$\tau_1\ \tau_2$ – rg** closed Sets in Bigeneralized Topological Spaces

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Abstract
In this paper we define regular generalized star star closed sets and regular generalized star star open sets in Bigeneralized topological spaces. Also we study the properties of these sets.

Key words - rg * *closed sets, rg* *open sets.

I. INTRODUCTION

In 1963, J.C.Kelly [3] defined the study of Bitopological spaces. K.Chandrasekhara Rao and N.Palaniappan[2] introduced the concepts of regular generalized star closed sets and regular generalized star open sets in a topological space. K.Chandrasekhara Rao and K.Kannan extended the above concept to Bitopological spaces. Chandrasekhara Rao and Palaniappan introduced the concept of regular generalized star star closed sets and regular generalized star star open sets in topological spaces and studied their properties. In this paper, we introduce the concepts of $\tau_1\ \tau_2$ – regular generalized star star closed sets ($\tau_1\ \tau_2$ – rg** closed sets) and $\tau_1\ \tau_2$ – regular generalized star star open sets ($\tau_1\ \tau_2$ – rg* * open sets) and study their properties in Bigeneralized topological spaces.

II. PRELIMINARIES

Throughout this paper, (X, τ_1 , τ_2) or simply X denote a Bigeneralized topological space. The intersection (resp.union) of all τ_i– closed sets containing A (resp. τ_i –open sets contained in A) is called the τ_i - closure (resp.τ_i – interior) of A, denoted by τ_i –cl(A) (resp. τ_i –int(A)). For any subset A $\subseteq$ X, τ_i -rint(A) and τ_i –rcl(A) denote the regular interior and regular closure of a set A with respect to the topology τ_i - respectively. The set of all τ_2 - regular closed sets in X is denoted by τ_2 - R.C(X, τ_1 , τ_2). The set of all $\tau_1\ \tau_2$ - regular open sets in X is denoted τ_2 - R.O(X, τ_1 , τ_2). A^C denotes the complement of A in X. We shall require the following known definitions and results.

Definition: 2.1

Let X be a non-empty set. A subset τ of P(X) is said to be generalized topology on X if $\phi \in \tau$ and arbitrary union of elements of τ belongs to τ.

Definition: 2.2

Let X be a non empty set and let τ_1 and τ_2 be generalized topologies on X. Then the triple (X, τ_1,τ_2) is said to be bigeneralized topological space .

Definition: 2.3

A subset A of a Bigeneralized topological space (X, τ_1, τ_2) is called

(a) $\tau_1\tau_2$ –regular closed if τ_1 –c1 [τ_2-int(A)] = A.
(b) $\tau_1\tau_2$ – regular open if τ_1 – int [τ_2-cl(A)] =A.
(c) $\tau_1\tau_2$ – regular generalized closed ($\tau_1\ \tau_2$ – rg closed) in X if τ_2 – cl (A) $\subseteq$ U whenever A $\subseteq$ U and U is $\tau_1\ \tau_2$ – regular open in X.

(d) $\tau_1\tau_2$ –regular generalized open ($\tau_1\tau_2$-rg open) in X if $F \subseteq \tau_2 - \text{int}(A)$ whenever $F \subseteq A$ and F is $\tau_1\tau_2$ – regular closed in X.

(e) $\tau_1 \tau_2$ – regular generalized star closed ($\tau_1 \tau_2$ – rg* closed) in X if $\tau_2 - \text{rcl}(A) \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1 \tau_2$ – regular open in X.

(f) $\tau_1 \tau_2$ – regular generalized star open ($\tau_1 \tau_2$ – rg* open) in X if its complement is $\tau_1 \tau_2$ – regular generalized star closed ($\tau_1\tau_2$ – rg* closed) in X.

III. REGULAR GENERALIZED STAR STAR CLOSED SETS

***Definition:** 3.1*

A subset A of a bigeneralized topological space (X, τ_1, τ_2) is called $\tau_1\tau_2$ – regular generalized star star closed ($\tau_1 \tau_2$ – rg** closed) in X if $\tau_2\text{-cl}[\tau_1\text{-int}(A)] \subseteq U$ whenever $A \subseteq U$ and U is $\tau_1\tau_2$-regular open in X.

Example 3.2 : Let X ={ a,b,c,d} τ_1 ={ ϕ, X, {a,b},{b,d} ,{a,b,d}} τ_2 ={ ϕ, X, {a},{d}, {b,d}, {a,d}, {a,b,d}}. Take U = {b, d}. Then U is $\tau_1\tau_2$ regular open in X. {b} and {d} $\tau_1\tau_2$-rg** closed.

***Theorem:** 3.3*

Let A be a subset of a bigeneralized topological space (X, τ_1, τ_2). If A is $\tau_1\tau_2$-rg** closed then $\tau_2\text{-cl}[(\tau_1\text{-int}(A)]-A$ does not contain non empty $\tau_1\tau_2$-regular closed set.

Proof :

Suppose that A is $\tau_1\tau_2$-rg** closed. Let F be a $\tau_1\tau_2$-regular closed set such that $F \subseteq \tau_2\text{-cl}[\tau_1\text{-int}(A)]- A$. Then $F \subseteq \tau_2\text{-cl}[\tau_1\text{-int}(A)] \cap A^C$. Since $F \subseteq A^C$, we have $A \subseteq F^C$. Since F is $\tau_1\tau_2$ – regular closed set, F^C is $\tau_1\tau_2$-regular open. Since A is $\tau_1\tau_2$-rg ** closed , we have $\tau_2\text{-cl}[(\tau_1\text{-int}(A)] \subseteq F^C$. Therefore, $F \subseteq [\tau_2\text{-cl}[\tau_1\text{-int}(A)]]^C$. Also $F \subseteq \tau_2\text{-cl}[\tau_1\text{-int}(A)]$. Hence $F \subseteq \phi$. Therefore $F = \phi$.

Theorem: 3.4

If A is $\tau_1\tau_2$-rg**closed and B is $\tau_1\tau_2$- rg closed, then $A \cup B$ is $\tau_1 \tau_2$-rg** closed.

Proof :

Let $A\cup B \subseteq U$ and U is $\tau_1\tau_2$ – regular open in X. Since $A \subseteq U$ and A is $\tau_1\tau_2$-rg** closed, we have $\tau_2\text{-cl}[\tau_1\text{-int}(A)] \subseteq U$. Since $B \subseteq U$ and B is $\tau_1\tau_2$-rg closed, we have $\tau_2\text{-cl}(B) \subseteq U$. Now $\tau_2\text{-cl}[\tau_1\text{-int}(A\cup B)] \subseteq \tau_2\text{-cl}[\tau_1\text{-int}[A\cup \tau_2\text{-cl}(B)]] \subseteq \tau_2\text{cl}\tau_1 \text{ int } U \subseteq U$. Therefore, $A\cup B$ is $\tau_1\tau_2$-rg** closed.

Theorem: 3.5

If a subset A is $\tau_1\tau_2$-rg closed then A is $\tau_1\tau_2$-rg** closed.

Proof :

Let $A \subseteq U$ and U is $\tau_1\tau_2$-regular open. Since A is $\tau_1\tau_2$-rg closed, we have $\tau_2\text{-cl}(A) \subseteq U$. Hence $\tau_2\text{-cl}\,[\tau_1\text{-int}(A)] \subseteq U$. Therefore A is $\tau_1\tau_2$-rg** closed.

***Theorem:** 3.6*

Let A and B be subsets of such that $A \subseteq B \subseteq \tau_2\text{-cl}[\tau_1\text{-int}(A)]$. If A is $\tau_1 \tau_2$-rg** closed, then B is $\tau_1 \tau_2$ – rg** closed.

Proof :

Let $B \subseteq U$ and U is $\tau_1 \tau_2$-regular open in X. Since $A \subseteq B$, we have $A \subseteq U$, Since A is $\tau_1\tau_2$-rg** closed, we have

τ_2-cl[τ_1-int(A)] $\subseteq$ U. Since B $\subseteq$ τ_2-cl[τ_1-int(A)], we have τ_2-cl[τ_1-int(B)] $\subseteq$ τ_2-cl(B) $\subseteq$ τ_2-cl[τ_1-int (A)] $\subseteq$ U. Therefore, B is τ_1 τ_2-rg** closed.

***Theorem:** 3.7*

Suppose that $\tau_1\tau_2$-R.O(X,τ_1,τ_2) $\subseteq$ τ_2-C(X,τ_1,τ_2). Then every subset of X is $\tau_1\tau_2$-rg**closed.

Proof :

Let A be a subset of X. Let A $\subseteq$ U and U is $\tau_1\tau_2$-regular open in X. Since $\tau_1\tau_2$ –R.O (X, τ_1,τ_2) $\subseteq$ τ_2C(X, τ_1, τ_2). we have U is τ_2-closed in X. Since A $\subseteq$ U, we have τ_2-cl(A) $\subseteq$ τ_2-cl(U) = U. Therefore, τ_2-cl[τ_1-int(A)] $\subseteq$ τ_2-cl[A] $\subseteq$ U. Hence A is $\tau_1\tau_2$ –rg** closed.

IV. REGULAR GENERALIZED STAR STAR OPEN SETS

Definition: 4.1

A subset A of a bigeneralized topological space (X,τ_1,τ_2) is called $\tau_1\tau_2$-regular generalized star star open ($\tau_1\tau_2$ –rg** open) in X if its complement is $\tau_1\tau_2$-regular generalized star star closed ($\tau_1\tau_2$-rg** closed) in X.

Example 4.2 :

Let X = {a,b,c,d},τ_1 ={ϕ,X,{a,c},{c,d},{a,c,d}}τ_2 = {ϕ,X,{a},{c},{a,c}, {b,c},{a,b,c}}. {a,b,c} and {a,b,d} are $\tau_1\tau_2$rg** open sets and {a,b} is not $\tau_1\tau_2$rg** open set is obtained in the next theorem.

Theorem: 4.3

A subset A of a bigeneralized topological space (X, τ_1,τ_2) is τ_1 τ_2-rg** open if and only if, F $\subseteq$ τ_2-int[τ_1-cl(A)] whenever F $\subseteq$ A and F is $\tau_1\tau_2$-regular closed in X.

Proof :

Necessity : Let F$\subseteq$ A and F is $\tau_1\tau_2$-regular closed in X. Then $A^C \subseteq F^C$ and F^C is $\tau_1\tau_2$-regular open in X. Since A is $\tau_1\tau_2$-rg** open, we have A^C is $\tau_1\tau_2$-rg** closed. Hence, τ_2-cl[τ_1-int(A^C)] $\subseteq F^C$. Consequently, [τ_2-int[τ_1-cl(A)]]$^C \subseteq F^C$. Therefore, F $\subseteq$ τ_2-int[τ_1-cl(A)].

Sufficiency : Let $A^C \subseteq$ U and U is $\tau_1\tau_2$-regular open in X. Then $U^C \subseteq$ A and U^C is $\tau_1\tau_2$-regular closed in X. By our assumption, we have $U^C \subseteq$ τ_2-int[τ_1-cl(A)]. Hence [τ_2-int[τ_1-cl(A)]]$^C \subseteq$ U. Therefore, τ_2-cl[τ_1-int(A^C)] $\subseteq$ U. Consequently A^C is $\tau_1\tau_2$-rg** closed. Hence A is $\tau_1\tau_2$-rg** open.

***Theorem:** 4.4*

Let A and B be subsets such that τ_2 – int[τ_1- cl(A)] $\subseteq$B$\subseteq$A. If A is $\tau_1\tau_2$-rg** open, then B is $\tau_1\tau_2$-rg** open.

Proof:

Let F $\subseteq$ B and F is $\tau_1\tau_2$-regular closed in X. Since B $\subseteq$ A, we have F $\subseteq$ A. Since A is $\tau_1\tau_2$-rg** open, we have, F $\subseteq$ τ_2-int[τ_1-cl(A)] by theorem 4.3. Since τ_2-int[τ_1-cl(A) $\subseteq$ B, we have τ_2-int[τ_2-int[τ_1-cl(A)]] $\subseteq$ τ_2-int(B) $\subseteq$ τ_2-int[τ_1-cl(B)]. Hence F $\subseteq$ τ_2-int[τ_1-cl(A)] $\subseteq$$\tau_2$-int [$\tau_1$-cl(B)]. Therefore, B is $\tau_1\tau_2$-rg**open.

Theorem 4.5 : If a subset A is $\tau_1\tau_2$-rg** closed, then τ_2-cl[τ_1-int(A)] – A is $\tau_1\tau_2$-rg**open.

Proof :

Let F $\subseteq$ τ_2-cl[τ_1-int(A)] – A and F is $\tau_1\tau_2$-regular closed. Since A is $\tau_1\tau_2$-rg** closed. We have τ_2-cl[τ_1-int(A)] – A does not contain nonempty $\tau_1\tau_2$-regular closed {by theorem 3.3} Therefore, F = ϕ , hence τ_2-cl[τ_1-int(A)] – A is $\tau_1\tau_2$-rg** open.

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