_3=-D\delta p-\frac{\nu\rho u_3}{K}+\frac{H_0}{\mu_0}(ikh_z-Dh_x)+\rho\left(\nu-\frac{ikc}{\phi}\nu'\right)(D^2-k^2)u_3+\frac{1}{\phi}K_1N(\mathrm{v}_3-u_3)-g\delta\rho, \qquad (42)$$

$$ik(U-c)\delta\rho+(D\rho)u_3=0, \qquad (43)$$

$$ik(U-c)h_x=-H_0Du_3+(DU)h_z, \qquad (44)$$

$$ik(U-c)h_y=ikH_0u_2, \qquad (45)$$

$$ik(U-c)h_z=ikH_0u_3, \qquad (46)$$

$$ikh_x+Dh_z=0, \qquad (47)$$

$$ik(U-c)\mathrm{v}_1+\frac{1}{\phi}(DU)\mathrm{v}_3=\frac{u_1-\mathrm{v}_1}{T} \qquad (48)$$

and
$$\frac{ik}{\phi}(U-c)\mathrm{v}_3=\frac{u_3-\mathrm{v}_3}{T}. \qquad (49)$$

Equation (49) yields

$$\mathrm{v}_3=\frac{u_3}{1+\frac{ik}{\phi}(U-c)T}$$

$$\Rightarrow \qquad \mathrm{v}_3=\frac{u_3}{A}, \text{ where } A=1+\frac{ik}{\phi}(U-c)T.$$

Using this value of v_3 in equation (48), we get

$$\mathrm{v}_1 = \frac{u_1}{A} - \frac{T(DU)}{\phi A^2} u_3 . \tag{50}$$

Further, we eliminate various physical quantities from equation (47) to (50) with the help of (50) and get one equation in one variable u_3 alone. Thus the final stability governing equation is given by

$$\frac{\rho k^2}{\phi^2}(U-c)\left(1+\frac{f}{A}\right)u_3 = D\left\{\frac{\rho}{\phi^2}(U-c)\left(1+\frac{f}{A}\right)Du_3 - \frac{\rho}{\phi^2}(DU)\left(1+\frac{f}{A}\right)u_3 - \frac{iv\rho}{Kk}Du_3 - \frac{i\rho}{k}\left(v-\frac{ikc}{\phi}v'\right)(D^2-k^2)u_3\right\} + \frac{iv\rho k}{K}u_3 - \frac{H_0^2}{\mu_0}(D^2-k^2)\left(\frac{u_3}{U-c}\right) - ik\rho\left(v-\frac{ikc}{\phi}v'\right)(D^2-k^2)u_3 - \frac{g(D\rho)u_3}{U-c} \tag{51}$$

where, $f = \dfrac{mN}{\rho}$.

Now, using the non-dimensional quantities defined by

$$\left.\begin{aligned} &(U, c, u_3) = U_0(U^*, c^*, u_3^*), \\ &(k, D) = \frac{1}{d}(k^*, D^*) \\ \text{and}\quad &T = \frac{d}{U_0}T^* \end{aligned}\right] \tag{52}$$

and dropping the (*) for convenience, equation after dropping stars, becomes

$$\rho k^2(U-c)\left(1+\frac{f}{A}\right)u_3 = D\left[\rho\left\{(U-c)\left(1+\frac{f}{A}\right)Du_3 - iR_D^{-1}k^{-1}Du_3 - (DU)\left(1+\frac{f}{A}\right)u_3 + (ik^{-1}R_e^{-1} + cR_{v_e}^{-1})(D^2-k^2)Du_3\right\}\right] + ik\rho R_D^{-1}u_3 - \rho S(D^2-k^2)\left(\frac{u_3}{U-c}\right) - \rho(ik^{-1}R_e^{-1} + cR_{v_e}^{-1})k^2(D^2-k^2)u_3 + \frac{\rho J}{(U-c)}u_3 \tag{53}$$

where, $R_D^{-1} = \dfrac{v\phi^2 d}{kU_0}$, inverse of the Darcy-Reynolds number

$R_e^{-1} = \dfrac{v\phi^2}{U_0 d}$, inverse of the Reynolds number

$R_{v_e}^{-1} = \dfrac{v'\phi}{d^2}$,

$J = -\dfrac{gD\rho\phi^2}{U_0^2\rho}$,

$S = \dfrac{H_0^2\phi^2}{\mu_0\rho U_0^2}$ is the magnetic force number and is a function of z because of the presence of ρ

and $A = 1 + \dfrac{ik}{\phi}(U - c)T$.

As already pointed out in the introduction of this chapter, the problem will be discussed for fine dust particles under the assumption that the applied magnetic field is weak.

Now, if the dust is fine, then the relaxation time T is small, i.e., $T << 1$. Taking $T = 0$, so that

$$A = 1 + \frac{ik}{\phi}(U - c)T = 1.$$

Under these approximation, equation reduces to

$$D\left[\rho\left\{(U-c)(1+f)Du_3 - iR_D^{-1}k^{-1}Du_3 - (DU)(1+f)u_3\right\}\right]$$
$$-\rho k^2(U-c)(1+f)u_3 + i\rho kR_D^{-1}u_3 - \rho(ik^{-1}R_e^{-1} + cR_{v_e}^{-1})(D^2-k^2)^2 u_3$$
$$+\frac{\rho Sk^2}{(U-c)}u_3 + \frac{\rho J}{(U-c)}u_3 = 0. \qquad (54)$$

The necessary boundary conditions are

$$u_3 = 0 = Du_3 \text{ at } z = 0 \text{ and } z = 1. \qquad (55)$$

IV. STATICALLY UNSTABLE SYSTEM $(D\rho > 0)$

This section deals with the stability of the system under the assumption that $D\rho > 0$ so that the density increases in the vertically upward direction and $J < 0$.

Discussion for Large Wave Numbers

Multiplying equation (54) by u_3^*, integrating the resulting equation over the vertical range of z and making use of the boundary conditions (55), we get

$$-\int \rho(U-c)(1+f)(|Du_3|^2+k^2|u_3|^2)dz + i\int R_D^{-1}k^{-1}\rho((|Du_3|^2+k^2|u_3|^2)dz$$

$$+\int \rho(DU)(1+f)u_3Du_3^*dz + i\int \rho R_e^{-1}k^{-1}(|D^2u_3|^2+2k^2|Du_3|^2+k^4|u_3|^2)dz$$

$$+\int cR_{v_e}^{-1}\rho(|D^2u_3|^2+2k^2|Du_3|^2+k^4|u_3|^2)dz + \int \frac{\rho J}{(U-c)}|u_3|^2\,dz$$

$$+\int \frac{k^2\rho S}{(U-c)}|u_3|^2\,dz = 0. \tag{56}$$

The imaginary part of equation after using Schwarz's inequality, yields

$$c_i\int \rho(|Du_3|^2+k^2|u_3|^2)dz + \int \frac{R_D^{-1}k^{-1}\rho}{(1+f)}(|Du_3|^2+k^2|u_3|^2)dz$$

$$-\frac{q}{2k}\int (|Du_3|^2+k^2|u_3|^2)dz + \int \frac{R_D^{-1}k^{-1}\rho}{(1+f)}(|D^2u_3|^2+2k^2|Du_3|^2+k^4|u_3|^2)dz$$

$$+c_i\int \frac{\rho R_{v_e}^{-1}}{(1+f)}(|D^2u_3|^2+2k^2|Du_3|^2+k^4|u_3|^2)dz$$

$$+c_i\int \frac{\rho}{(1+f)}\frac{(J+k^2S)}{|U-c|^2}|u_3|^2\,dz \le 0 \tag{57}$$

where, $w = \max.|\rho DU|$.

For $J<0$, inequality (57) can be written as

$$c_i\int \rho(|Du_3|^2+k^2|u_3|^2)dz + \int \frac{1}{k}\left(\frac{\rho R_D^{-1}}{(1+f)} - \frac{q}{2}\right)(|Du_3|^2+k^2|u_3|^2)dz$$

$$+\int \frac{R_e^{-1}k^{-1}\rho}{(1+f)}(|D^2u_3|^2+2k^2|Du_3|^2+k^4|u_3|^2)dz$$

$$+c_i\int \frac{\rho R_{v_e}^{-1}}{(1+f)}(|D^2u_3|^2+2k^2|Du_3|^2+k^4|u_3|^2)dz$$

$$+c_i\frac{\int \frac{\rho}{(1+f)}(k^2S-|J|)|u_3|^2\,dz}{|U-c|^2} \le 0. \tag{58}$$

If $\frac{2\rho R_D^{-1}}{q(1+f)} \geq 1$, everywhere in the flow region, (59)

then it follows from that for the wave number

$$k^2 \geq \frac{|J|}{S} \tag{60}$$

$c_i < 0$ necessarily implying thereby that the system is stable.

V. A Particular Case

If the density and velocity profiles are such that

everywhere in (0, 1).

Then it has been shown that for some wave numbers, the modes are oscillatory.

The real part of equation , on using the fact that

$$\int f(z)\, u_3\, Du_3^* dz = \frac{1}{2}\int f'(z)\,|u_3|^2\, dz$$

where $f(z)$ is a real function, becomes

$$\int \rho(U-c_r)(|Du_3|^2 + k^2|u_3|^2)dz + \frac{1}{2}\int D(\rho DU)|u_3|^2\, dz - \frac{c_r R_{v_e}^{-1}}{1+f}\int \rho(|D^2u_3|^2$$

$$+2k^2|Du_3|^2 + k^4|u_3|^2)dz - \int \frac{\rho S k^2 (U-c_r)}{(1+f)|U-c|^2}|u_3|^2\, dz$$

$$-\int \frac{\rho J(U-c_r)|u_3|^2\, dz}{(1+f)(U-c)^2} = 0\,. \tag{61}$$

For $J < 0$, the terms in equation (61) are rearranged as

$$\int \rho(U-c_r)(|Du_3|^2 + k^2|u_3|^2)dz + \frac{1}{2}\int D(\rho DU)|u_3|^2\, dz - \frac{c_r R_{v_e}^{-1}}{(1+f)}\int \rho(|D^2u_3|^2$$

$$+2k^2|Du_3|^2 + k^4|u_3|^2)dz + \int \frac{\rho(|J| - k^2 S)(U-c_r)}{(1+f)|U-c|^2}|u_3|^2\, dz = 0\,. \tag{62}$$

It follows that if

$D(\rho DU) \geq 0$ and $k^2 < \frac{|J|}{S}$ anywhere in (0, 1), then c_r is non-zero necessarily which guarantees the existence of oscillatory modes.

VI. Bounds on c_r for $D(\rho DU) = 0$:

It we are restricted to the case, where the validity of the condition

$D(\rho DU) = 0$ everywhere in (0, 1) (63)

Therefore, (ρU) = Constant

Equation holds when the density and velocity profiles are chosen of the exponential type as given below

$$\left.\begin{aligned} &\rho(z) = \rho_0 e^{\alpha z} \\ \text{and} \quad &U(z) = U_0 e^{-\alpha z}, \ \alpha > 0 \end{aligned}\right\} \quad (64)$$

In view of the situation explained above, we should either take

$$\nu(z) = \nu_0 \ e^{\alpha z}$$

so that ν is constant.

or α should be taken as a very small number so that $e^{\alpha z}$ becomes a slowly varying function of z. Then if $U_{\max}$ and $U_{\min}$ be respectively the upper and lower bounds of U in the flow domain,

$$\int \left[\rho(U - c_r)(|Du_3|^2 + k^2 |u_3|^2) + \frac{|J| - k^2 S}{(1+f)|U-c|^2} |u_3|^2 \right] dz + \frac{c_r R_{v_e}^{-1}}{(1+f)} \int \rho(|D^2 u_3|^2$$

$$+2k^2 |Du_3|^2 + k^4 |u_3|^2) dz = 0$$

which for the wave numbers

$$k^2 < \frac{|J|}{S}$$

show that $U - c_r$ must vanish somewhere in the flow domain on the other words

$$U_{\min} < c_r < U_{\max}$$

This result was proved by Miles [15] and Howard [16] for a non-porous medium. Miles assumed in his proof the monoatomicity of the velocity profile together with the analyticity of the velocity and density profiles. While Howard [16] obtained the same result without imposing any restriction on density ad velocity profiles like Miles [15], it is to be noted that whereas Miles [15] and Howard [16] were able to prove this result for arbitrary velocity and density profiles irrespective of whether the system is statically stable or statically unstable, we have only obtained for the case when $\rho(z)$ and $U(z)$ are both exponential given by and the system is statically unstable.

VII. STABILITY STABLE SYSTEM $(D\rho < 0)$:

In the following analysis, some results are obtained for the statically stable arrangement of the fluid.

Sufficient Conditions for Stability

Inequality can be rearranged as follows :

$$c_i \int \rho(|Du_3|^2 + k^2|u_3|^2)dz + \int \frac{1}{k}\left(\frac{\rho R_D^{-1}}{(1+f)} - \frac{q}{2}\right)(|Du_3|^2 + k^2|u_3|^2)dz$$

$$+\int \frac{\rho(R_e^{-1}k^{-1} + c_i R_{v_e}^{-1})}{(1+f)}(|D^2u_3|^2 + 2k^2|Du_3|^2 + k^4|u_3|^2)dz$$

$$+ c_i \int \frac{\rho}{(1+f)} \frac{(J + k^2 S)}{|U - c|^2} |u_3|^2 \, dz \le 0. \qquad (65)$$

Clearly, if the condition

$$\frac{2\rho R_D^{-1}}{q(1+f)} \ge 1 \qquad (66)$$

hold everywhere in (0, 1), then $c_i < 0$, necessarily, i.e., the system is stable.

$$\frac{2\rho R_D^{-1}}{q(1+f)} < 1.$$

Therefore, it is important to obtain, as we have done in the next discussion, an estimate on c_i associated with these unstable modes.

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