The First and Second Zagreb Indices of Degree Splitting of Graphs

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Abstract

Let G be simple graph. The first Zagreb index is the sum of squares of degree of vertices and second Zagreb index is the sum of the products of the degrees of pairs of adjacent vertices. In this paper, we compute the first and second Zagreb index of degree splitting of standard graphs.

Keywords - *Zagreb Indices, Degree Splitting of graph.*

I. INTRODUCTION

The first and second Zagreb indices first appeared in a topological formula for the total -energy of conjugated molecules, were introduced by Gutman and Trinajstić in 1972. Since then these indices have been used as branching indices. The Zagreb indices are found to have applications in QSPR and QSAR studies.

Let G be a simple connected graphs, i.e.) connected graphs without loops and multiple edges. For a graph G, $V(G)$ and $E(G)$ denote the set of all vertices and edges respectively. For a graph G, the degree of a vertex v is the number of edges incident to v and denoted by $deg(v)$. The first Zagreb index M_1 (G) is equal to the sum of squares of the degrees of the vertices, and the second Zagreb index M_2 (G) is equal to the sum of the product of the degrees of pairs of adjacent vertices of the underlying molecular graph G.

They are defined as:

$$M_1(G) = \sum_{v \epsilon V(G)} d(v)^2$$

$$M_2(G) = \sum_{uv \in E(G)} d(u)d(v)$$

Let G = (V, E) be a graph with $V = S_1 \cup S_2 \cup \cdots \cup S_t \cup T$ where each S_i is a set of vertices having at least two vertices and having the same degree and $T = V - \cup S_i$. The degree splitting graph of G is denoted by DS(G) is obtained from G by adding vertices $w_1, w_2, \cdots, w_t$ and joining w_i to each vertex of $S_i (1 \le i \le t)$.

My research is to find the first and second Zagreb index of degree splitting of graphs like Paths, Cycle, Wheel, Complete Graph, Star,Complete Bi-partite Graphs, Path Corona and Cycle corona.

Theorem 1: For a PathP_n,$n \geq 3, M_1(DS(P_n)) = n^2 + 5n - 6, M_2(DS(P_n)) = 3n^2 - 3n + 5.$

Proof:

Let $v_1, v_2, \ldots, v_n$ be the vertices of P_n. Let w_1 and w_2 be the degree splitting vertices in which w_1 is adjacent to the end vertices and w_2 is adjacent to the remaining vertices.

Then, $d(v_i) = \begin{cases} 2 & i = 1, n \\ 3 & otherwise \end{cases}$, $d(w_1) = 2, d(w_2) = n - 2.$

$$M_1(DS(P_n)) = \sum_{i=1}^{n} d(v_i)^2 + \sum_{i=1}^{2} d(w_i)^2$$

$$= d(v_1)^2 + \sum_{i=2}^{n-2} d(v_i)^2 + d(v_n)^2 + d(w_1)^2 + d(w_2)^2$$

$$= 2^2 + \sum_{i=2}^{n-2} 3^2 + 2^2 + 2^2 + (n-2)^2$$

$$= n^2 + 5n - 6.$$

$$M_2(\,DS(P_n)) \;=\sum_{uv\in E} d(u)d(v)$$

$$= \;d(v_1)\,d(v_2) + d(v_{n-1})d(v_n) + \sum_{i=1}^{n-1} d(v_i)d(v_{i+1}) + \sum_{i\neq 1,n} d(v_i)d(w_2)$$

$$= 2(2) + 2(2) + 2(3) + [n-3]3(3) + 2(3) + [n-2]3(n-2)$$

$$= 3n^2 - 3n + 5.$$

Note: For n = 2, there is only one degree splitting vertex. Therefore, $M_1(\,DS(P_2)) = M_2(\,DS(P_2)) = 12$.

Theorem 2: For Cycle, $C_{n,}n \geq 3, M_1(\,DS(C_n)) = n^2 + 9n$, $M_2(\,DS(C_n)) = 9n + 3n^2$.

Proof:

Let v_1, v_2, …,v_n be the vertices of C_n. Let w_1 be the degree splitting vertex.

Then $d(v_i) = 3$ for all i, $d(w_1) = n$.

$$M_1(\,DS(C_n)) \;=\sum_{i=1}^{n} d(v_i)^2 + d(w_1)^2$$

$$= \sum_{i=1}^{n} 3^2 + n^2$$

$$= 3^2(n) + n^2$$

$$= n^2 + 9n.$$

$$M_2(\,DS(C_n)) \;=\sum_{uv\in E} d(u)d(v)$$

$$= \sum_{i=1}^{n} 3(3) + \sum_{i=1}^{n} 3(n)$$

$$= 9n + 3n^2.$$

Theorem 3: For Wheel $W_{n,}n \geq 4, M_1(\,DS(W_n)) = 2n^2 + 18n + 2$, $M_2(\,DS(W_n)) = 8n^2 + 21n + 1$.

Proof:

Let v_1, v_2, …,$v_n$and u be the vertices of W_n. Let $w_1$and $w_2$be the degree splitting vertices.

Then $d(v_i) = 4$ for all i $d(u) = n + 1, d(w_1) = n, d(w_2) = n$..

$$M_1(\,DS(W_n)) \;=\sum_{i=1}^{n} d(v_i)^2 + d(u)^2 + d(w_1)^2 + d(w_2)^2$$

$$= \sum_{i=1}^{n} 4^2 + (n+1)^2 + n^2 + 1^2$$

$$= 4^2(n) + 2n^2 + 2n + 2$$

$$= 2n^2 + 18n + 2.$$

$$M_2(\,DS(W_n)) = \sum_{uv\in E} d(u)d(v)$$

$$= \sum_{i=1}^{n} 4(4) + \sum_{i=1}^{n} 4(n+1) + \sum_{i=1}^{n} 4(n) + (n+1)$$

$$= 16n + 4n(n+1) + 4n^2 + n + 1$$

$$= 8n^2 + 21n + 1.$$

Note: For n =3, there is only one degree splitting vertex, $M_1(\,DS(W_3)) = 80$,$M_2(\,DS(W_n)) = 160$.

Theorem 4: For the Complete graphK_n, $n \geq 3, M_1(\,DS(K_n)) = n^2(n+1)$,$M_2(\,DS(K_n)) = n^3\left(\frac{n+1}{2}\right)$.

Proof:

Let $v_1, v_2, \ldots, v_n$ be the vertices of K_n. Let w_1 be the degree splitting vertex.

Then $d(v_i) = n$ for all i $d(w_1) = n$.

$$M_1(\,DS(K_n)) = \sum_{i=1}^{n} d(v_i)^2 + d(w_1)^2$$

$$= \sum_{i=1}^{n} n^2 + n^2$$

$$= n^2(n) + n^2$$

$$= n^2(n+1).$$

$$M_2(\,DS(K_n)) = \sum_{uv\in E} d(u)d(v)$$

$$= \sum_{i=1}^{nC_2} n(n) + \sum_{i=1}^{n} n(n)$$

$$= n^2.\frac{n(n-1)}{2} + n^3$$

$$= n^3\left(\frac{n+1}{2}\right).$$

Theorem 5: For the Star$K_{1,n}$, $n \geq 2$,$M_1\left(\,DS\left(K_{1,n}\right)\right) = 2n^2 + 6n + 2$, $M_2\left(\,DS(K_{1,n})\right) = 4n^2 + 3n + 1$.

Proof:

Let v and $u_1, u_2, \ldots, u_n$ be the vertices of $K_{1,n}$ and w_1 and w_2 be the degree splitting vertices. Then,d(v) $= n+1, d(u_i) = 2$ for all i and $d(w_1) = 1, d(w_2) = n$

$$M_1\left(\,DS(K_{1,n})\right) = d(v)^2 + \sum_{i=1}^{n} d(u_i)^2 + d(w_1)^2 + d(w_1)^2$$

$$= (n+1)^2 + \sum_{i=1}^{n} 2^2 + 1^2 + n^2$$

$$= (n+1)^2 + n(2)^2 + 1 + n^2$$

$$= 2n^2 + 6n + 2.$$

$$M_2(\,DS(K_{1,n})) = \sum_{uv\in E} d(u)d(v)$$

$$= \sum_{i=1}^{n} 2(n+1) + \sum_{i=1}^{n} 2(n) + (n+1)1$$

$$= 2n(n+1) + 2n^2 + n + 1.$$

$$=4n^2 + 3n + 1.$$

Theorem 6: For the Complete Bi-partite graph $K_{m,n}$, $m, n \geq 2$

$$M_1(\,DS(K_{m,n})) = m^2(n+1) + n^2(m+1) + 4mn + m + n,$$

$$M_2(\,DS(K_{m,n})) = [(m+1)(n+1)]mn + (n+1)m^2 + (m+1)n^2.$$

Proof:

Let $v_1, v_2, \ldots, v_m$ and $u_1, u_2, \ldots, u_n$ be the vertices of $K_{m,n}$ and w_1 and w_2 be the degree splitting vertices. Then $d(v_i) = n+1, d(u_i) = m+1$ for all i and $d(w_1) = m, d(w_2) = n$.

$$M_1(\,DS(K_{m,n})) = \sum_{i=1}^{m} d(v_i)^2 + \sum_{i=1}^{n} d(u_i)^2 + d(w_1)^2 + d(w_1)^2$$

$$= \sum_{i=1}^{m} (n+1)^2 + \sum_{i=1}^{n} (m+1)^2 + m^2 + n^2$$

$$= m(n+1)^2 + n(m+1)^2 + m^2 + n^2$$

$$= m^2(n+1) + n^2(m+1) + 4mn + m + n.$$

$$M_2(\,DS(K_{m,n})) = \sum_{uv\in E} d(u)d(v)$$

$$= \sum_{i=1}^{mn} (m+1)(n+1) + \sum_{i=1}^{m} (n+1)m + \sum_{i=1}^{n} (m+1)n$$

$$= [(m+1)(n+1)]mn + (n+1)m^2 + (m+1)n^2.$$

Theorem 7: For a Path Corona $P_n{}^+$, $n \geq 3$, $M_1(\,DS(P_n{}^+)) = 2n^2 + 16n - 6$, $M_2(\,DS(P_n{}^+)) = 6n^2 + 8n$.
Proof:

Let $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$ be the vertices of $P_n{}^+$. Let w_1, w_2 and w_3 be the degree splitting vertices.

Then, $d(v_i) = \begin{cases} 3 & i = 1, n \\ 4 & otherwise \end{cases}$, $d(u_i) = 2 \; for\; all\; i, d(w_1) = 2, d(w_2) = n - 2$ and $d(w_3) = n$.

$$M_1(\,DS(P_n{}^+)) = \sum_{i=1}^{n} d(v_i)^2 + \sum_{i=1}^{n} d(u_i)^2 + d(w_1)^2 + d(w_1)^2 + d(w_3)^2$$

$$= 3^2 + \sum_{i=2}^{n-2} 4^2 + 3^2 + \sum_{i=1}^{n} 2^2 + 2^2 + (n-2)^2 + n^2$$

$$= (n-2)16 + n(4) + 2n^2 - 4n + 26$$

$$= 2n^2 + 16n - 6.$$

$$M_2(\,DS(P_n{}^+)) = \sum_{uv\in E} d(u)d(v)$$

$$= d(v_1)\,d(v_2) + \sum_{i=1}^{n-1} d(v_i)d(v_{i+1}) + d(v_{n-1})d(v_n) + d(v_1)\,d(w_1) + d(v_n)d(w_1)$$

$$+ \sum_{i\neq 1,n} d(v_i)d(w_3) + d(v_1)\,d(u_1) + d(v_n)d(u_n) + \sum_{i=2}^{n-1} d(v_i)d(u_i) + \sum_{i=1}^{n} d(u_i)d(w_2)$$

$$= 3(4) + [n-3]4(4) + 3(4) + 3(2) + 3(2) + [n-2]4(n-2) + 3(2) + 3(2) + [n-2]4(2) + n2(n)$$

$$= 6n^2 + 8n.$$

Note: For n = 2, there is only one degree splitting vertex. Therefore, $M_1(\,DS(P_2{}^+)) = 34, M_2(\,DS(P_2{}^+)) = 41$.

Theorem 8: For Cycle Corona, $C_n{}^+ n \geq 3, M_1\big(\,DS(C_n{}^+)\big) = 2n^2 + 20n$, $M_2\big(\,DS(C_n{}^+)\big) = 6n^2 + 24n.$

Proof:

Let $v_1, v_2, \ldots, v_n$ and $u_1, u_2, \ldots, u_n$be the vertices of $C_n{}^+$. Let $w_1$and $w_2$be the degree splitting vertices. Then $d(v_i) = 4, d(u_i) = 2$for all i $d(w_1) = d(w_2) = n.$

$$M_1\big(\,DS(C_n{}^+)\big) = \sum_{i=1}^{n} d(v_i)^2 + \sum_{i=1}^{n} d(u_i)^2 + d(w_1)^2 + d(w_1)^2$$

$$= \sum_{i=1}^{n} 4^2 + \sum_{i=1}^{n} 2^2 + n^2 + n^2$$

$$= 4^2(n) + 2^2(n) + 2n^2$$

$$= 2n^2 + 20n.$$

$$M_2\big(\,DS(C_n{}^+)\big) = \sum_{uv\in E} d(u)d(v)$$

$$= \sum_{i=1}^{n} 4(4) + \sum_{i=1}^{n} 4(n) + \sum_{i=1}^{n} 4(2) + \sum_{i=1}^{n} 2(n)$$

$$= 16n + 4n^2 + 8n + 2n^2$$

$$= 6n^2 + 24n.$$

II. CONCLUSION

In this paper, I have computed the first and second Zagreb indices of some standard graphs. Further work is going on some special types of graphs and molecular structures.

III. ACKNOWLEDGEMENT

This work is supported by the Institutional Fund, Mannar Thirumalai Naicker College, Madurai.

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