

Functions on α^*g –Open Set in Topological Spaces

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Abstract

The purpose of this paper is to introduce α^*g –open maps and α^*g –closed maps in topological spaces and discuss its properties. Additionally, we relate and compare these functions with some other functions in topological spaces.

Keywords - α^*g –open maps and α^*g –closed maps.

I. INTRODUCTION

Generalized closed mappings were introduced and studied by Malghan. In 1983, A.S.Mashour et al and I.A. Hasanein introduced α –open maps and α –closed maps, recently P. Anbarasi Rodrigo and S.Pious Missier introduced α^* –open maps and α^* –closed maps in Topology. In this paper, we introduce α^*g –open maps and α^*g –closed maps also we relate and compare these functions with some other functions in topological spaces.

II. PRELIMINARIES

Throughout this paper (X, τ) , (Y, σ) and (Z, η) or X, Y, Z represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset A of a space (X, τ) , $\text{cl}(A)$ and $\text{int}(A)$ denote the closure and the interior of A respectively.

The power set of X is denoted by $P(X)$.

Definition 2.1

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a **open map** if image of each open set in X is open in Y .

Definition 2.2

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a **closed map** if image of each closed set in X is closed in Y .

Definition 2.3

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a **α –open map**[6] if image of each open set in X is α –open in Y .

Definition 2.4

A subset A of a topological space X is said to be **α^* –open**[7] if $A \subseteq \text{int}^*(\text{cl}(\text{int}^*(A)))$.

Definition 2.5

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a **g –open map**[6] if image of each open set in X is g –open in Y .

Definition 2.6

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a **$g\alpha$ –open map**[6] if image of each open set in X is $g\alpha$ –open in Y .

Definition 2.7

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a **αg –open map**[6] if image of each open set in X is αg –open in Y .

Definition 2.8

A subset A of a topological space X is said to be **generalized closed** (briefly g -closed) [3] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X .

Definition 2.9

A subset A of a topological space X is said to be **generalized α -closed set** [5] (briefly $g\alpha$ -closed) if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is α -open in (X, τ) .

Definition 2.10

A subset A of a topological space X is said to be **α generalized-closed set** [4] if $\alpha cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in (X, τ) .

III. α^*g -OPEN MAPS AND α^*g -CLOSED MAPS

Definition 3.1:

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called **α^*g -open map** if image of each open set in X is α^*g -open in Y .

Theorem 3.2:

Every open map is α^*g -open map

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an open map. Since f is an open map, the image of each open set in X is open in Y . Since every open set is α^*g -open. Hence, f is α^*g -open map.

Remark 3.3:

The following example supports that the converse of the above theorem is not true in general.

Example 3.4:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$, $\alpha^*gO(X, \tau) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\alpha^*gO(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = a, f(b) = b, f(c) = c$. Clearly, f is α^*g -open map. But $f(\{a, c\}) = \{a, c\}$ is not open in Y . Therefore, f is not an open map.

Theorem 3.5:

Every α -open map is α^*g -open map.

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a α -open map. Since f is a α -open map, the image of each open set in X is α -open in Y . Since every α -open set is α^*g -open. Hence, f is α^*g -open map.

Remark 3.6:

The converse of above theorem need not be true.

Example 3.7:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$, $\alpha^*gO(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $\alpha O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\alpha^*gO(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$, $\alpha O(Y, \sigma) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an identity map. Clearly, f is α^*g -open map. But $f(\{b\}) = \{b\}$ is not α -open in Y . Therefore, f is not α -open map.

Theorem 3.8:

Every g -open map is α^*g -open map

Proof:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be a g -open map. Since f is a g -open map, the image of each open set in X is g -open in Y . Since every g -open set is α^*g -open. Hence, f is α^*g -open map.

Remark 3.9:

The converse of above theorem need not be true.

Example 3.10:

Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b, c, d\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, Y\}$, $\alpha * gO(X, \tau) = \{\emptyset, \{a\}, \{b, c, d\}, X\}$, $gO(X, \tau) = P(X)$ and $\alpha * gO(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, Y\}$, $gO(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = f(b) = a$, $f(c) = d$, $f(d) = b$. Clearly, f is $\alpha * g$ -open map. But $f(\{b, c, d\}) = \{a, b, d\}$ is not g -open in Y . Therefore, f is not g -open map.

Theorem 3.11:

Every $\alpha * g$ -open map is $g\alpha$ -open map

Proof:

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\alpha * g$ -open map. Since f is $\alpha * g$ -open map, the image of each open set in X is $\alpha * g$ -open in Y . Since every $\alpha * g$ -open set is $g\alpha$ -open. Hence, f is $g\alpha$ -open map.

Remark 3.12:

The converse of above theorem need not be true.

Example 3.13:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a, b\}, Y\}$, $\alpha * gO(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $g\alpha O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, and $\alpha * gO(Y, \sigma) = \{\emptyset, \{a, b\}, Y\}$, $g\alpha O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Y\}$.

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Clearly, f is $g\alpha$ -open map. But $f(\{a\}) = \{a\}$ is not $\alpha * g$ -open in Y . Therefore, f is not $\alpha * g$ -open map.

Theorem 3.14:

Every $\alpha * g$ -open map is αg -open map.

Proof:

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a $\alpha * g$ -open map. Since f is $\alpha * g$ -open map, the image of each open set in X is $\alpha * g$ -open in Y . Since every $\alpha * g$ -open set is αg -open. Hence, f is αg -open map.

Remark 3.15:

The converse of above theorem need not be true.

Example 3.16:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$, $\alpha * gO(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$, $\alpha g O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$, and $\alpha * gO(Y, \sigma) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$, $\alpha g O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, Y\}$.

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = c$, $f(b) = a$, $f(c) = b$. Clearly, f is αg -open map. But $f(\{a\}) = \{c\}$ is not $\alpha * g$ -open in Y . Therefore, f is not $\alpha * g$ -open map.

Theorem 3.17:

A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is $\alpha * g$ -open if and only if $f(\text{int}(A)) \subseteq \alpha * g \text{ int} (f(A))$ for each set A in X .

Proof:

Suppose that f is a $\alpha * g$ -open map. Since $\text{int} (A) \subseteq A$, then $f(\text{int} (A)) \subseteq f(A)$. By hypothesis, $f(\text{int} (A))$ is a $\alpha * g$ -open and $\alpha * g \text{ int} (f(A))$ is the largest $\alpha * g$ -open set contained in $f(A)$. Hence $f(\text{int}(A)) \subseteq \alpha * g \text{ int} (f(A))$. Conversely, suppose A is an open set in X . Then $f(\text{int}(A)) \subseteq \alpha * g \text{ int} (f(A))$. Since $\text{int} (A) = A$, then $f(A) \subseteq \alpha * g \text{ int} (f(A))$. Therefore, $f(A)$ is a $\alpha * g$ -open set in (Y, σ) and f is $\alpha * g$ -open map.

Theorem 3.18:

Let (X, τ) , (Y, σ) and (Z, η) be three topologies spaces $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two maps. Then

1. If $(g \circ f)$ is $\alpha * g$ -open and f is continuous, then g is $\alpha * g$ -open.
2. If $(g \circ f)$ is open and g is $\alpha * g$ -continuous, then f is $\alpha * g$ -open map.

Proof:

1. Let A be an open set in Y . Then, $f^{-1}(A)$ is an open set in X . Since $(g \circ f)$ is $\alpha * g$ -open map, then $(g \circ f)^{-1}(A) = g(f^{-1}(A)) = g(A)$ is $\alpha * g$ -open set in Z . Therefore, g is a $\alpha * g$ -open map.

2. Let A be an open set in X . Then, $g(f(A))$ is an open set in Z . Therefore, $g^{-1}(g(f(A))) = f(A)$ is a α^* - g -open set in Y . Hence, f is a α^* - g -open map.

Remark 3.19:

The concept of α^* - g -open map and semi- g -open map are independent as can be seen from the following examples.

Example 3.20:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$,
 $\alpha^*gO(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, X\}$, $SO(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ and $\alpha^*gO(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$, $SO(Y, \sigma) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, Y\}$.
 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Clearly, f is α^* - g -open map. But, $f(\{b\}) = \{b\}$ is not semi- g -open in Y . Hence, f is not semi- g -open map.

Example 3.21:

Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, Y\}$,
 $\alpha^*gO(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, \{a, b, d\}, X\}$, $SO(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$ and $\alpha^*gO(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}, Y\}$,
 $SO(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, Y\}$.
 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = a$, $f(b) = b$, $f(c) = c$, $f(d) = d$. Clearly, f is semi- g -open map. But, $f(\{b, c\}) = \{b, c\}$ is not α^* - g -open in Y . Hence, f is not α^* - g -open map.

Remark 3.22:

The concept of α^* - g -open map and semi- g^* -open map are independent as can be seen from the following examples.

Example 3.23:

Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a, b\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b, c\}, Y\}$, $\alpha^*gO(X, \tau) = \{\emptyset, \{a, b\}, \{a, b, c\}, \{a, b, d\}, X\}$, $S^*O(X, \tau) = \{\emptyset, \{a, b\}, X\}$ and $\alpha^*gO(Y, \sigma) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, Y\}$,
 $S^*O(Y, \sigma) = \{\emptyset, \{a\}, \{a, d\}, \{a, b, c\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map. Clearly, f is α^* - g -open map. But $f(\{a, b\}) = \{a, b\}$ is not semi- g^* -open in Y . Hence, f is not semi- g^* -open map.

Example 3.24:

Let $X = Y = \{a, b, c, d\}$, $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, Y\}$, $\alpha^*gO(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$, $S^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, d\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}, X\}$ and
 $\alpha^*gO(Y, \sigma) = \{\emptyset, \{a\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, Y\}$, $S^*O(Y, \sigma) = P(X)$
 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map. Clearly, f is semi- g^* -open map. But $f(\{b\}) = \{b\}$ is not α^* - g -open in Y . Hence, f is not α^* - g -open map.

Remark 3.25:

The concept of α^* - g -open map and g^* -open map are independent as can be seen from the following examples.

Example 3.26:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$, $\alpha^*gO(X, \tau) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$, $g^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, X\}$ and $\alpha^*gO(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$,
 $g^*O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, Y\}$. Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map. Clearly, f is α^* - g -open map. But $f(\{a, c\}) = \{a, c\}$ is not g^* -open in Y . Hence, f is not g^* -open map.

Example 3.27:

Let $X = Y = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, X\}$ and $\sigma = \{\emptyset, \{b\}, \{a, b\}, \{b, c\}, Y\}$, $\alpha^*gO(X, \tau) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$, $g^*O(X, \tau) = \{\emptyset, \{a\}, \{b\}, \{c\}, X\}$ and $\alpha^*gO(Y, \sigma) = \{\emptyset, \{b\}, Y\}$, $g^*O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{c\}, Y\}$.
 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a map defined by $f(a) = a$, $f(b) = b$, $f(c) = c$. Clearly, f is g^* -open map. But $f(\{a\}) = \{a\}$ is not α^* - g -open in Y . Hence, f is not α^* - g -open map.

Theorem 3.28:

If a map $f : (X, \tau) \rightarrow (Y, \sigma)$ is open and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is α^*g -open, then $(g \circ f)$ is α^*g -open map.

Proof:

Let O be an open set in X . Since f is an open map, $f(O)$ is open in Y and we know that g is α^*g -open map then $(g \circ f)(O) = g(f(O))$ is α^*g -open in Z . Therefore, $(g \circ f)$ is α^*g -open map.

Remark 3.29:

The composition of two α^*g -open maps need not be α^*g -open maps as it can be seen from the following examples.

Example 3.30:

Let $X = Y = Z = \{a, b, c\}$, $\tau = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$ and $\sigma = \{\emptyset, \{a\}, \{a, b\}, Y\}$ and $\eta = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Z\}$, $\alpha^*g O(X, \tau) = \{\emptyset, \{a\}, \{a, b\}, \{a, c\}, X\}$, $\alpha^*g O(Y, \sigma) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, Y\}$ and $\alpha^*g O(Z, \eta) = \{\emptyset, \{a\}, \{b\}, \{a, b\}, Z\}$. Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an identity map. Clearly, f is α^*g -open map. Consider the map $g : (Y, \sigma) \rightarrow (Z, \eta)$ defined by $g(a) = a$, $g(b) = b$, $g(c) = c$. Clearly, g is α^*g -open map. Here, f and g are α^*g -open maps. But $(g \circ f)(\{a, c\}) = g(f(\{a, c\})) = g(\{a, c\}) = \{a, c\}$ is not α^*g -open in Z . Therefore, $(g \circ f)$ is not α^*g -open map.

IV. α^*g -CLOSED MAPS

Definition 4.1:

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is called a α^*g -closed map if image of each closed set in X is α^*g -closed in Y .

Theorem 4.2:

A map $f : (X, \tau) \rightarrow (Y, \sigma)$ is α^*g -closed if and only if $\alpha^*g \text{ cl}(f(A)) \subseteq f(\text{cl}(A))$ for each set A in X .

Proof:

Suppose that f is a α^*g -closed map. Since for each set A in X , $\text{cl}(A)$ is closed set in X , then $f(\text{cl}(A))$ is α^*g -closed set in Y . Since, $f(A) \subseteq f(\text{cl}(A))$, then $\alpha^*g \text{ cl}(f(A)) \subseteq f(\text{cl}(A))$

Conversely, suppose A is a closed set in X . Since $\alpha^*g \text{ cl}(f(A))$ is the smallest α^*g -closed set containing $f(A)$, then $f(A) \subseteq \alpha^*g \text{ cl}(f(A)) \subseteq f(\text{cl}(A)) = f(A)$. Thus, $f(A) = \alpha^*g \text{ cl}(f(A))$. Hence, $f(A)$ is a α^*g -closed set in Y . Therefore, f is a α^*g -closed map.

Theorem 4.3:

If $f : (X, \tau) \rightarrow (Y, \sigma)$ is g -closed map and $g : (Y, \sigma) \rightarrow (Z, \eta)$ is α^*g -closed and (Y, σ) is $T_{1/2}$ spaces. Then the composition $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ is α^*g -closed map.

Proof:

Let O be a closed set in (X, τ) . Since f is g -closed, $f(O)$ is g -closed in (Y, σ) and g is α^*g -closed which implies $g(f(O))$ is α^*g -closed in Z and $g(f(O)) = g \circ f(O)$. Therefore, $g \circ f$ is α^*g -closed.

Theorem 4.4:

Let $f : (X, \tau) \rightarrow (Y, \sigma)$ and $g : (Y, \sigma) \rightarrow (Z, \eta)$ be two mappings such that their composition $g \circ f : (X, \tau) \rightarrow (Z, \eta)$ be α^*g -closed mapping. Then the following statements are true.

1. If f is continuous and surjective, then g is α^*g -closed.
2. If g is α^*g -irresolute and injective, then f is α^*g -closed.
3. If f is g -continuous, surjective and (X, τ) is a $T_{1/2}$ spaces, then g is α^*g -closed.
4. If g is strongly α^*g -continuous and injective, then f is α^*g -closed.

Proof:

1. Let O be a closed set in (Y, σ) . Since, f is continuous, $f^{-1}(O)$ is closed in (X, τ) . Since, $g \circ f$ is α^*g -closed which implies $g \circ f(f^{-1}(O))$ is α^*g -closed in (Z, η) . That is $g(O)$ is α^*g -closed in (Z, η) , since f is surjective. Therefore, g is α^*g -closed.

2. Let O be a closed set in (X, τ) . Since $g \circ f$ is α^*g -closed, $g \circ f(O)$ is α^*g -closed in (Z, η) . Since g is α^*g -irresolute, $g^{-1}(g \circ f(O))$ is α^*g -closed in (Y, σ) . That is $f(O)$ is α^*g -closed in (Y, σ) . Since f is injective. Therefore, f is α^*g -closed.

3. Let O be a closed set of (Y, σ) . Since, f is g -continuous, $f^{-1}(O)$ is g -closed in (X, τ) and (X, τ) is a $T_{1/2}$ spaces, $f^{-1}(O)$ is closed in (X, τ) . Since, $g \circ f$ is α^*g -closed which implies, $g \circ f(f^{-1}(O))$ is α^*g -closed in (Z, η) . That is $g(O)$ is α^*g -closed in (Z, η) , since f is surjective. Therefore, g is α^*g -closed.

4. Let O be a closed set of (X, τ) . Since, $g \circ f$ is α^*g -closed which implies, $g \circ f(O)$ is α^*g -closed in (Z, η) . Since, g is strongly α^*g -continuous, $g^{-1}(g \circ f(O))$ is closed in (Y, σ) . That is $f(O)$ is closed in (Y, σ) . Since g is injective, f is α^*g -closed.

Theorem 4.5:

Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a bijective map. Then the following are equivalent:

- (1) f is a α^*g -open map.
- (2) f is a α^*g -closed map.
- (3) f^{-1} is a α^*g -continuous map.

Proof:

(1) \Rightarrow (2) Suppose O is a closed set in X . Then $X \setminus O$ is an open set in X and by (1) $f(X \setminus O)$ is α^*g -open in Y . Since, f is bijective, then $f(X \setminus O) = Y \setminus f(O)$. Hence, $f(O)$ is a α^*g -closed set in Y . Therefore, f is a α^*g -closed map.

(2) \Rightarrow (3) Let f be a α^*g -closed map and O be closed set in X . Since, f is bijective then $(f^{-1})^{-1}(O) = f(O)$ which is a α^*g -closed set in Y . Therefore, f is a α^*g -continuous map.

(3) \Rightarrow (1) Let O be an open set in X . Since, f^{-1} is a α^*g -continuous map then $(f^{-1})^{-1}(O) = f(O)$ is a α^*g -open set in Y . Hence, f is α^*g -open map.

Theorem 4.6:

A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is α^*g -open if and only if for any subset O of (Y, σ) and any closed set of (X, τ) containing $f^{-1}(O)$, there exists a α^*g -closed set A of (Y, σ) containing O such that $f^{-1}(A) \subset F$.

Proof:

Suppose f is α^*g -open. Let $O \subset Y$ and F be a closed set of (X, τ) such that $f^{-1}(O) \subset F$. Now $X - F$ is an open set in (X, τ) . Since f is α^*g -open map, $f(X - F)$ is α^*g -open set in (Y, σ) . Then, $A = Y - f(X - F)$ is α^*g -closed set in (Y, σ) . Note that $f^{-1}(O) \subset F$ implies $O \subset A$ and $f^{-1}(A) = X - f^{-1}(X - F) \subset X - (X - F) = F$. That is, $f^{-1}(A) \subset F$. Conversely, let B be an open set of (X, τ) . Then, $f^{-1}((f(B))^c) \subset B^c$ and B^c is a closed set in (X, τ) . By hypothesis, there exists a α^*g -closed set A of (Y, σ) such that $(f(B))^c \subset A$ and $f^{-1}(A) \subset B^c$ and so $B \subset (f^{-1}(A))^c$. Hence, $A^c \subset f(B) \subset f((f^{-1}(A))^c)$ which implies $f(B) = A^c$. Since, A^c is a α^*g -open. $f(B)$ is α^*g -open in (Y, σ) and therefore f is α^*g -open map.

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