Some Topological Concepts In Penta Topological Space

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Abstract:

The purpose of this paper is to introduce the concept of generalized closed set to penta topological space. Further we studied some fundamental properties of penta topological space.

Keyword: *Penta topological space,* $\tau_{1,2,3,4,5}$ *open,* $\tau_{1,2,3,4,5}$ *closed,* $\alpha(1,2,3,4,5)$ *open,* $\alpha(1,2,3,4,5)$ *closed,* $\beta(1,2,3,4,5)$ *open,* $\beta(1,2,3,4,5)$ *closed, semi* $(1,2,3,4,5)$ *open, semi* $(1,2,3,4,5)$ *closed , regular* $(1,2,3,4,5)$ *open and regular* $(1,2,3,4,5)$ *closed.*

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I. INTRODUCTION

General topology plays a vital role in mathematics and other branches of science. Starting from single topology it extended to bitopology and tritopology etc. with usual definitions. In 1983, J.C Kelly[1] introduced bitopological space (X,τ_1,τ_2), where X is a non empty set together with two topologies τ_1 and τ_2. As an extension of bitopological space, tritopological space$(X,\tau_1,\tau_2,\tau_3)$was first initiated by Martin M. Kovar[2] in 2000, where X is a non empty set together with three topologiesτ_1,τ_2 and τ_3.Quad topological space$(X,\tau_1,\tau_2,\tau_3,\tau_4)$was investigated by Mukundan[3], where X is a non empty set together with four topologiesτ_1,τ_2,τ_3 and τ_4. Penta topological space$(X,\tau_1,\tau_2,\tau_3,\tau_4,\tau_5)$ was introduced by Muhammad Shahkar Khan and Gulzar Ali Khan[4] where X is a non empty set together with five topologies$\tau_1,\tau_2,\tau_3,\tau_4$ and τ_5. In this paper we have investigated some properties of penta topological space and also we have analysed how the generalized closed set act on penta topological space.

II. PRELIMINARIES

Definition: 2.1

Let X be a non empty set together with five topologies $\tau_1,\tau_2,\tau_3,\tau_4$ and τ_5 is called penta topological space and it is denoted by $(X,\tau_1,\tau_2,\tau_3,\tau_4,\tau_5)$.

Example: 2.2

$$X=\{a,b,c,d\}\tau_1=\{\varphi,\{a\},X\}\tau_2=\{\varphi,\{a\},\{b\},\{a,b\},X\}\tau_3=\{\varphi,\{b\},\{c\},\{b,c\},X\}$$

$\tau_4=\{\varphi,\{c\},\{d\},\{c,d\},X\}\tau_5=\{\varphi,\{a\},\{c\},\{a,c\},X\}$. Then $(X,\tau_1,\tau_2,\tau_3,\tau_4,\tau_5)$ is a penta topological space.

Definition: 2.3

Let $(X,\tau_1,\tau_2,\tau_3,\tau_4,\tau_5)$ be a penta topological space. A subset A of X is called penta-open if

$A\in\tau_1\cup\tau_2\cup\tau_3\cup\tau_4\cup\tau_5$ and its complement is said to be penta-closed

Remark: 2.4

Penta open sets satisfy all the axioms of topology.

Example: 2.5

$$X=\{a,b,c,d\}\tau_1=\{\varphi,\{a\},X\}\tau_2=\{\varphi,\{b\},X\}\tau_3=\{\varphi,\{c\},X\}\tau_4=\{\varphi,\{d\},X\}\tau_5=\{\varphi,\{a,b\},X\}$$

$\{\varphi,X,\{a\},\{b\},\{c\},\{d\},\{a,b\}\}$ are penta open sets and $\{\varphi,X,\{c,d\},\{a,b,c\},\{a,b,d\},\{a,c,d\}\}$ are penta closed sets.

Remark: 2.6

φ and X are both penta open and penta closed.

Theorem: 2.7

Arbitrary union of penta open sets is penta open.

Theorem: 2.8

Arbitrary intersection of penta closed sets is penta closed.

III. BASIC PROPERTIES OF $\tau_{1,2,3,4,5}$ OPEN SETS

Definition: 3.1

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $S \subset X$ is $\tau_{1,2,3,4,5}$ open if $S \subset A \cup B \cup C \cup D \cup E$ where $A \in \tau_1, B \in \tau_2, C \in \tau_3, D \in \tau_4, E \in \tau_5$.

Result: 3.2

φ and X are $\tau_{1,2,3,4,5}$ open.

Theorem: 3.3

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $S_1, S_2 \subset X$. If S_1 and S_2 are $\tau_{1,2,3,4,5}$ open then

$S_1 \cup S_2 \subset X$ is $\tau_{1,2,3,4,5}$ open.

Proof:

S_1 and S_2 are $\tau_{1,2,3,4,5}$ open

$$S_1 = A_1 \cup B_1 \cup C_1 \cup D_1 \cup E_1$$

$S_2 = A_2 \cup B_2 \cup C_2 \cup D_2 \cup E_2$ where $A_1, A_2 \in \tau_1; B_1, B_2 \in \tau_2; C_1, C_2 \in \tau_3; D_1, D_2 \in \tau_4$ and $E_1, E_2 \in \tau_5$

$S_1 \cup S_2 = (A_1 \cup A_2) \cup (B_1 \cup B_2) \cup (C_1 \cup C_2) \cup (D_1 \cup D_2) \cup (E_1 \cup E_2)$ is $\tau_{1,2,3,4,5}$ open.

Theorem: 3.4

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $S_\alpha \in X$ where $\alpha \in I$. If S_α is $\tau_{1,2,3,4,5}$ open for each α then $\cup S_\alpha$ is $\tau_{1,2,3,4,5}$ open.

Proof:

$S_\alpha = A_\alpha \cup B_\alpha \cup C_\alpha \cup D_\alpha \cup E_\alpha$ where $A_\alpha \in \tau_1, B_\alpha \in \tau_2, C_\alpha \in \tau_3, D_\alpha \in \tau_4, E_\alpha \in \tau_5$.

$$\cup_{\alpha \in I} S_\alpha = \cup (A_\alpha \cup B_\alpha \cup C_\alpha \cup D_\alpha \cup E_\alpha)$$

$$= (\cup A_\alpha) \cup (\cup B_\alpha) \cup (\cup C_\alpha) \cup (\cup D_\alpha) \cup (\cup E_\alpha)$$

$\cup A_\alpha \in \tau_1, \cup B_\alpha \in \tau_2, \cup C_\alpha \in \tau_3, \cup D_\alpha \in \tau_4, \cup E_\alpha \in \tau_5$.

Hence $\cup S_\alpha$ is $\tau_{1,2,3,4,5}$ open.

Definition: 3.5

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X$. If A is $\tau_{1,2,3,4,5}$ open then the complement of A is said to be $\tau_{1,2,3,4,5}$ closed.

Definition: 3.6

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space.

$A \subset X$. $\tau_{1,2,3,4,5} cl\ A = \cap \{F / F \supset A \text{ and } F \text{ is } \tau_{1,2,3,4,5} \text{ closed}\}$

$A \subset X$. $\tau_{1,2,3,4,5} int\ A = \cap \{F/F \supset A\ and\ F\ is\ \tau_{1,2,3,4,5}\ open\}$

Theorem: 3.7

Arbitrary intersection of $\tau_{1,2,3,4,5}$ closed sets is$\tau_{1,2,3,4,5}$ closed in penta topological space.

Proof:

Follows from theorem 3.4

Theorem: 3.8

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X$. A is $\tau_{1,2,3,4,5}$ open iff $\tau_{1,2,3,4,5} int\ A = A$.

Proof:

Follows from the fact that arbitrary union of $\tau_{1,2,3,4,5}$ open sets is $\tau_{1,2,3,4,5}$ open.

Theorem: 3.9

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X$. A is $\tau_{1,2,3,4,5}$ closed iff $\tau_{1,2,3,4,5} clA = A$.

Proof:

Follows from the fact that arbitrary intersection of $\tau_{1,2,3,4,5}$ closed sets is$\tau_{1,2,3,4,5}$ closed.

Theorem: 3.10

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. For any $A \subset X$, $(\tau_{1,2,3,4,5} int\ A)^c = \tau_{1,2,3,4,5} cl\ A^c$.

Proof:

$$(\tau_{1,2,3,4,5} int\ A)^c = [\cup \{G/G \subseteq A\ \&\ G\ is\ \tau_{1,2,3,4,5}\ \text{open}\ ]^c$$

$$= \cap \{G^c/G^c \supseteq A^c \& G^c\ is\ \tau_{1,2,3,4,5}\ \text{closed}\ \}$$

$= \cap \{F = G^c/F \supseteq A^c \& F\ is\ \tau_{1,2,3,4,5}\ \text{closed}\}$where $F = G^c$

$= \tau_{1,2,3,4,5} cl\ A^c$.

Remark: 3.11

φ and X are $\tau_{1,2,3,4,5}$ closed.

Result: 3.12

The intersection of two $\tau_{1,2,3,4,5}$ open sets need not be $\tau_{1,2,3,4,5}$ open.

Example: 3.13

$$X = \{a, b, c\} \tau_1 = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\} \tau_2 = \{\varphi, \{a\}, \{a, c\}, X\} \tau_3 = \{\varphi, \{b\}, \{b, c\}, X\}$$

$$\tau_4 = \{\varphi, \{a\}, X\} \tau_5 = \{\varphi, \{b\}, \{b, c\}, X\}$$

$A = \{a, c\}$ is $\tau_{1,2,3,4,5}$ open.

$B = \{b, c\}$ is $\tau_{1,2,3,4,5}$ open.

$A \cap B = \{c\}$ which is not $\tau_{1,2,3,4,5}$ open.

$\{b\}$ cannot be written as $B_1 \cup B_2 \cup B_3 \cup B_4 \cup B_5$ where B_i is $\tau_i$open for all $i = 1\ to\ 5$

Remark: 3.14

Union of two $\tau_{1,2,3,4,5}$ closed sets need not be $\tau_{1,2,3,4,5}$ closed.

Example: 3.15

In the example 3.13 A and B are $\tau_{1,2,3,4,5}$ open.

$\Rightarrow A^c$ and B^c are $\tau_{1,2,3,4,5}$ closed.

A^c={ b} and B^c= { a}

$A^c \cup B^c = \{a, b\}$ is not $\tau_{1,2,3,4,5}$ closed, since $\{a, b\}^c = \{c\}$ is not $\tau_{1,2,3,4,5}$ open.

Hence $\{a, b\}$ is not $\tau_{1,2,3,4,5}$ closed.

Result: 3.16

The set of all $\tau_{1,2,3,4,5}$ open sets contains $\tau_1 \cup \tau_2 \cup \tau_3 \cup \tau_4 \cup \tau_5$.

Theorem: 3.17

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. The set of all $\tau_{1,2,3,4,5}$ open sets is a generalized topology on X.

***Proof*:**

Follows from Result 3.2, Theorem 3.4 and Result 3.12

Definition: 3.18

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called semi(1, 2,3,4, 5)open if

$A \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5} int\ A.$

Definition: 3.19

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called semi(1, 2,3,4, 5) closed if A^c is semi(1,2,3,4,5) open.

Definition: 3.20

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called pre(1, 2,3,4, 5) open if

$A \subset \tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5} cl\ A.$

Definition: 3.21

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called pre(1, 2,3,4, 5) closed if A^c is pre(1,2,3,4,5) open.

Definition: 3.22

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called α(1, 2,3,4, 5) open if

$A \subset \tau_{1,2,3,4,5} int\ \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5} int\ A.$

Definition: 3.23

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called α(1, 2,3,4, 5) closed if A^c is α(1, 2,3,4, 5) open.

Definition: 3.24

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called $\beta(1, 2,3,4, 5)$ open if

$A \subset \tau_{1,2,3,4,5} cl\tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5} clA$.

Definition: 3.25

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called $\beta(1, 2,3,4, 5)$ closed if A^c is $\beta(1, 2,3,4, 5)$ open.

Definition: 3.26

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called regular(1, 2,3,4, 5) open if

$A = \tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5} cl\ A$.

Definition: 3.27

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called regular(1, 2,3,4, 5) closed if $A = \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5} intA$.

Example:3.28

$$X = \{a, b, c, d\}\tau_1 = \{\varphi, \{a\}, \{b\}, \{a, b\}, X\}\tau_2 = \{\varphi, \{a\}, \{c\}, \{a, c\}, X\}$$

$$\tau_3 = \{\varphi, \{a\}, \{b\}, \{a, b, c\}, X\}\tau_4 = \{\varphi, \{b\}, \{c\}, \{b, c\}, X\}\tau_5 = \{\varphi, \{a\}, \{c\}, \{a, b, c\}, X\}$$

$\tau_{1,2,3,4,5}$open sets are $\{\varphi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}, X\}$

$\tau_{1,2,3,4,5}$closed sets are $\{\varphi, \{b, c, d\}, \{a, c, d\}, \{a, b, d\}, \{c, d\}, \{b, d\}, \{a, d\}, \{d\}, X\}$

Take $A = \{a, c\}$

$$\tau_{1,2,3,4,5} intA = \{a, c\}$$

$$\tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5} intA = \tau_{1,2,3,4,5}\ cl\{a, c\} = \{a, c, d\}$$

$A \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5} intA$. Hence A is semi (1,2,3,4,5) open.

Hence $\{b, d\}$ is semi (1,2,3,4,5) closed.

$$\tau_{1,2,3,4,5}\ clA = \{a, c, d\}$$

$$\tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5} cl\ A = \tau_{1,2,3,4,5} int\{a, c, d\} = \{a, c\}$$

$A \subset \tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5} cl\ A$.Hence A is pre (1,2,3,4,5) open.

$\{b, d\}$ is pre (1,2,3,4,5) closed.

$$\tau_{1,2,3,4,5} int\ \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5} int\ A = \tau_{1,2,3,4,5} int\{a, c, d\} = \{a, c\}$$

$A \subset \tau_{1,2,3,4,5} int\ \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5} int\ A$. Hence A is $\alpha(1,2,3,4,5)$ open.

$\{b, d\}$ is $\beta(1,2,3,4,5)$ open.

Theorem: 3.29

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X$. A is semi (1, 2,3,4, 5) open iff there exist a $\tau_{1,2,3,4,5}$ open set O such that $O \subset A \subset \tau_{1,2,3,4,5}\ clO$.

Proof:

Let A be semi(1,2,3,4,5) open. Then $A \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$.

Now $\tau_{1,2,3,4,5}int\ A \subset\ A \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$. Take $O = \tau_{1,2,3,4,5}int\ A$.

Then O is $\tau_{1,2,3,4,5}$ open and $O \subset\ A \subset \tau_{1,2,3,4,5}\ clO$.

Conversely, suppose there exist $\tau_{1,2,3,4,5}$ open with $O \subset\ A \subset \tau_{1,2,3,4,5}\ clO$.

Now O is $\tau_{1,2,3,4,5}$ open and $O \subset\ A$ and hence $O \subset \tau_{1,2,3,4,5}int\ A$.

Therefore $\tau_{1,2,3,4,5}\ clO \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$.

Now $A \subset \tau_{1,2,3,4,5}\ clO$ and hence $A \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$.

Theorem: 3.30

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X$. A is semi (1, 2,3,4, 5) open iff $\tau_{1,2,3,4,5}\ clA = \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$.

Proof:

Suppose A is semi (1, 2,3,4, 5) open. Then $A \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$.

Always $\tau_{1,2,3,4,5}int\ A \subset\ A$.

$\Longrightarrow \tau_{1,2,3,4,5}\ cl[\tau_{1,2,3,4,5}int\ A] \subset \tau_{1,2,3,4,5}\ clA$ ---------------------------- 1

Also $\tau_{1,2,3,4,5}\ clA \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$ ---------------------------- 2

1and 2 $\Longrightarrow \tau_{1,2,3,4,5}\ clA = \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$.

Conversely, suppose $\tau_{1,2,3,4,5}\ clA = \tau_{1,2,3,4,5}int\ A$.

Claim: A is semi (1,2,3,4,5) open.

$$A \subset \tau_{1,2,3,4,5}\ clA = \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$$

$A \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A$.

Therefore A is semi (1,2,3,4,5) open.

Theorem: 3.31

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called regular (1, 2,3,4, 5) open iff A^c is regular(1, 2,3,4, 5) closed.

Proof:

Suppose A is regular (1, 2,3,4, 5) open.

$$\Leftrightarrow\ A = \tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5}cl\ A$$

$$\Leftrightarrow A^c = [\tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5}cl\ A]^c$$

$$\Leftrightarrow A^c = \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ A^c$$

$\Leftrightarrow A^c$ is regular(1,2,3,4,5) closed.

Theorem:3.32

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X$.

$[semi(1,2,3,4,5)intA]^c = semi(1,2,3,4,5)clA^c$.

Proof:

$$[semi(1,2,3,4,5)intA]^c = [\cup \{G/G \subset A \,\&\, G \; is \; semi(1,2,3,4,5)open\}]^c$$

$$= \cap \{G^c/G^c \supset A^c \& G^c \; is \; semi(1,2,3,4,5)closed\}$$

$$= \cap \{F/F \supset A^c \,\&\, F \; is \; semi(1,2,3,4,5)closed\} \text{ where } F = G^c.$$

$$= semi(1,2,3,4,5)cl\, A^c$$

Hence $[semi(1,2,3,4,5)intA]^c = semi(1,2,3,4,5)clA^c$.

Theorem:3.33

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X. A$ is semi(1,2,3,4,5) closed iff $A \supset \tau_{1,2,3,4,5}\; int\tau_{1,2,3,4,5}cl\; A$.

Proof:

A is semi (1, 2,3,4, 5) closed

$\Leftrightarrow A^c$ is semi(1,2,3,4,5) open

$$\Leftrightarrow A^c \subset \tau_{1,2,3,4,5}\; cl\tau_{1,2,3,4,5}int\; A^c$$

$$\Leftrightarrow A \supset [\tau_{1,2,3,4,5}\; cl\tau_{1,2,3,4,5}int\; A^c]^c$$

$$\Leftrightarrow A \supset \tau_{1,2,3,4,5}int[\tau_{1,2,3,4,5}int\; A^c]^c$$

$\Leftrightarrow A \supset \tau_{1,2,3,4,5}\; int\tau_{1,2,3,4,5}cl\; A$.

Theorem: 3.34

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X. A$ is pre(1,2,3,4,5) closed iff

$A \supset \tau_{1,2,3,4,5}\; cl\tau_{1,2,3,4,5}int\; A$.

Proof:

$$A \supset \tau_{1,2,3,4,5}\; cl\tau_{1,2,3,4,5}int\; A.$$

A is pre(1,2,3,4,5) closed $\Leftrightarrow A^c$ is pre(1,2,3,4,5) open.

$$\Leftrightarrow A^c \subset \tau_{1,2,3,4,5}\; int\tau_{1,2,3,4,5}cl\; A^c$$

$$\Leftrightarrow [A^c]^c \supset \left[\tau_{1,2,3,4,5}\; int\tau_{1,2,3,4,5}cl\; A^c\right]^c$$

$$\Leftrightarrow A \supset \tau_{1,2,3,4,5}\; cl\tau_{1,2,3,4,5}int\; A.$$

Theorem: 3.35

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X. A$ isα(1,2,3,4,5) closed iff

$A \supset \tau_{1,2,3,4,5}\; cl\tau_{1,2,3,4,5}int\tau_{1,2,3,4,5}\; cl\; A$.

Proof:

A is α(1,2,3,4, 5) closed $\Leftrightarrow A^c$ is α(1,2,3,4,5) open

$$\Leftrightarrow A^c \subset \tau_{1,2,3,4,5}\; int\tau_{1,2,3,4,5}cl\; \tau_{1,2,3,4,5}\; intA^c$$

$$\Leftrightarrow [A^c]^c \supset \left[\tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5}cl\tau_{1,2,3,4,5}\ int\ A^c\right]^c$$

$\Leftrightarrow A \supset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\tau_{1,2,3,4,5}\ cl\ A.$

Theorem: 3.36

Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. Let $A \subset X. A$ is $\beta(1,2,3,4,5)$ closed iff $A \supset \tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5}cl\tau_{1,2,3,4,5}\ int\ A.$

Proof:

A is $\beta(1,2,3,4,5)$ closed $\Leftrightarrow$ A^c is $\beta(1,2,3,4,5)$ open.

$$\Leftrightarrow A^c \subset \tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\ \tau_{1,2,3,4,5}\ cl\ A^c$$

$$\Leftrightarrow [A^c]^c \supset \left[\tau_{1,2,3,4,5}\ cl\tau_{1,2,3,4,5}int\tau_{1,2,3,4,5}\ cl\ A^c\right]^c$$

$\Leftrightarrow A \supset \tau_{1,2,3,4,5}\ int\tau_{1,2,3,4,5}cl\tau_{1,2,3,4,5}\ int\ A.$

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