Shock Wave’s Existance, Salient Features And Importance

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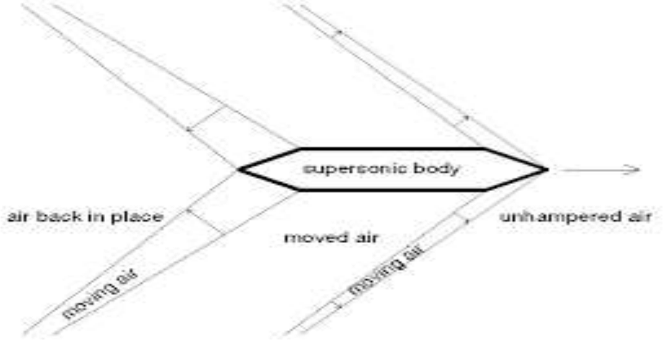
I. SHOCK WAVES

Shock wave is a type of propagating disturbance, which is often produced in nature during explosion, earth quake, electric discharge and when an object is moving with supersonic speed. Like an ordinary wave shock carries energy and can propagate through a medium (solid, liquid, gas or plasma) or in some cases in the absence of a material medium, through a field such as an electromagnetic field.

Suppose the continuous motion of the piston is approximated by a set of forward-moving pulses, each of short duration. When the piston makes the first short movement forward, a small disturbance is propagated forwards into the gas at the speed of sound. This small amplitude wave (or sound wave) heats the gas slightly and because of the square of the local speed of sound is proportional to the temperature, the second pulse will be propagated as another sound wave at a speed slightly in excess of first one. Similarly the third pulse will be propagated at a speed slightly in excess of second and so on. Thus the discrete pulses cause a train of sound waves of ever increasing velocity to be propagated through the gas. The tendency is for faster moving rearmost waves to catch up with the slower moving foremost ones. In so doing the sound waves coalesce to form a more powerful shock front moving at a speed which is in excess of the local speed of sound.

Shock waves are the most conspicuous phenomena occurring in non-linear wave propagation. Even without being caused by initial discontinuities, they may appear and be propagated. The underlying mathematical fact is that, unlike linear partial differential equations, non-linear equations often do not admit solutions which can be continuously extended whenever the differential equations themselves remain regular. The problem of shock waves has a bearing on many problems outside of supersonic aeronautics, for example, detonation waves, but also has great importance for several practical aeronautical problems. In fact, shock waves may cause sudden change in the aerodynamic behaviour of high speed aerocrafts affecting not only their balance and stability but also control producing undesirable vibrations.

Rayleigh [1] and Hugoniot [2] first pointed that an adiabatic reversible transition in a shock would violate the principle of conservation of energy. In fact, Hugoniot [2] showed that in the absence of viscosity and heat-conduction (outside the shock) conservation of energy implies conservation of entropy in continuous flow and also implies a change of entropy across a shock. Rayleigh [1] pointed out that the entropy must increase in crossing a shock front and that, for this reason, a rarefaction shock can not occur in a perfect gas.



It is true, of course, that a shock wave is not discontinuity in the strict sense. It has a finite thickness across which the physical properties change continuously. If this thickness is small compared with some appropriate macroscopic dimension of the flow-field, such as the radius of curvature of a curved shock, the physical relationships may be obtained by an analysis which treats the discontinuity as strict. The assumption that the discontinuity thickness is small compared with a macroscopic dimension is a fundamental one. The term ‘structure’ as applied to a shock wave, refers to the values of the physical properties of the fluid within the small but finite thickness of the discontinuity. If thermodynamic equilibrium in a substance is disturbed a characteristic time must elapse before equilibrium can be approximately re-established. This time and the velocity of the fluid, define a characteristic distance which is of the order of a molecular mean free path or greater. If the physical and chemical changes, occurring in the discontinuity are sufficiently slow, so that the thickness of discontinuity is large compared with this characteristic distance, the concept of their thermodynamic quasi-equilibrium may be considered. In this case, the Navier-Stoke’s equations are applicable. If the discontinuity is thin, with physical and chemical changes occurring rapidly, i.e. detonation front, the essential absence of thermodynamic equilibrium must be taken into account ([3]).

Viscosity and heat conduction are negligible, in most of the cases, but where large gradients of temperature and velocity develop, they become important. Viscosity and heat conduction have the effect of smoothing out the discontinuity. Both viscosity and thermal conductivity present themselves as diffusion phenomena, arising respectively from the molecular transfer of momentum and energy. The former manifests itself in diffusion of vorticity and the later in the diffusion of heat.
Relative to the shock wave, the flow on the upstream side must be supersonic, on the down stream side the flow relative to the shock wave may be either supersonic or subsonic, depending on the inclination to the incident stream of the normal to the wave. If the normal to the wave is parallel to the incident stream, the flow behind is always subsonic relative to the wave. By now, the theory of shock waves in non-conducting and conducting media has been much developed ([4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29,]).

II. SHOCK WAVES IN MAGNETOGASDYNAMICS

If a conducting fluid moves in a magnetic field, electric fields are induced and electric currents flow. The magnetic field exerts forces on these currents which considerably modify the flow. In many problems the energy in the electric field is much smaller than that in magnetic field. In these cases, we may express all the electromagnetic quantities in terms of magnetic field. As a result, we consider only the interaction between the magnetic field and the gas dynamic field. This analysis forms the subject matter of the well known ‘magnetogasdynamics’ and this interaction is of prime importance in most of the astrophysical and geophysical problems and in the behaviour of interstellar gaseous masses. As done in many problems, we have ignored Maxwell’s displacement current. We also assume as usual, that the dissipative mechanisms such as viscosity and thermal conductivity are absent.

The equations of motion for one dimensional magnetogasdynamic flow in a perfectly conducting fluid are ([8, 10,]as under:

(i) The continuity equation is

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{i\rho u}{r} = 0 \ , \qquad (2.1)$$

where i = 0, 1, 2 for planar, cylindrically and spherically symmetric flows, respectively; and u and ρ are fluid velocity and density at time t and at distance r from the plane, axis or centre of symmetry.
(ii) The momentum equation in planar symmetry with magnetic field h perpendicular to the flow, is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu h}{\rho}\frac{\partial h}{\partial r}=0\ , \qquad (2.2)$$

where μ is the magnetic permeability of the medium, and p is the pressure.

The momentum equation in cylindrical or spherical symmetry with azimuthal magnetic field h, is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu h}{\rho r}\frac{\partial (hr)}{\partial r}=0\ . \qquad (2.3)$$

The momentum equation in cylindrical symmetry with axial magnetic field h, is

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\mu h}{\rho}\frac{\partial h}{\partial r}=0\ . \qquad (2.4)$$

If the fluid is not perfectly conducting, but weakly conducting, the momentum equation in cylindrical symmetry takes the form,

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+\frac{1}{\rho}\frac{\partial p}{\partial r}+\frac{\sigma\mu^2h^2u}{\rho}=0\ , \qquad (2.5)$$

where σ is the electrical conductivity and h is axial or azimuthal magnetic field.

(iii) The magnetic field equation is

$$\frac{\partial h}{\partial t}+u\frac{\partial h}{\partial r}+h\frac{\partial u}{\partial r}+\frac{jhu}{r}=0\ , \qquad (2.6)$$

where, for planar symmetry, j = 0; for cylindrical symmetry with azimuthal magnetic field (h), j = 0; for cylindrical symmetry with axial magnetic field (h), j = 1; and for spherical symmetry with azimuthal magnetic field (h), j = 1.

(iv) The energy equation is

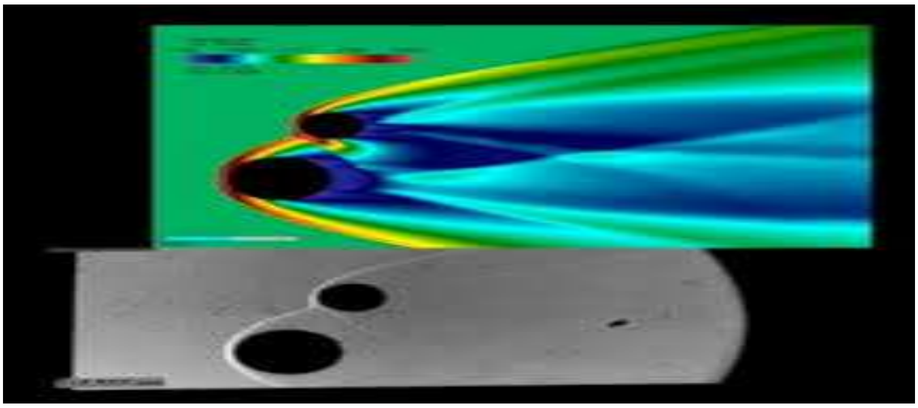
$$\frac{\partial}{\partial t}\left(p\rho^{-\gamma}\right)+u\frac{\partial}{\partial r}\left(p\rho^{-\gamma}\right)=0\ , \qquad (\ 27)$$

where γ is the ratio of specific heats of the fluid.

If the fluid is not perfectly conducting, but weekly conducting, the energy equation takes the form

$$\frac{\partial}{\partial t}\left(p\rho^{-\gamma}\right)+u\frac{\partial}{\partial r}\left(p\rho^{-\gamma}\right)=\sigma(\gamma-1)\rho^{-\gamma}\mu^2h_0^2u^2\ , \qquad (\ 2.8)$$

where h_0 is the initial axial or azimuthal magnetic field.



Shock Wave In Magnetic Field

The study of magnetogasdynamic shock wave was systematically begun in the year 1950 with the paper of Hoffmann and Teller [25]. The basic properties of magnetogasdynamic shock waves, as determined by the conservation laws (the Rankine- Hugoniot relations), have been developed further by Lee and Chen [21], Christer and Helliwell [37], Kanwal [42], Pai [43], Greenspan [44], Bhutani [45], Ray [46], Summers [47], Rosenau and Frankenthal [48], Verma and Vishwakarma [49], Singh and Singh [50], Ganguly and Jana [51], Ohsaki and Yoshida [52], Vishwakarma [53], Vishwakarma and Yadav [54], Vishwakarma and Vishwakarma [55], Vishwakarma et al. [56] and many others; although the more complex question of their existence in nature has yet to be exhaustively treated. The relations connecting the flow variables on the two sides of the shock surface (the generalized Rankine-Hugoniot relations) in the coordinate system in which the velocity in front of the shock wave is zero, are as follows ([8, 35]):

$$h_2\left(U-u_2\right)=h_1U\ , \tag{2.9}$$

$$\rho_2\left(U-u_2\right)=\rho_1U\ , \tag{2.10}$$

$$p_2+\frac{1}{2}\mu h_2^2+\rho_2\left(U-u_2\right)^2=p_1+\frac{1}{2}\mu h_1^2+\rho_1U^2\ , \tag{2.11}$$

$$\frac{1}{2}\left(U-u_2\right)^2+\frac{\gamma p_2}{(\gamma-1)\rho_2}+\frac{\mu h_2^2}{\rho_2}=\frac{1}{2}U^2+\frac{\gamma p_1}{(\gamma-1)\rho_1}+\frac{\mu h_1^2}{\rho_1}\ , \tag{2.12}$$

where the subscripts '1' and '2' correspond to the values of the quantities just ahead and just behind the shock surface, respectively, and U is the shock velocity. If the fluid is weakly conducting, the magnetic field is continuous across the shock ([10]).

III. FUNDAMENTAL EQUATIONS OF A NON-IDEAL GAS

When a strong explosion takes place, the character of the motion of the substance depends essentially on its equation of state. Such a motion was studied originally for the case of an ideal gas; subsequently, examples of solutions of the explosion problem were given for certain real, thermodynamically imperfect media ([7]. It should however be noted that the study of explosions in the media differing from the ideal gas did, as a rule, involve empirical equations of state, which only describe the behaviour of the medium satisfactorily in a certain, limited interval of densities. Almost every one of these empirical equations was found to be incorrect for the low density region and, in the limit when $\rho\to 0$, it either did not reduce to the equation of state for the ideal gas, or yield an incorrect first term of the so-called virial expansion for the pressure in powers of density.

As we know from statistical physics, at low densities the equation of state can be written in the form [59]

$$p = \rho RT\left[1+\rho B(T)+\rho^2 C(T)+\ldots\ldots\right],$$

where R is the gas constant, and B(T) and C(T) are virial coefficients which can be determined if the molecules interaction potential is known. In the high temperature range, the coefficients B(T) and C(T) tend to constant values equal to $\bar{b}$ and (5/8) $\bar{b}^2$ respectively. For gases $\bar{b}\rho$ << 1, $\bar{b}$ being the internal volume of the molecules, and therefore it is sufficient to consider the equation of the state in the form (Anisimov and Spiner [60], Ranga Rao and Purohit [61])

$$p = \rho RT\left(1+\bar{b}\rho\right), \qquad (3.1)$$

Wu and Roberts [62] and Roberts and Wu [63] have used an equivalent equation of state to study the shock wave theory of sololuminescence

From thermodynamics, we have

$$\left(\frac{\partial E}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_T - p, \qquad (3.2)$$

where E is the internal energy per unit mass of the gas and v is the specific volume.

Using the equation of state (3.1) in equation (3.2), we get $\left(\partial E/\partial v\right)_T = 0$, which shows that the internal energy E is a function of temperature T only. Therefore,

$$E = C_v T, \qquad (3.3)$$

where C_v is the specific heat at constant volume.
Using equation (3.2) in the first law of thermodynamics, we have

$$C_p - C_v = T\left(\frac{\partial p}{\partial T}\right)_v\left(\frac{\partial v}{\partial T}\right)_p, \qquad (3.4)$$

where C_p is the specific heats of the gas at constant pressure.
Using equation (3.1) in equation (3.4), we get

$$C_p - C_v = \frac{R\left(1+\bar{b}\rho\right)^2}{1+2\bar{b}\rho} \cong R, \qquad (3.5)$$

neglecting second and higher powers of $\bar{b}\rho$.

Equation (3.5) implies that

$$C_v = \frac{R}{\gamma-1}. \qquad (3.6)$$

Then equations (3.1), (3.3) and (3.6) give the internal energy E as a function of p and ρ, in the form

$$E = \frac{p}{\rho\left(1+\bar{b}\rho\right)\left(\gamma-1\right)} \quad . \tag{3.7}$$

The speed of sound 'a_n' may be calculated from equation (3.1) as follows :

$$a_n^2 = \frac{dp}{d\rho} = \frac{\left(1+2\bar{b}\rho\right)}{\left(1+\bar{b}\rho\right)}\frac{\gamma p}{\rho} \quad . \tag{3.8}$$

The equation of the energy of the non-ideal gas whose equation of state is in the form of equation (3.1) and which is non-conducting or perfectly conducting, is given by ([64])

$$\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)-a_n^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=0 \ , \tag{3.9}$$

where a_n^2 is given in equation (3.8). If the gas is weakly conducting, it takes the form

$$\left(\frac{\partial p}{\partial t}+u\frac{\partial p}{\partial r}\right)-a_n^2\left(\frac{\partial \rho}{\partial t}+u\frac{\partial \rho}{\partial r}\right)=\left(\gamma-1\right)\sigma\mu^2 h_1^2 u^2 \ , \tag{3.10}$$

where h_1 is the initial axial or azimuthal magnetic field.
The equation of continuity, magnetic field and momentum are the same as for a perfect gas.

IV. FUNDAMENTAL EQUATIONS OF MIXTURE OF A GAS AND SMALL SOLID PARTICLES

In the present paper we are concerned only with the two-phase flow of mixture of a gas and small solid particles (Marble [25], Murray [66], Soo [67], Pai [68], Rudinger [69]). When a large number of small solid particles flow in a fluid and the velocity of the fluid is sufficiently high, the behaviour of such solid particles is similar to ordinary fluid. We may consider these solid particles as a pseudo-fluid. The solid particles are assumed to
be spheres of identical mass m_p, radius r_p, and specific heat C_s. We may consider the mixture as a mixture of two fluids: one is the real fluid, gas or liquid and the other is the pseudo-fluid of solid particles. For each species r in the mixture, we would like to know its velocity vector $\bar{q}_r$, its temperature T_r, its pressure p_r and its density ρ_r. In this case, we have two definitions for density: the species density and the partial density. We consider an element of the mixture of a fluid f and solid particles p with total mass $M = M_f + M_p$ and with total volume $V = V_f + V_p$, where subscript "f " refers to the value for the fluid, gas or liquid, and subscript "p" refers to that of solid particles.

It is convenient to introduce the number density of the solid particles n_p which is the number of solid particles per unit volume at a point in the flow-field. The volume occupied by the solid particles V_p is

$$V_p = n_p V \bar{\tau}_p \quad , \tag{4.1}$$

where $\bar{\tau}_p = \frac{4}{3}\pi r_p^3$ is the volume of a solid particle in the mixture. Without subscript, we refer to the value of the mixture as a whole.
The mass of the solid particles in the volume V of the mixture is

$$M_p = m_p n_p V \quad . \tag{4.2}$$

The species density of the solid particles is defined as

$$\rho_{sp} = \frac{M_p}{V_p} = \frac{m_p}{\bar{\tau}_p} \quad . \qquad (4.3)$$

Hence the species density of solid particles is a constant for a given problem. The partial density of pseudo-fluid of solid particles is defined as

$$\bar{\rho}_p = \frac{M_p}{V} = m_p \, n_p = Z\rho_{sp} = \rho_{sp} \, \bar{\tau}_p n_p \quad , \qquad (4.4)$$

where Z represents the fraction of volume of solid particles in the mixture which is one of the important variables in the treatment of two-phase flow of a gas and small solid particles. From equation (4.4) we have the volume fraction of solid particles as follows

$$Z = \frac{V_p}{V} = n_p \bar{\tau}_p \quad . \qquad (4.5)$$

The partial density of the pseudo-fluid of solid particles one is of the fundamental variables in our analysis and it is proportional to Z or $n_{p.}$

$\bar{\rho}_p$ Similarly we have also the species density of the gas and partial density of the gas, too. The species density of the gas or the fluid is defined as

$$\rho_f = \frac{M_f}{V_f} \quad \text{or} \quad \rho_g = \frac{M_g}{V_g} \qquad (4.6)$$

and the partial density of gas is defined as

$$\bar{\rho}_g = \frac{M_g}{V} = \frac{M_g}{V_g}\frac{V_g}{V} = \frac{M_g}{V_g}\left(\frac{V - V_p}{V}\right) = (1 - Z)\,\rho_g \quad . \qquad (4.7)$$

(i) Equation of State and Thermodynamics

For each species in the mixture of a gas and a pseudo-fluid of solid particles, we have one equation of state. For the gas, we may use the perfect gas law which is

$$p_g = R\,\bar{\rho}_g T_g = R(1 - Z)\rho_g T_g = (1 - Z)\,p \quad , \qquad (4.8)$$

where p_g is the partial pressure of the gas in the mixture and T_g is the partial temperature of the gas and R is the gas constant. The total pressure of the mixture is p which is obtained, from (4.8), as

$$p = R\,\rho_g T_g \quad . \qquad (4.9)$$

Since the total pressure of the mixture is the sum of the partial pressure of the gas p_g and partial pressure of the pseudo-fluid of solid particles p_p, we have $p = p_g + p_p$. With the help of the equation (4.8) and (4.9), we find that the partial pressure of the solid particles must be

$$p_p = Z\,p \quad . \tag{4.10}$$

The equation of state for the pseudo-fluid of solid particles is simply

$$\rho_{sp} = \text{constant} \quad . \tag{4.11}$$

We consider the thermodynamic equilibrium condition such that $T_p = T_g = T$. The density of the mixture as a whole is given by

$$\rho = Z\,\rho_{sp} + (1-Z)\,\rho_g = \bar{\rho}_p + \bar{\rho}_g \quad . \tag{4.12}$$

The mass concentration of the pseudo-fluid of solid particles is defined as

$$K_p = \frac{\bar{\rho}_p}{\rho} = \frac{Z\,\rho_{sp}}{\rho} \quad . \tag{4.13}$$

In equilibrium flow, K_p is a constant in the whole flow field. Therefore, from equation (4.13)

$$(Z/\rho) = \text{constant}, \tag{4.14}$$

in the whole flow field, for equilibrium flow.

Also, from equations (4.12) and (4.13), we get

$$Z = \frac{K_p}{\left(1-K_p\right)G + K_p} \quad , \tag{4.15}$$

where $G = \dfrac{\rho_{sp}}{\rho_g}$.

The pressure of the mixture is

$$p = p_p + p_g \; . \tag{4.16}$$

Now, we shall write $T_g = T$ for simplicity and T_p may be equal to or different from T. In thermodynamic equilibrium condition, however, we have $T_p = T_g = T$. From equations (4.9) and (4.16) we find, the following relation between the pressure and density of the mixture as a whole

$$p = \frac{(1-K_p)}{(1-Z)}\rho R T = \frac{\rho R_m T}{(1-Z)} \quad , \tag{4.17}$$

where $R_m = (1-K_p)\,R$; and R_m may be considered as an effective gas constant of the mixture and ρ is given by equation (4.12). It is interesting to notice that if the volume fraction Z of the solid particles is negligibly small, the perfect gas law holds for the mixture as a whole when an effective gas constant R_m is used. For many engineering problems, we do have a very small value of Z but K_p is not negligible in comparison with unity. On the other hand, if Z is not negligibly small in comparison to unity, the volume fraction Z of solid particles does affect the equation of state of the mixture as a whole because Z is a function of ρ.

The internal energy of the mixture per unit mass U_m is related to the internal energies of the two species by following relation :

$$\rho\, U_m = Z\, \rho_{sp} C_{sp} T_p + (1-Z)\, \rho_g\, C_v\, T \qquad (4.18)$$

or

$$U_m = K_p C_{sp} T_p + (1-K_p) C_v T \quad , \qquad (4.19)$$

where C_v is the specific heat of the gas at constant volume, $C_{sp} = C_s + C_{vp}$ and we assume that C_{sp} and C_v are constants for simplicity, C_{vp} being an effective specific heat at constant volume of the pseudo-fluid of solid particles due to random translational motion and C_s is the specific heat of the solid particles due to the internal degree of freedom. For thermodynamic equilibrium condition, we have the specific heat of the mixture at constant volume C_{vm} as follows :

$$C_{vm} = k_p C_{sp} + \left(1-k_p\right) C_v . \qquad (4.20)$$

The enthalpy of the mixture per unit mass is

$$H_m = U_m + \frac{p}{\rho} = K_p \left(C_{sp} T + \frac{\rho}{\rho_{sp}}\right) + (1-K_p)\, C_p T \quad ,$$

where C_p is the specific heat of the gas at constant pressure. For thermodynamic equilibrium condition, the specific heat of the mixture at constant pressure is then

$$C_{pm} = K_p C_{sp} + (1-K_p)\, C_p \quad . \qquad (4.21)$$

The specific heats of the mixture are independent of the volume fraction Z but depend on the mass concentration K_p of the solid particles. The ratio of specific heats of the mixture is

$$\Gamma = \frac{C_{pm}}{C_{vm}} = \frac{(1-K_p) C_p + K_p C_{sp}}{(1-K_p) C_v + K_p C_{sp}} = \gamma \frac{\left(1+\eta \frac{\delta}{\gamma}\right)}{(1+\eta\delta)} \quad , \qquad (4.22)$$

where

$$\gamma = \frac{C_p}{C_v}, \quad \delta = \frac{C_{sp}}{C_v}, \quad \text{and} \quad \eta = \frac{K_p}{1 - K_p} \quad . \qquad (4.23)$$

The ratio Γ is always smaller than γ of the gas if K_p is different from zero. As $K_p = 0$, $\Gamma = \gamma$. Subtracting equation (4.20) from equation (4.21) and using $C_p - C_v = R$, we obtain

$$C_{pm} - C_{vm} = (1 - K_p)R \, . \tag{4.24}$$

From equations (4.22) and (4.24), we get

$$C_{vm} = \frac{(1-K_p)R}{(\Gamma - 1)} \quad \text{and} \quad C_{pm} = \frac{(1-K_p)R\Gamma}{(\Gamma - 1)} \quad .$$

Therefore,

$$U_m = C_{vm} T = \frac{(1-K_p)RT}{(\Gamma - 1)}$$

or, using equation (4.17),

$$U_m = \frac{(1-Z)}{(\Gamma - 1)} \frac{p}{\rho} \quad . \tag{4.25}$$

If we consider the mixture as a homogeneous medium, the first law of thermodynamics for the mixture gives:

$$dQ = dU_m - \frac{1}{\rho^2} \; p \, d\rho \quad , \tag{4.26}$$

where dQ is the heat addition to the mixture. Equation (4.26) is the energy equation of the mixture as a whole.

For isentropic change of state of gas-particle mixture, we have dQ = 0, equation (4.26) gives

$$\frac{1}{\Gamma - 1} \frac{dT}{T} = \frac{1}{1-Z} \frac{d\rho}{\rho} \quad . \tag{4.27}$$

Since $Z = \left(K_p \rho / \rho_{sp}\right)$, for constant K_p and $T_p = T$, from the integration of (4.27), we have :

$$T \left[\frac{\rho}{1-Z} \right]^{-(\Gamma - 1)} = \text{constant} \quad . \tag{4.28}$$

If Z << 1, the isentropic change of state of the mixture has similar relation as that for a pure gas with an effective ratio of specific heats Γ. In general, the volume fraction Z has some influence on the isentropic change of the mixture.

Similarly from equation (4.17) for a given K_p and $T_p = T$, we have

$$\frac{dp}{p} = \frac{dT}{T} + \frac{1}{(1-Z)} \frac{d\rho}{\rho} \quad . \tag{4.29}$$

From equations (4.27) and (4.29), we have

$$\frac{dp}{p} = \frac{\Gamma}{(1-Z)} \frac{d\rho}{\rho} \tag{4.30}$$

or
$$p\left(\frac{\rho}{1-Z}\right)^{-\Gamma} = \text{constant} . \qquad (4.31)$$

Again, if Z << 1, equation (4.31) is identical in form for the corresponding relation of pure gas but with an effective ratio of specific heats.

We may calculate the so-called equilibrium speed of sound of the mixture "a_m" from (4.31) as follows:

$$a_m^2 = \frac{dp}{d\rho} = \frac{\Gamma(1-K_p)RT}{(1-Z)^2} = \frac{\Gamma R_m T}{(1-Z)^2}$$

or

$$a_m^2 = \frac{\Gamma p}{\rho(1-Z)} \quad . \qquad (4.32)$$

(ii) Equation of Continuity:

For each species in the mixture, we have one equation of continuity which gives the conservation of mass of that species. Combining equations of continuity of both the species, we have the equation of continuity as a whole ([68, 70]) for one dimensional motion

$$\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{i\rho u}{r} = 0 \quad , \qquad (4.33)$$

where i = 0, 1 and 2 correspond to the plane, cylindrical and spherical symmetry, respectively; ρ is the density and u the flow velocity of the mixture.

(iii) Equation of Motion:

For each species the conservation of momentum gives the corresponding equation of motion for that species. If we combine equations of motion of both the species, we may obtain an equation of motion for the mixture as a whole.
The equation of motion for the unsteady one-dimensional flow of the mixture can be written as ([68, 70])

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0 \quad , \qquad (4.34)$$

where $p = p_g + p_p$ is the total pressure of the mixture. Here, it is assumed that the viscous stress and heat conduction of the mixture are negligible.

(iv) Equation of Energy:

For each species, the conservation of energy gives the corresponding equation of energy for that species. Combining the energy equations of both the species we may obtain an energy equation of the mixture as a whole.The energy equation of unsteady one-dimensional flow of the mixture in which the viscous stress and heat conduction are assumed to be negligible can be written as ([68, 70])

$$\frac{\partial U_m}{\partial t} + u\frac{\partial U_m}{\partial r} - \frac{p}{\rho^2}\left(\frac{\partial\rho}{\partial t} + u\frac{\partial\rho}{\partial r}\right) = 0 \quad , \qquad (4.35)$$

where U_m is the internal energy of the mixture of gas and small solid particles.

Shock waves in a mixture of small solid particles and a gas have been studied by several authors, for example, Jena and Sharma [23], Vishwakarma [24], Steiner and Hirschler [28], Vishwakarma and Haidar [29], Pai et al. [70], Miura and Glass [18, 71], Suzuki et al. [72], Higashino and Suzuki [73], Gretler and Regenfelder [74, 75], Vishwakarma and Pandey [31]. The generalized shock conditions, in this case, are

$$\rho_2\left(U-u_2\right)=\rho_1 U \quad , \tag{4.36}$$

$$p_2+\rho_2\left(U-u_2\right)^2=p_1+\rho_1 U^2 \quad , \tag{4.37}$$

$$U_{m_2}+\frac{p_2}{\rho_2}+\frac{1}{2}\left(U-u_2\right)^2=U_{m_1}+\frac{p_1}{\rho_1}+\frac{1}{2}U^2 \quad , \tag{4.38}$$

$$\frac{Z_2}{\rho_2}=\frac{Z_1}{\rho_1} \quad . \tag{4.39}$$

Here, it is assumed that the viscous stress and heat conduction of the mixture are negligible

$$q_i=-\left(\frac{4\pi}{3\alpha_1}\right)\frac{\partial B}{\partial x_i} \quad , \tag{5.6}$$

transparent limit (thin gas)

$$\frac{\partial q_i}{\partial x_i}=4\pi\alpha_1 B. \tag{5.7}$$

Traugott [75] has drawn attention to the fact that it is not strictly possible to introduce a single meaningful absorption coefficient in relating a real to a grey gas for, in the limiting cases above, two distinct meaningful averages exist. If the gas is transparent at

all frequencies, the Planck mean, α_p is appropriate, but if it is opaque, the Rosseland

mean, α_R, should be used. For a grey gas, one sets $\alpha_p=\alpha_R=\alpha_1$. For a more realistic gas, both α_p and α_R should be incorporated in the theory, so that in the opaque and transparent limits the appropriate absorption coefficient may arise, viz, $\alpha_1=\alpha_R$ in (3.6) and $\alpha_1=\alpha_p$ in (3.7). Traugott has indicated how this may be done by a simple remodeling of equation (3.5) so that it reads

$$\frac{\partial}{\partial x_i}\left(\frac{1}{\alpha_p}\frac{\partial q_i}{\partial x_i}\right)-3\alpha_R q_i-4\pi\frac{\partial B}{\partial x_i}=0 \quad . \tag{5.8}$$

VI. LAW OF CONSERVATION OF ENERGY IN PRESENCE OF THERMAL CONDUCTION

In an ideal fluid the law of conservation of energy is expressed by [59]

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2+\rho e\right)=-\operatorname{div}\left[\rho\vec{v}\left(\frac{1}{2}v^2+w\right)\right]. \qquad (6.1)$$

The expression on the left is the rate of change of the energy in unit volume of the fluid, while that on the right is the divergence of the energy flux density, w is the heat function and e the internal energy per unit mass. In a viscous fluid the law of conservation of energy still holds, of course: the change per unit time in the total energy of the fluid in any volume must still be equal to the total flux of energy through the surface bounding that volume. The energy flux density, however, now has a different form. Besides the flux $\rho\vec{v}\left(\frac{1}{2}v^2+w\right)$ due to the simple transfer of mass by the motion of the fluid, there is also a flux due to process of internal friction (viscosity). There is, moreover, another term that must be included in the energy flux. If the temperature of the fluid is not constant throughout its volume, there will be, besides the two means of energy transfer
indicated above, a transfer of heat by what is called thermal conduction. This signifies
the direct molecular transfer of energy from points where the temperature is high to those where it is low. It does not involve macroscopic motion, and occurs even in a fluid at rest.

We denote by q_C the heat flux density due to thermal conduction. The flux q_C is related to the variation of temperature through the fluid. This relation can be written down atonce in cases where the temperature gradient in the fluid is not large; in phenomena of thermal conduction we are almost always concerned with such cases. We can then expand q_C as a series of powers of the temperature gradient, taking only the first terms of the expansion. The constant term is evidently zero, since q_C must vanish when grad T does so. Thus, we have

$$q_C=-K\ \text{grad}\ T. \qquad (6.2)$$

The constant K is called the thermal conductivity. It is always positive, as we see atonce from the fact that the energy flux must be from points at a high temperature to those at a low temperature, i.e. q_C and grad T must be in opposite directions. The coefficient K is in general a function of temperature and pressure.

Thus the total energy flux in a fluid when viscosity is neglected and thermal conduction is taken into account is $\rho\vec{v}\left(\frac{1}{2}v^2+w\right)-K$ grad T. Accordingly, the law of conservation of energy is given by the equation

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\rho v^2+\rho e\right)=-\operatorname{div}\left[\rho\vec{v}\left(\frac{1}{2}v^2+w\right)-K\ \text{grad}\ T\right]. \qquad (6.3)$$

It is convenient, however, to put it in the following form by transforming it with the aid of equation of motion

$$\rho T\left(\frac{\partial S}{\partial t}+\vec{v}\ .\ \text{grad}\ S\right)=\operatorname{div}\ (K\ \text{grad}\ T) \qquad (6.4)$$

whe re S is the entropy per unit mass of the fluid.

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