Effect of Variable Viscosity on Linear Thermal Convection in Ferromagnetic Liquids under Terrestrial Gravity Condition

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Abstract: *The stability analyses of linear theory of ferroconvection in ferromagnetic liquid in the presence of variable viscosity have been studied under terrestrial gravity condition using the generalized Lorenz model.*
The obtained results show that the stability of the studied ferrofluid system depends on several parameters, namely, magnetic field dependent viscosity V, buoyancy M_1 *and non-buoyancy force* M_3*. Furthermore, the effects of various parameters on the critical stability parameters* Ra_s *and* k_c *are discussed in more detail through graphical and tabular illustrations.*

Keywords *— Thermal convection, Ferromagnetic liquid, Variable viscosity, Generalized Lorenz model.*

I. Introduction

Ferromagnetic liquids that are synthesized liquids have smart-liquid applications. It is well known that many chemists and physicists made a big effort to synthesize the stable magnetic ferrofluids to motivate some important technology. An ample material is now available on certain specific characteristic of ferrofluids. The study of thermo magnetic convective instability in ferromagnetic liquids has gained importance, because of several attractive features of the liquid. Two such properties of the liquid are the temperature-sensitivity of its viscosity as well as magnetic field sensitivity. Finlayson [4] was one of the firsts to make a detailed study of convective instability in a ferromagnetic liquid. Results on ferroconvection were studied by Gupta and Gupta [5], Gotoh and Yamada [6]. Stavans et al. [7] made the chaotic behavior in the system experimentally. Kaloni and Lou [8] made a theoretical investigation of stationary and oscillatory convection of a magnetic liquid layer placed horizontally which is heated from the bottom.. Siddheshwar and Abraham [9] analytically investigated the natural convection in a micro polar magnetic fluid layer and discussed micro polar magnetic liquids are more reliable than a Newtonian ferromagnetic liquid. Sunil and Mahajan [10] studied problems on thermal convection using ferrofluid and he showed that, the viscosity is constant and is not dependent on the magnetic field intensity. Sekhar et al. [11] studied the elastic effects on Benard convection in liquids with temperature-dependent viscosity in the thermal convection in ferromagnetic liquids for variable viscosity. They analyzed the convection for various parameters with heat source/sink. Loroze et al.[12] examined chaotic convection in ferromagnetic liquids.

The above works address the problem of natural convection in Newtonian liquids. An attempt was made under terrestrial gravity condition using the Lorenz model, how the variable viscosity has an effect on thermal convection. The entire paper is made up of 6 sections, methodology, basic equations are presented for ferromagnetic liquid in section I, and the generalized Lorenz model derivation is explained in section II, sections III includes linear stability. The paper finally culminates in the section of results and discussion.

II. Methodology

In the present paper, a generalized Lorenz model is used to get first order partial differential equations. Linear stability is studied by using ND solver in MATHEMATICA 12.0

III. Basic Equations

A large horizontal extension of ferrofluid of thickness 'd' in the xy-plane is considered in a ferromagnetic liquid layer, subjected to a vertical temperature and $\vec{g} = -g\,\hat{z}$ the gravitational field, followed by an application of vertical magnetic field. The lower and upper surfaces are maintained at constant temperature $T_0 + \Delta T$ and T_0

respectively. Within the Boussinesq approximation, non-dimensional equations for the perturbation from the basic state can be written as

$$\nabla.\mathbf{v} = \mathbf{0}, \tag{1}$$

$$\frac{1}{Pr} d_t \mathrm{v} = -\nabla \mathbf{p}_{\mathbf{eff}} + Ra \sum(\theta, \Phi) + \nabla.[\mu(\mathbf{H},\mathbf{T})(\nabla\vec{\mathbf{q}}\,\nabla.\vec{\mathbf{q}}^{\mathbf{Tr}})], \tag{2}$$

$$\mathbf{d_t}\theta = \nabla^2\theta + v_z, \tag{3}$$

$$\frac{\partial^2\Phi}{\partial z^2} + \mathbf{M_3}\left(\frac{\partial^2\Phi}{\partial x^2} + \frac{\partial^2\Phi}{\partial y^2}\right) - \frac{\partial\theta}{\partial z} = 0 \tag{4}$$

Where the non - dimensional perturbations of velocity, temperature and magnetic scalar potential are v, θ and Φ respectively.

Here $d_t\mathbf{f} = \partial_t\mathbf{f}$ + v. ∇f, is a material derivative, $\mathbf{p}_{\mathbf{eff}}$, effective pressure and magnetic force

$\sum \boldsymbol{\pi}_1\,(\theta, \Phi)\hat{\mathbf{z}} + \boldsymbol{M}_1\nabla\theta\,\partial_z\Phi$ with $\boldsymbol{\pi}_1 = (\mathbf{1} + \boldsymbol{M}_1)\theta - \boldsymbol{M}_1\partial_z\Phi$. The Rayleigh number, $Ra = \frac{\alpha_T g \beta \mathbf{d}^4}{\kappa\upsilon}$, the Prandtl number , $Pr = \frac{\upsilon}{\kappa}$, the buoyancy magnetization parameter, $\mathbf{M_1} = \frac{\beta\,\chi_1{}^2\mathbf{H_0}^2}{\alpha_T\,\rho_0\,g(1+\chi_m)}$, the non- buoyancy magnetization parameter, $\mathbf{M_3} = \frac{1+\chi}{(1+\chi+\chi\mathbf{H_0}^2)}$.

IV. Generalized Lorenz System Equation

The convective flow is limited to two dimensions in the xz- plane. We now express velocity in respect of stream function, ψ, defined by $v = (\partial_z\psi, \mathbf{0}, \partial_x\psi)$ and hence the set of equations (2) - (4) on eliminating the pressure and non dimensionalizing as in Loroze et al. [12].

$$\frac{1}{Pr} d_t\,\nabla^2\Psi = -Ra\,(\mathbf{1} + \boldsymbol{M}_1)\,\frac{\partial\theta}{\partial x} - Ra\boldsymbol{M}_1\left(\frac{\partial^2\Phi}{\partial x\partial z}\right) + Ra\mathbf{M_1}\left[\frac{\partial\theta}{\partial x}\frac{\partial^2\Phi}{\partial z^2} - \frac{\partial\theta}{\partial z}\,\frac{\partial^2\Phi}{\partial x\partial z}\right] +$$

$$g_1(z)\nabla^4\Psi + 2\,D\,[g_1(z)]\,\nabla^2\Psi + D^2\,[g_1(z)]\left(\frac{\partial^2\psi}{\partial z^2}\right) - D^2\,[g_1(z)]\left(\frac{\partial^2\psi}{\partial x^2}\right) \tag{5}$$

$$d_t\theta = \nabla^2\theta + \frac{\partial\psi}{\partial x} \tag{6}$$

$$\frac{\partial^2\Phi}{\partial z^2} + \boldsymbol{M}_3\left(\frac{\partial^2\Phi}{\partial x^2}\right) - \frac{\partial\theta}{\partial z} = 0, \tag{7}$$

Where $D = \frac{d}{dz}$, $\quad d_t\mathbf{f} = \frac{\partial f}{\partial t} + \frac{\partial\psi}{\partial x}\frac{\partial f}{\partial z} - \frac{\partial\psi}{\partial z}\frac{\partial f}{\partial x}$, $\quad \nabla^2\mathbf{f} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial z^2}$ and $\quad \nabla^4\mathbf{f} = \frac{\partial^4 f}{\partial x^2} + \mathbf{2}\frac{\partial^4 f}{\partial x^2\partial z^2} + \frac{\partial^4 f}{\partial z^2}$

Effective viscosity μ_b and the $g_1(z)$ are

$$\mu_b = \mu_0\,[\,1 + \delta_H(H_b - H_0)^2 - \delta_T(T_b - T_0)^2\,]$$

$$= \mu_0\left[1 - (\delta_T(\Delta T)^2(1-z)^2 + \left(\frac{\delta_H\,k_l{}^2}{1+\chi_m}\right)(\Delta T)^2(1-z)^2)\right]$$

$g_1(z) = [1 - V(1-z)^2]$

Where $V = \left(\delta_T - \frac{\delta_H k_l^2}{1+\chi_m}\right)(\Delta T)^2$

We consider the following boundary condition for temperature, stream function and scalar magnetic potential.

$$\theta = \psi = \frac{\partial^2 \psi}{\partial z^2} = \frac{\partial \Phi}{\partial z} = 0, \text{ at } z = 0, 1. \tag{8}$$

Finite amplitude convection in ferromagnetic liquid is studied by applying double Fourier series and is given by

$$\Psi = \frac{1}{k} a_1(t) Sin(kx) Sin(\pi z), \tag{9}$$

$$\theta = a_2(t) Cos(kx) Sin(\pi z) + a_3(t) Sin(2\pi z), \tag{10}$$

$$\Phi = a_4(t) Cos(kx)\, Cos(\pi z) + a_5(t) Cos(2\pi z), \tag{11}$$

Substituting these trial functions in Eqs. (5) - (7) followed by multiplying with orthogonal eigen functions and integrating $\int_{\frac{\pi}{k}}^{\frac{\pi}{k}} \int_0^1 dxdz$, we obtain

$$\frac{1}{Pr} a_1'(t) = -q^2 a_1(t) - q^4 r\, a_2(t)\; [1 - M_{13} a_3(t)] - f(v)\, a_1(t) \tag{12}$$

$$a_2'(t) = -q^2 a_2(t) - a_1(t) - \pi a_1(t) a_3(t) \tag{13}$$

$$a_3'(t) = -4q^2 a_3(t) + \frac{\pi}{2} a_1(t) a_2(t) \tag{14}$$

Where $f(V) = 1 + \frac{V}{6\pi^2 q^4} [(3-2\pi^2) q^4 + 12\pi^4 - 12k^2q^2 - 6\pi^2q^2]$, $r = \frac{Ra}{Ra_s}$, $q^2 = \pi^2 + k^2$,

$M_{13} = \frac{\pi^2 k^2 M_1 M_3}{[\pi^2 + k^2(1+M_1)M_3]}$

In obtaining Eq. (11), we have written down $a_4(t)$ and $a_5(t)$ in terms of $a_2(t)$ and $a_3(t)$ respectively. The critical Rayleigh number Ra_s of stationary convection is obtained in the next section.

V. Linear Stability

To perform a linear stability analysis, the linearized steady - state version of Eqs. (12) - (13) is considered to obtain,

$$\begin{bmatrix} -q^2 f(v) & Ra_s \frac{k_c^2[\pi^2 + k_c^2(1+M_1)M_3]}{q^2(k_c^2 M_3 + \pi^2)} \\ 1 & q^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix} = 0 \tag{15}$$

For a non- trivial solution to the Eq. (15), we require Ra_s to take the form,

$$Ra_s = \frac{q^6(k_c{}^2 M_3+\pi^2)}{k_c{}^2(k_c{}^2 M_3+k_c{}^2 M_1 M_3+\pi^2)}\,\mathbf{f}(\mathbf{v}) , \qquad (16)$$

where $f(V) = 1+\frac{V}{6\,\pi^2 q^4}\,[(3-2\pi^2)\,q^4 + 12\pi^4 - 12k^2q^2 - 6\pi^2q^2]$

Results and Discussion

The convergence is obtained by using algebra package of Mathematica 12.0 to compute numerically, the minimum value of $\mathrm{Ra_s}$ $corrresponding\ k_c$, for given values of various parameters $V,\ M_1\ and\ \ M_3$. In the case of linear stability analysis, Fig. 1.(a) and (b) are the plot of $Ra_s\ and\ k_c$ versus V for M_1 = 1, varying with M_3 and Fig. 2.(a) and (b) are the plot of $Ra_s\ and\ k_c$ versus V for M_3 = 1, varying with M_1. From these figures, we can see that the effect of increasing or decreasing the variable viscosity parameter V is to decrease Ra_s . This implies that the varying viscosity effect is to destabilize the system with increasing values of $M_1$and M_3, therefore M_1 has a destabilizing effect on the ferrofluid. M_3 is the departure from the linearity of the magnetic equation of the state. As M_3 large, the fluid layer is slightly destabilized. The problem of gravity driven convection with non- existence of magnetic buoyancy maagnetization parameter (M_1 = 0) , the results of Ra_s is usual to the viscous fluid case. But it is seen that other values of M_1 (M_1 = 0) destabilizes a ferrofluid system. Figs. 1 to 2 shows that, increasing in the non-linearity of magnetization parameter M_3 with Ra_s. Hence it is destabilizes the system. Also it reveals that as the magnetic equation of the state departure from linearity when M_3 is very large, the system is slightly destabilized.

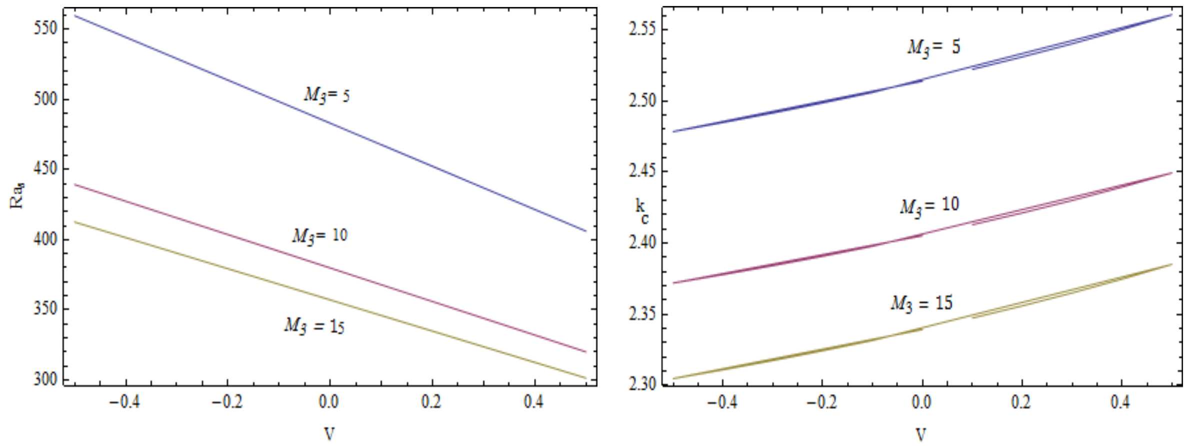

Figure 1. Variation of $Ra_s\ and\ k_c$ on V by varing M_3 and with a fixed value of M_1 = 1.

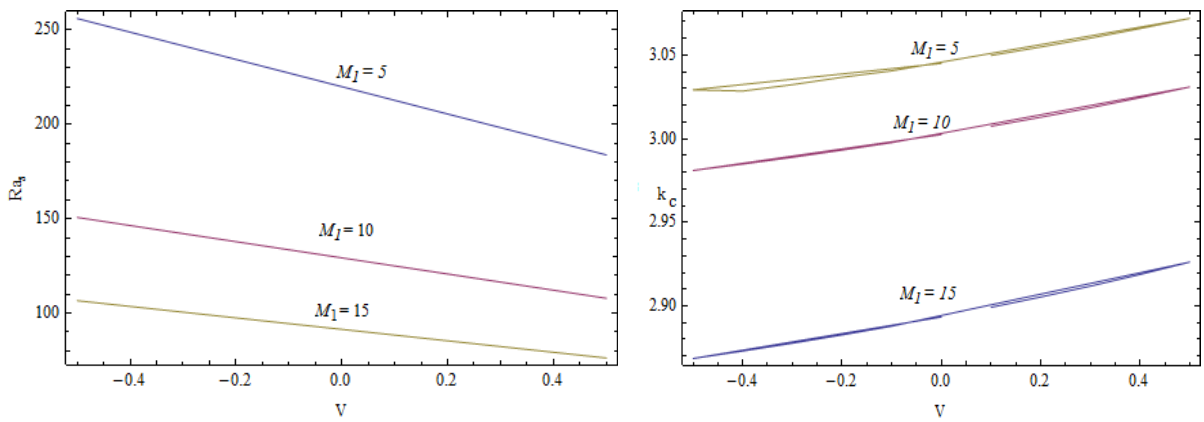

Figure 2. Variation of $Ra_s\ and\ k_c$ on V by varing M_1 and with a fixed value of M_3 = 1.

Table 1: Comparison of critical Rayleigh number R_s and corresponding wave number k_c for the limiting case V = 0.

Boundary condition	Sekhar et al. [13]		Present study	
Free- free	R_s	k_c	R_s	k_c
	657.51380	2.22086	657.51300	2.22078

Conclusion

- The stability analyses of linear and non - linear ferromagnetic fluid convection in the presence of a variable viscosity dependent on a magnetic field are studied.
- Increasing the variable viscosity V has a destabilizing effect and the system stabilizes when variable viscosity decreases.
- It is an advantage to use the Lorenz model viscosity variable when parameters such as V, M_1 , M_3 of ferromagnetic fluid and magnetic field, it thus delays the onset of convection.

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