

Assessing School Students' Mathematic Ability Using DINA and DINO Models

Mohammad Nasim Wafa

Department of Mathematics, Ghor Institute of Higher Education, Firozkoh city, Ghor, Afghanistan

Abstract : *Cognitive diagnosis models (CDMs) are restricted latent class models that can be used to analyze response data from educational or psychological tests. This article focuses on evaluating the application of CDM in identifying school students' mathematic abilities in grade 8 at four different schools in Afghanistan. In addition, this research aims at determining the 8th grade students' level of mathematics at the school level. Followed by the analysis of a set of data from Trends in International Mathematics and Science Study (TIMSS) 2011 mathematics assessment is used to examine the Mathematical abilities of students in Grade 8, which measures 13 attributes and includes 32 questions. A sample size of 274 includes 129 girls and 145 boys, and the students are selected based on the multistage cluster sampling method from Ghor province. Under the cognitive diagnosis assessment framework, the deterministic, inputs, noisy, "and" gate (DINA) model and the deterministic, inputs, noisy, "or" gate (DINO) model are used.*

Keywords: *cognitive diagnosis models, item response theory, DINA, DINO, TIMMS 2011*

I. Introduction

Cognitive Diagnosis Models (CDMs) are statistical and psychometric models developed to identify the examinees' ability to master fine-grained skills based on a pre-specified matrix. Cognitive diagnostic tests can be used to identify the skill combinations that the examinee is likely to either possess or not possess or have all attributes or not (Su, 2013). The objective of each of these models is to classify examinees according to their mastery of skills assumed to be required. The use of CDMs methods has the advantage of not being available in other methods. First of all, most other models including IRT assumes the statistical one-dimensionality of the dataset and they need it as a precondition for calibrating queries and estimating parameters. In most models, one-dimensionality is essential as a prerequisite for determining the location of the subjects along a hypothetical continuum. One of the important features of CDMs is that it doesn't need to be one-dimensional. One-dimensionality seems to be somewhat problematic in educational settings because research has shown that academic tools typically consider sets of traits or sub-skills, each of which can create a separate statistical dimension. (Afzali, 2016).

CDMs are of growing interest in test development and in the measurement of human performances (Huang, 2014). Mathematics is one of the main courses in educational programs. The mastery of mathematics is a precondition for entering higher academic levels. One way to incorporate the substantive definition into the modeling process is to introduce specially designed items targeted to measure specific levels of the attributes (PG-DINA; Chen & de la Torre, 2013). In recent years, educational research was characterized by increasing demand for complex information on students' achievement.

There are many reasons why it is difficult to learn Mathematics for Afghans students. So far, coherent research in evaluating and determining the strengths and weaknesses of Afghanistan students in attributes and skills in mathematics has not been investigated. Hence the main issue in this study is the strengths and weaknesses of Afghanistan Mathematical education. There are some old curriculums in the past four decades, the learning

environment was not good in Afghan, actually inferior, but improved over the past decade than in the previous years, including the presence of girls and boys in schools. Teaching in Afghan schools portrays a very traditional approach, which promotes the rote learning of the subject, Curriculum problems are the main factor of declining class size in this study(Mansory, 2010). Consequently, the development and evaluation of CDMs (Rupp, Templin, & Henson, 2010) for Mathematics in the high schools, to structure knowledge and special skills processed in students. The goal is to provide information about students' weaknesses and their strengths to provide credible educational information that can be used effectively by the teacher (Afshin, 2016).

In this study, we attempt to find the level or surface of the slipping and guessing of the students of Grade 8 using the DINA and DINO models. In particular, the International Mathematics and Science Study (TIMSS), a quadrennial assessment administered by the International Association for the Evaluation of Educational Achievement (IEA) since 1995, evaluates the mathematics and science abilities of fourth and eighth-graders. The TIMSS has taken an exam every four years in many countries, for example, 1999, 2003, 2007, 2011, 2015 and 2019, Afghanistan is not eligible for this competition.

Thus, in Afghanistan, there is no study about the evaluation of students' Mathematical abilities using cognitive diagnosis models. There are only a few limited kinds of research on undergraduate Mathematics Education in Afghanistan. So, this article is addressing the following objectives and questions.

A. Research Objectives

1. To evaluate the application of CDMs in identifying school students' mathematic abilities at grade 8
2. To determine the 8th grade students' level of mathematics at the school level.

B. Research questions

1. Which cognitive diagnosis model can be used to model students' Mathematical abilities?
2. What is Afghan secondary school students' level of Mathematical abilities?

In Afghanistan, most students in the Department of Mathematics, who have basic Mathematics skills, have distinct disadvantages, and a significant part of students fail to learn higher-level subjects. The academic failure in this course and the weak results in national and international tests are due to this weakness. So, this study pays more attention to construct the hierarchy of this course at the time of developing educational programs and follow a cognitive diagnosis model in the process of learning as well as planning to ensure reciprocity. Prerequisite knowledge for curriculum concepts and planning before training high-level skills can have a significant impact on the quality of education and learning of Mathematics in Afghanistan middle schools.

II. The study background

In recent literature, several CDMs have been used to parameterize latent attribute space to model relationships between attributes and help improve parameter estimation performance. These approaches include log-linear (Xu & von Davier, 2008), unstructured tetra-choric correlation (Hartz, 2002), and tetra-choric structural correlation (de la Torre & Douglas, 2004). There are several extensive reviews of CDMs, they have appeared in the literature, include

cognitive diagnosis models (or cognitively diagnostic models) (de la Torre, 2009; de la Torre & sun Lee, 2013; Henson & Douglas, 2005; Huebner & Wang, 2011; Tatsuoaka, 1995), diagnostic classification models (DCM) (Rupp & Templin, 2008a; Kunina et al., 2012).

Research on the impact of cognitive theory on test design was very limited as mentioned in (Gierl & Zhou, 2008; Leighton et al., 2004). Most CDMs application examples in the literature are limited to no more than eight attributes (Hartz, 2002; Rupp & Templin, 2008b) because of the long computing time for models with larger numbers of attributes and items. If the number of latent classes can be reduced from 2^K , the sample size needed to obtain stable parameter estimates from CDMs calibrations will decrease. This will also result in faster computing time. One solution to decrease the number of latent classes is to impose hierarchical structures (Leighton et al., 2004) on skills. The resulting approach is able to assess and analyze more attributes by reducing the number of possible latent classes and the sample size requirement (de la Torre, 2008, 2009; de la Torre & Lee, 2010). Two methods to estimate attributes with hierarchical structures could be as de la Torre (2012) suggested: First, keeping the EM algorithm as is, but without any gain in efficiency, the prior value of attribute patterns not possible under the hierarchy can be set to 0, and second, for greater efficiency, but requiring minor modifications of the EM algorithm, attribute patterns not possible under the hierarchy can be dropped.

A. DINA model: The DINA model (de la Torre & Douglas, 2004; Haertel, 1989; Junker & Sijtsma, 2001; Macready & Dayton, 1977) is a noncompensatory CDM that assumes that lack of one attribute cannot be reimbursed by the existence of another attribute. DINA works with a conjunctive condensation rule, which means that to have a high probability of responding an item correctly, an individual need to master all the attributes required by that item. The main limitation of the DINA model is that it does not make a distinction between respondents who did not master only one or more than one attribute.

The DINA model estimates the probability of a correct response to item i for all the respondents in latent class c as follows:

$$\pi_{ij} = P[X_{ij} = 1 | \xi_{ij}] = (1 - s_j)^{\xi_{ij}} \cdot g_j^{(1-\xi_{ij})} = \begin{cases} 1 - s_j & \text{for } \xi_{ij} = 1 \\ g_j & \text{for } \xi_{ij} = 0 \end{cases}$$

where π_{ij} is the probability of correct response, X_{ij} is the observed response, ξ_{ij} is the attribute mastery indicator, and s_j and g_j are, respectively, the slipping and the guessing parameters. The slipping parameter, s_j , is defined as the probability of responding an item incorrectly for a respondent who has mastered all the required attribute:

$$s_j = P(X_{ij} = 0 | \eta_{ij} = 1), \quad j = 1, 2, \dots, J$$

The guessing parameter, g_j , is the probability of responding an item correctly for a respondent who has not mastered at least one required attribute:

$$g_j = P(X_{ij} = 1 | \eta_{ij} = 0), \quad j = 1, 2, \dots, J$$

If a respondent masters all the required attributes, $\xi_{ij} = 1$, the probability of responding the item correctly is equal to the probability of not slipping for the item, $1 - s_j$. On the other hand, if the respondent fails to master at least one of the required attributes, $\xi_{ij} = 0$, the probability of responding the item correctly drops to the probability of guessing for the item, g_j . The DINA model order-constrains the slipping and guessing parameters: $1 - s_j$ is assumed to be greater than g_j ; thus, the probability of responding an item correctly is guaranteed to be always higher for the respondents who mastered all the measured attributes than the respondents who lacked at least one of the measured attributes, regardless of the magnitudes of slipping and guessing parameters (Rupp et al., 2010).

The attribute mastery indicator is formulated $\xi_{ij} = \prod_{k=1}^A \alpha_{jk}^{q_{ik}}$, where A is the total number of attributes measured, and q_{ik} indicates whether attribute k is measured by item i . The possible values that q_{ik} takes are 0 or 1. The other indicator α_{jk} identifies whether the respondent in latent class j mastered attribute k , which takes values of 0 or 1 as well. Since the attribute mastery indicator, ξ_{ij} , is created through multiplication of each alpha for every measured attribute, lack of a single measured attribute would cause the value of ξ_{ij} to be 0.

B. DINO model : The DINO models also separate the latent classes into two groups for each item. It is supposing that an item can be answered correctly if at least one of the required attributes involved in the item has been mastered. Given the slipping and guessing parameters s'_j and g'_j , its item response function (IRF) is written as $P(X_j = 1 | \alpha_j) = P(X_j = 1 | \zeta_{jl}) = g_j^{(1-\zeta_{jl})} (1 - s'_j)^{\zeta_{jl}}$ where $\zeta_{jl} = 1 - \prod_{k=1}^K (1 - \alpha_{jk})^{q_{jk}}$ is the deterministic component of the model. The slipping parameter s'_j is the supposition that the examinees in latent class l whose $\zeta_{jl} = 1$ will slip and inaccurately answer the item j , and the guessing parameter g'_j is the supposition that the examinees in furtive class l whose $\zeta_{jl} = 0$ will guess and accurately answer the item. Officially, s'_j and g'_j are defined as $s'_j = P(X_j = 0 | \zeta_{jl} = 1)$ and $g'_j = P(X_j = 1 | \zeta_{jl} = 0)$.

C. The Duality of the DINA Model and the DINO Model: As Y. Liu et al. (2011) discovered and proved, the DINA model and the DINO model are technically identical under certain transformations of (a) the examinees' attribute profiles, (b) their observed item scores, and (c) the model parameters. This means that one model can be expressed in terms of the other and both models can be fitted by the same software. (As an aside, note that the characterization of the special relationship between the DINA model and the DINO model as “dual” deviates from the well-defined meaning of this term in operations research; for details, consult Papadimitriou & Steiglitz, 1998.) Model is two popular cognitive diagnosis models (CDMs) for educational assessment. They represent different views on how the mastery of cognitive skills and the probability of a correct item response are related. Recently, however, Liu, Xu, and Ying demonstrated that the DINO model and the DINA model share a “dual” relation and which of the two models is fitted to a given data set is essentially irrelevant because the results are identical.

D. Q-matrix: The analysis of most CDMs is based on an item-attribute incidence matrix called a *Q*-matrix (Tatsuoka, 1983). The diagnostic power of CDMs relies on the construction of a *Q*-matrix with attributes that are theoretically appropriate and empirically supported (Lee & Sawaki, 2009). Studies on the *Q*-matrix can be normally categorized as exploratory approaches intend to discover the *Q*-matrix from the data when the whole *Q*-matrix is unknown. Confirmatory approaches aim to purify a certain *Q*-matrix in which some elements of the *Q*-matrix are assumed to be known. Although an entirely exploratory approach obtains no information about the number of attributes in advance, an approach given the number of attributes is still regarded as exploratory here as long as it estimates the whole *Q*-matrix (Chung, 2014). After defining, determining and identify the *Q*-matrix for measuring the test, the next step is to construct the *Q*-matrix.

In this study to form a *Q*-matrix, after translating the protocol or the codebook of the questions, encode a copy of the Mathematic questions for Grade 8 of the TIMMS 2011 with attributes and the coding protocol and provide it to 3 Math teachers with bachelor degree, who had 6-year, 8-year and 10-year training experiences, respectively. They are asked for constructing the *Q*-matrix separately and independently. In a two-dimensional matrix in which the columns contained those skills and each question measures attributes in the rows of the question, by specifying either 1 or 0. Attributes are explained in the analysis section, Table, 3

Table 1. *Q*-Matrix for Each Content Domain.

Item	attribute												
	A_{N1}	A_{N2}	A_{N3}	A_{N4}	A_{A1}	A_{A2}	A_{A3}	A_{A4}	A_{G1}	A_{G2}	A_{G3}	A_{G4}	A_{D1}
Item 1	1	1	1	0	0	0	0	0	0	0	0	0	0
Item 2	0	0	0	1	0	0	0	0	0	0	0	0	1
Item 3	1	0	0	0	1	0	0	1	0	0	0	0	0
Item 4	0	1	1	0	0	0	0	0	0	0	0	0	0
Item 5	0	1	1	0	0	0	0	0	0	0	0	0	0
Item 6	0	0	0	1	0	0	0	0	0	0	0	0	0
Item 7	0	0	0	1	0	0	0	0	0	0	0	0	0
Item 8	0	0	0	1	0	0	0	0	0	0	0	0	0
Item 9	0	1	1	1	0	0	0	0	0	0	0	0	0
Item 10	0	0	0	1	0	0	0	0	0	0	0	0	0
Item 11	0	0	0	0	0	0	0	0	0	1	0	1	0
Item 12	0	0	0	0	0	0	0	0	0	1	0	1	0
Item 13	0	0	0	0	0	0	0	0	0	0	1	1	0
Item 14	0	0	0	0	0	0	0	0	1	1	1	1	0
Item 15	0	0	0	0	1	0	0	0	0	0	0	0	0
Item 16	0	0	0	0	0	1	0	0	0	0	0	0	0
Item 17	0	0	0	0	1	1	0	1	0	0	0	0	0
Item 18	0	0	0	0	0	0	1	0	0	0	0	0	0
Item 19	0	0	0	0	0	1	1	0	0	0	0	0	0
Item 20	0	0	0	0	0	1	0	0	0	0	0	0	0

Item 21	0	0	0	0	0	0	1	0	0	0	0	0	0
Item 22	0	0	0	0	1	1	1	1	0	0	0	0	0
Item 23	0	0	0	0	0	1	1	0	0	0	0	0	0
Item 24	0	0	0	0	0	0	0	0	1	1	1	0	0
Item 25	0	0	0	0	0	0	0	0	1	0	1	0	0
Item 26	0	0	0	0	0	0	0	0	0	0	1	0	0
Item 27	0	0	0	0	0	0	0	0	0	0	1	0	0
Item 28	0	0	0	0	0	0	0	0	0	0	0	0	1
Item 29	0	0	0	0	1	0	0	1	0	0	0	0	0
Item 30	0	0	0	0	0	0	0	1	0	0	0	0	0
Item 31	0	0	0	0	0	0	0	1	0	0	0	0	0
Item 32	0	0	0	0	0	0	0	1	0	0	0	0	1

Table 1 presents the original Q -matrix for this example. For the 32 items in this assessment, the vector of skill requirements for each item forms the Q -matrix. A_{N1}, A_{N2}, A_{N3} and A_{N4} , are the attributes of the *Number* domain; A_{A1}, A_{A2}, A_{A3} and A_{A4} are the attributes of the *Algebra* domain; A_{G1}, A_{G2}, A_{G3} and A_{G4} are the attributes of the *Geometry* domain; and A_{D1} are the attributes of the *Data and Chance* domain.

III. Method

A quantitative research approach was used to collect data for the current research. A total of 274 Afghan students within 4 schools participated. In each classroom, 16 different classes of Afghanistan mathematics tests were assigned randomly to students.

A. Research participants and research tool: The R package CDMs is used to fit the response data. A simple form in this research we study the mathematical experts’ opinions, contains 8 linear hierarchical traits, it is given annually at approximately 4 test centers in Ghor province of Afghanistan high schools, it there was 275 students in different areas of Firozkouh city 145 boys and 129 girls. the average of examinees is around 11-17 years old.

The same questionnaire was used as in Taiwan, each student was requested to answer only one out of eight booklets and only Booklets 1, 3, 5, and 7 were used for the current study. These booklets were selected based on the criterion that each attribute analyzed in the study had to be included in at least three items (Corter&Tatsuoka, 2002). For the purpose of comparisons across subgroups, these schools were selected into the rural and urban groups. Schools located in a geographically isolated area and in the village or rural area, and there were also some schools located in the middle of Firozkoh city.

Table 2. Demographic data of the participants

Age	Gender		Grade	Number of classes
11-17	Male	Female	8	16
	129	145		

B. Research analysis: Mathematical response datasets of the students in Grade 8 in Afghanistan were analyzed in this study. Students responded to the multiple-choice and constructed response items, which assessed four content

domains: Data and Chance, Geometry, Algebra, Algebra, and Number. The DINA model and DINO model were used to fit the response data. The test was composed of 32 items, including 15 multiple-choice and 17 constructed response items. There were 129 female and 145 male participants in this study.

Quantitative analyses were carried out in the process of test development and *Q*-matrix construction. the data was analyzed using TIMSS 2011 with eighth grade mathematics data-sets from the students of Afghanistan who were compared in this study. Students responded to the multiple-choice and constructed response items, assessing four content domains: Number, Algebra, Geometry, and Data and Chance. analyzed together with the *Q*-matrix using the R. Improving the teaching and learning of Mathematics and Science through providing data on student progress in relation to different types of curricula, educational practices and educational environments, or schools (Mullis& Martin 2003). Since the TIMSS mathematics items included multiple-choice and constructed responses, I dichotomized (0 = wrong answer, 1 = correct answer) those items for the dichotomous DINA model in this study.

Table 3. Marginal skill probability percentage of mastering

attribute	Title	probability
attribute1	Possesses understanding of fraction equivalence and ordering; uses equivalent fractions as a strategy to add and subtract fractions.	0.2417
attribute 2	Understands decimal notation for fractions, and compares decimal fractions; performs operations with decimals.	0.3481
attribute 3	Understands ratio concepts, and uses ratio reasoning to solve problems; finds a percent of a quantity as a rate per 100	0.3481
attribute 4	Understanding the whole number, uses the equivalent expression, prime factors, express of as power.	0.269
attribute 5	Applies and extends previous understandings of arithmetic to algebraic expressions; solves real-life and mathematical problems using numerical and algebraic expressions and equations.	0.2232
attribute 6	Reasons about and solves one-variable equations and inequalities; uses properties of operations to generate equivalent expressions.	0.1914
attribute 7	Analyzes and solves linear equations and pairs of simultaneous linear equations.	0.1701
attribute 8	Uses the four operations with whole numbers to solve problems; identifies and explains patterns in arithmetic.	0.2349
attribute 9	Draws, constructs, and describes geometrical figures	0.1772
attribute10	Solves real-life and mathematical problems involving angle measure, area, surface area, and volume.	0.2517
attribute11	Recognizes perimeter, understands concepts of area, and relates area to multiplication and addition.	0.2008
attribute12	Describes geometrical figures, and describes the relationships between them.	0.2636
attribute13	Represents and interprets data; draws informal comparative inferences about two populations	0.13

Table 3 shows the Marginal probability of mastering each of the thirteen attributes. According to the results in the table, the highest probability of mastery in the attribute belongs to the attribute 4 at (0.4836) and the lowest probability belong to attribute 24 and 32 which is (0.12).

Table 4. Marginal skill probability percentage of mastering each Item

Item	Probability	Item	probability
Item1	0.2153	Item17	0.2846
Item2	0.208	Item18	0.2956
Item3	0.2226	Item19	0.1569
Item4	0.489	Item20	0.1167
Item5	0.4014	Item21	0.0875
Item6	0.2372	Item22	0.197
Item7	0.3321	Item23	0.1204
Item8	0.3686	Item24	0.1496
Item9	0.2956	Item25	0.2481
Item10	0.2043	Item26	0.2627
Item11	0.3357	Item27	0.2846
Item12	0.3357	Item28	0.1934
Item13	0.2007	Item29	0.2554
Item14	0.2007	Item30	0.3686
Item15	0.1605	Item31	0.2116
Item16	0.2919	Item32	0.1204

Table 4 shows the marginal probability of the mastering of each item. According to the results in the table, the highest probability of mastery belongs to item four at (0.489) and the lowest probability belongs to attribute 21 which is (0.0875).

Table 5 shows the guessing and slipping parameters based on the DINA model. According to Table 5, the lowest guessing parameter of DINA models belongs to Item #32 and the highest guessing coefficient belongs to Item #4, and the lowest slipping coefficient belongs to Item #2 and the highest slipping coefficient belongs to Item #14. The coefficient of lowest indicates the possibility of incorrectly responding to those who possess the skills needed to answer the question. The smaller the guessing and slipping parameters, the better the fit between the diagnostic measurement and experimental data in the CDMs (Ravand, Barati, & Widhiarso. 2012)

Table 5. Guessing and Slipping parameters in the DINA and DINA

iteme	DINA				DINO			
	Guess est.	Guess SE	Slip est.	Slip SE	Guess est.	Guess SE	Slip est.	Slip SE
Item 1	0.1623	0.0222	0.4182	0.1265	0.1184	0.0208	0.574	0.0662
Item 2	0.0909	0.0144	0	0	0.0117	0.0028	0.3264	0.0543
Item 3	0.1701	0.0242	0.2703	0.0808	0.0868	0.0206	0.579	0.0563
Item 4	0.364	0.0323	0.1761	0.0706	0.3742	0.0338	4.2E-16	1.2E-16
Item 5	0.1903	0.0219	0.0274	0.0104	0.2917	0.0305	0.1088	0.0265
Item 6	0.0846	0.0189	0.2713	0.0455	0.1238	0.0196	0.1724	0.0727
Item 7	0.203	0.0327	0.2541	0.044	0.2324	0.0272	0.1514	0.0671
Item 8	0.2634	0.0395	0.2959	0.0532	0.2843	0.0296	0.197	0.0868

Item 9	0.2738	0.0287	0.5875	0.1265	0.2639	0.0336	0.6108	0.0653
Item10	0.1836	0.0347	0.7315	0.0623	0.1782	0.0249	0.6628	0.1171
Item11	0.002	0.0002	5.1E-11	1.6E-11	0.2116	0.0268	0.2765	0.0555
Item12	0.3332	0.0341	0.6628	0.0895	0.2638	0.0309	0.4415	0.0745
Item13	0.1749	0.0227	0.4759	0.1374	0.1484	0.0272	0.7211	0.0567
Item14	0.1979	0.0249	0.7533	0.1189	0.0362	0.0131	0.5494	0.0559
Item15	5.5E-12	1.73E-12	0.5017	0.0655	0.0135	0.0025	0.3016	0.0764
Item16	0.2217	0.0372	0.3551	0.0572	0.2308	0.0282	0.3949	0.1027
Item17	0.2412	0.027	0.2611	0.0979	0.2464	0.0361	0.6491	0.0512
Item18	0.1881	0.0326	0.2547	0.0432	0.2716	0.0318	0.4105	0.0864
Item19	0.0815	0.0163	0.0561	0.0186	0.0813	0.0214	0.4923	0.0658
Item20	0.0289	0.013	0.4362	0.063	0.0308	0.0089	0.4365	0.1066
Item21	0.027	0.0118	0.6576	0.0699	0.0164	0.0054	0.001	0.0002
Item22	0.1857	0.0236	0.6375	0.1544	0.1822	0.0319	0.7787	0.0445
Item23	0.0718	0.017	0.373	0.0955	0.0289	0.0112	0.4535	0.0633
Item24	0.0744	0.0137	0	0	0.0751	0.0179	0.624	0.0677
Item25	0.2035	0.0266	0.5505	0.1138	0.2188	0.0294	0.6652	0.0808
Item26	0.2483	0.0412	0.7135	0.0583	0.2298	0.028	0.5368	0.1212
Item27	1.8E-08	4.02E-09	0.197	0.0344	0.17	0.0227	0	0
Item28	0.1694	0.0253	0.6363	0.1109	0.1401	0.0247	0.5903	0.0709
Item29	0.1424	0.0213	0.274	0.0754	0.1329	0.0246	0.4998	0.0646
Item30	0.1865	0.0305	0.0908	0.0178	0.2215	0.027	0	0
Item31	9.10E-16	2.58E-16	0.1567	0.0276	0.0291	0.0059	0	0
Item32	3.2E-110	9.5E-111	0	0	1.08E-145	4.7E-146	0.385	0.0573
Mean	0.1537		0.3462		0.1185		0.3462	

The average values of the guessing and slipping parameters in the DINA model are 0.1537 and 0.3462. The mean guessing parameter indicates that for the students who have not mastered all the required skills for an item, there is still, on average, a 15.37 percent chance that they will choose the correct response and the average slipping parameter indicates that for the students who have mastered all the skills required for an item, there is still, on average, a 34.61 percent chance that they will choose the incorrect response. The most informative items on a test are the ones whose slipping and guessing probabilities are low (Rupp et al., 2010). Generally speaking, small guessing and slipping parameters indicate a good fit between the diagnostic assessment design, the response data, and the postulated DINA model. The table above shows each item guess and slip parameters based on the DINA model, the information in this table has the lowest guessing coefficients for item 32 and 31 with 3.2E-110 and 9.10E-16 the highest guessing coefficients its belong to the item#4 and 12 with values of 0.364 and 0.3332 these coefficients are likely to answer the question correctly for students demonstrates that they do not have the skills needed to answer the question. Also, the lowest slip value is related to items #2, 24 and 32 with values all are equal

to the 0 and the highest slip coefficient is related to items#14 and 10 with values of 0.7533 and 0.7315 This coefficient indicates the probability of students answering the question incorrectly have the skills needed to answer the question. And also, the item of guess and slip parameters based on the DINO model, the information in this table has the lowest guessing coefficients for item 32 with $1.08E-145$ and the highest guessing coefficients it belongs to the item#4 with values of 0.3742. Also, the lowest slip value is related to items #27, 30 and 31 with values all are equal to the 0 and the highest slip coefficient is related to items#22 with values of 0.7787 This coefficient indicates the probability of students answering the question incorrectly have the skills needed to answer the question.

IV. Discussion

This research aimed at evaluating the application of two popular core Cognitive Diagnosis Models, the Deterministic Input Noisy “And” gate (DINA) and Deterministic Input Noisy “Or” gate (DINO) by identifying school students’ mathematic abilities at grade 8. The analysis was done to show the level of probability in every attribute in the questionnaire. The results demonstrated that the highest probability of mastery belonged to attribute 4 at (0.4836). However, the lowest probability belonged to attribute 24 and 32 which is (0.12). Then, another descriptive analysis was done to show the level of probability in every item in the questionnaire. The results showed that the highest probability of mastery belonged to item four at (0.489). However, the lowest probability belonged to item 21, which is (0.0875). The same analysis was calculated on the DINO model to demonstrate each item’s guess and slip parameters. Results show that the lowest guessing coefficients for item 32 with $1.08E-145$ and the highest guessing coefficients belonged to the item#4 with values of 0.3742. in addition, the lowest slip value related to items #27, 30 and 31 with values all equals to the 0 and the highest slip coefficient is related to items#22 with values of 0.7787. This coefficient indicates the probability of students answering the question incorrectly has the skills needed to answer the question.

This result is in line with Afzaly. et al. (2016) they found that eight basic attributes explain the mathematical performance of first- grade high school students. Rahimi, et al. (2018) also found that most of the attribute was not mastered in each skill, but the status of the individuals in the SUM skill. In addition, de la Torre and Sun Lee (2010) focused on one CDM, the deterministic inputs, noisy “and” gate (DINA) model, and the invariance property of its parameters. Using simulated data involving different attribute distributions, they found that the DINA model parameters are absolutely invariant when the model perfectly fits the data. Another related study was conducted by Ravand (2016) which demonstrated the application of the G-DINA to the reading comprehension data of a high-stakes test. The study showed Syntax was the easiest and Inference was the most difficult attribute. The second most difficult attribute was the Main Idea, followed by Detail and Vocab. The same results were also found by (Grabe& Stoller, 2002; Lumley, 1993). Moreover, the findings of this study are in line with those of Baghaei and Ravand (2015) who applied the linear logistic test model to these data . Further, Yi Chiuand Ko`hn (2015) prove that the ACTCD also found that an extension to the statistical framework of the ACTCD, originally developed for test data conforming to the Reduced Reparameterized Unified Model or the General Diagnostic Model is valid also for both the DINA model and the DINO. Additionally, Kaya and Leite (2017) present longitudinal models for CDM. They

indicate that the proposed models provide adequate convergence and correct classification rates. Finally, Yamaguchi and Okada (2018) examined which CDMs better fit the actual data by comparatively fitting representative CDMs to (TIMSS, 2007) assessment data across seven countries. First, CDMs were shown to have a better fit than did the item response theory models. Second, the main effects models generally had a better fit than other parsimonious or saturated models. Related to the second finding, the fit of the traditional parsimonious models such as the DINA and DINO models were not optimal.

Thus, related studies show that CDM has been applied in different contexts such as mathematics and language contexts. However, studies also show that there are not enough studies conducted in the mathematic context.

V. Conclusion

This research aimed at evaluating the application of two popular core Cognitive Diagnosis Models, the Deterministic Input Noisy “And” gate (DINA) and Deterministic Input Noisy “Or” gate (DINO) by identifying school students’ mathematic abilities at grade 8. This research also tried to determine the 8th grade students’ level of mathematics at the school level. The research applied Trends in International Mathematics and Science Study (TIMSS) 2011 mathematics assessment in order to evaluate DINA and DINO models by examining the Mathematical abilities of students in Grade 8. It measured 13 attributes which included 32 questions.

First, a descriptive analysis was done on the DINA model to show the level of probability in every attribute in the questionnaire. The results demonstrated that the highest probability of mastery belonged to attribute 4 at (0.4836). However, the lowest probability belonged to attribute 24 and 32 which is (0.12). Then, another descriptive analysis was done to show the level of probability in every item in the questionnaire. The results showed that the highest probability of mastery belonged to item four at (0.489). However, the lowest probability belonged to item 21, which is (0.0875). Secondly, the same analysis was calculated on the DINO model to demonstrate each item's guess and slip parameters. Results show that the lowest guessing coefficients for item 32 with 1.08E-145 and the highest guessing coefficients belonged to the item#4 with values of 0.3742. in addition, the lowest slip value related to items #27, 30 and 31 with values all equals to the 0 and the highest slip coefficient is related to items#22 with values of 0.7787. This coefficient indicates the probability of students answering the question incorrectly has the skills needed to answer the question.

On the other hand, the R software analyses were done to show the levels of Guess and Slip in the DINA and DINO models. The results on average values of the guessing and slipping parameters are 0.1537 and 0.3461. The mean guessing parameter shows that the participants who did not master all the required skills for an item chose the correct response. However, the participants who mastered all the required skills for an item chose the incorrect response.

In addition, a calculation was done to show values related to each item in Guess and Slip parameters based on the DINA model. Findings on Guess showed that the lowest guessing coefficients belonged to item #32 with 3.19E-110 respectively. However, the highest guessing coefficients belonged to item 4 with values of 0.364 respectively. So,

these coefficients might answer the question correctly for students who tended not to have the skills needed to answer the question.

Findings on Slip on the other hand showed that the lowest slip value was related to items #2, 24 and 32 with values all equal to the 0 while the highest slip coefficient is related to items#14 with values of 0.7533. So, this coefficient specifies the probability of students answering the question incorrectly, who had the skills needed to answer the question.

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