On Harmonic Mean of Certain Triangular Line Graphs

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Abstract
We investigate the harmonic mean of several standard line graphs such as path, Triangular line snakes graphs, Quadrilateral lien snakes graphs.etc.

Keywords * Graph, Harmonic mean graph, line graphs, triangular snake graphs, quadrilateral snake graph.
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I. INTRODUCTION

In this paper we consider simple graphs. Let G= (V, E) be a graph with p vertices and q edges. For all terminologies and notations we follow Harary [1] . There are several types of labeling and detailed survey can be found in . The concept of mean labeling has been introduced in [1] and the Harmonic mean labeling was introduced in [4]. The concept of Double Triangular snake and Double Quadrilateral snake has been proved in [5] and [6]. In this paper we prove that the Harmonic mean behaviour of Triangular line snake graphs , Quadrilateral snake line graphs. Also we wish to investigate Harmonic mean of such graphs.
The following definitions are necessary for our present investigation

Definition 1.1 : A graph G=(V,E) with p vertices and q edges is called Harmonic mean graph if its possible to label the vertices $x \in V$ with distant lables from 1,2,3......q+1 in such that when edge e=uv is labled with $f(e = uv) = \frac{2f(u).f(v)}{f(u) + f(v)}$. Then the edge labels of distinct ,then "f" is called Harmonic mean labelling inn Graph G.

Definition1.2 : Line graphs of a triangular snake graphs
The line graph L(G) of an undirected graph G is another graph L(G) that represent the adjacent between edges of G. In other words given graph G, its line graph L(G) is a graph such that
(1). Each vertex of L(G) represents an edge of G.
(2). The vertices of L(G) are adjacent iff, there corresponding edges are adjacent to G.

Definition 1.3 : A Triangular snake Tn is obtained from a path u1u2…..un by joining ui and ui+1 to a new vertex vi for 1≤i≤n-1. That is every edge of a path is replaced by a triangles C3

Definition 1.4 : A Quadrilateral snake Qn is obtained from a path u1u2..un by joining ui and ui+1 to new vertices vi, wi respectively and then joining vi and wi, 1≤i≤n-1. That is, every edge of a path is replaced by cycle C4.

II. MAIN RESULTS

Theorem 1.1 : Triangular snakes are Harmonic graphs.
Proof : Consider the following two conditions
Case 1 If the triangular starts from the vertex u_2,
Then we define a function $f : V(T_n) \to \{1,2,3 .... q+1\}$. Let us consider the function $f(u_i, u_{i+1}) = 2i - 1,\ for\ all\ q = 1,2,3....(n-1).\ f(u_i, v_i) = (2i-2)\ for\ all\ i = 2,4,6.......(n-2).$
$f(v_i, u_{i+1}) = 2i\ for\ all\ i = 2,4,6..........\ ....\ ...(n-2).$

Therefore f is Harmonic mean labelling.
Case 2 If the triangular starts from $u_{1,}$

Let us define a function $f : V(T_n) \rightarrow \{1,2,3 .... \quad .... \quad .q+1\}$

By this we can $f(u_i) = 2i-1, \quad for \quad all \quad i = 1,2,3 ...... \; n.$

$f(u_i, v_i) = 2i-1, \quad for \quad i = 1,3.5 .... \quad .... \quad .... \quad ..(n-1).$

$f(v_i, v_{i+1}) = 2i+1, \quad for \quad all \quad i = 1,3,5 .....(n-1).$

From case 1 and case 2 .
Hence the proof.

III. DEFINITION OF QUADRILATERAL SNAKE GRAPHS

Let us consider the path $u_1, u_2, u_3, ... \quad ..... \; u_n$ by using $u_{i,} u_{i+1}$ to new vertices v_i, w_i respectively and the joining theses vertices which gives a quadrilateral snake graph.

Theorem 1.2 : Quadrilateral snake graph Q_n is a Harmonic mean on such graphs.
Proof : Let us consider the quadrilateral snake graphs of line graphs Q_n.
Case 1 : Let us consider the quadrilateral snake graphs from u_2.
Let us define the new function
such that $f(u_1) = 1, f(u_2) = 2, f(u_3) = 5. \quad f(u_i) = f(u_{i-2}) \; for \quad all \quad i = 4,5 ....(n-1).$

$f(v_i) = 3, f(v_2) = 4. \quad f(v_i) = f(v_{i-2}) + 5 \quad for \quad all \quad i = 3,4,5 .... \quad .....(n-1).$

Then labeing for each edges of graph as $f(u_1, u_2), \; f(u_{2,} u_3) .... \quad .... \quad ... \; f(u_i, v_i) = f(u_{i-2}, v_{i-2}) + 5 \quad for \quad all \; i = 3,4,5 ... \; n.$

Therefore f is harmonic labeing.
Case 2 : Starts from u_1

$f : v(Q_n) = \{1,2,3 .... \quad .... \; q+1\}$ by $f(u_1) = 1, f(u_2) = 3, f(u_3) = 4.$

$f(u_i) = f(u_{i-2}) + 1, \; i = 3,4 .... \; n.$

$f(u_i) = f(u_i) + 1, \; i = 3,4 ....(n-1).$

$f(v_i) = f(u_i) - 1, i = 2,4, ... \quad .... \; n.$

From Case 1 and Case 2 , therefore f is Harmonic.

REFERENCES

[1] J.A.Gallian, A dynamic survey of graph labeling. The Electronic Journal of combinators 2. F.Harary, Graph theory, Narosa publishing House New Delhi.
[2] S.Somasundram and R.Ponraj, Mean labeling of graphs, National Academy of Science letters vol.26, p210-2013.
[3] S.Somasundaram and S.S.Sandhya Harmonic Mean Labeling of graphs, Communicated to Journal of Combinatorial Mathematics and Combinatorial Computing.
[4] C.David Raj, C. Jayasekaran and S.S.Sandhya, 'Harmonic Mean Labeling On Double Triangular Snakes' Global Journal of Theoretical and Applied Mathematics Sciences Volume 3, Number 2 (2013) pp.67-72.
[5] C.Jayasekaran, S.S.Sandhya and C. David Raj, 'Harmonic Mean Labeling On Double Quadrilateral Snakes' International Journal of Mathematics Research, Volume 5, Number 2 (2013), pp.251-256.
[6] Balakrishnan R.and Ranganathan K, A Text Book of Graph Theory, Springer, (2000).
[7] J.A. Bondy and U.S.R. Murty, graph theory with applications, Elsevier science publishing, new York, 1976.
[8] A.T. Balaban (ed), Chemical Applications of Graph Theory, Academic Press, NewYork (1976).
[9] K. T. Bali´nska, L. V. Quintas, Random Graphs with Bounded Degree, PublishingHouse of the Pozna´n University of Technology, Pozna´n (2011)
[10] J. Gross, J. Yellen, Graph Theory and its Applications, CRC Press, Boca Raton, Florida (1999).
[11] Harary F.Graph Theory, Narosa Publishing House, (1988). R. Neville, Graphs whose vertices are forests with bounded degree, Graph Theory Notes of New York, New York Academy of Sciences, LIV (2008) 12–21. L. V. Quintas, J.
[12] G. V. Ghodasara, Mitesh J. Patel "Some bistar related square sum graphs", International Journal of Mathematics Trends and Technology (IJMTT). V47(3):172-177 July 2017. ISSN:2231-5373. www.ijmttjournal.org. Published by Seventh Sense Research Group. Some bi-star related square sum graphs.