Cube Root Cube Mean Labeling of Graphs

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Abstract
A function f is called cube root cube mean labeling of a graph $G(V,E)$ with p vertices and q edges if $f: V(G) \to \{1,2,\dots,q+1\}$ is injective and the induced function f^* defined as $f^*(uv) = \left\lfloor \sqrt[3]{\frac{f(u)^3+f(v)^3}{2}} \right\rfloor$ for all $uv \in E(G)$, is bijective. Then the resulting edge labels are distinct. A graph that admits a cube root cube mean labeling f is called a cube root cube mean graph. In this paper, we introduce cube root cube mean labeling and investigate cube root cube mean labeling of PathP_n, Comb, Ladder Quadrilateral snake and Fish.

Keywords: *Mean labeling of graphs, Cube Root Cube Mean labeling of graphs.*

I. INTRODUCTION

By a graph $G = (V, E)$ we mean a finite undirected graph without loops or parallel edges. For all detailed survey of graph labeling we refer to Gallian[2]. For all other standard terminology and notations we follow Harary[4]. The concept of Mean labeling has been introduced by Somasundaram and Ponraj[5]. Root Square Mean Labeling of Graphs has been introduced by Sandhya, Somasundaram and Anusa[6]. Root Cube Mean Labeling of Graphs has been introduced by Gowri and Vembarasi[3]. Motivated the above works we introduced a new type of labeling called Cube Root Cube Mean Labeling.

In this paper we investigate the Cube Root Cube Mean Labeling of Path, Comb, Ladder, Quadrilateral snake and Fish graphs.

II. PRELIMINARIES

Definition: 2.1

A walk in which vertices are distinct is called a path. A path on n vertices is denoted by P_n.

Definition: 2.2

The graph obtained by joining a single pendent edge to each vertex of a path is called a Comb graph.

Definition: 2.3

The Cartesian product of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is a graph G= $(V$,$E)$ with V=$V_1 \times V_2$ and two vertices u=$(u_1 u_2)$ and v=$(v_1 v_2)$ are adjacent in $G_1 \times G_2$ whenever (u_1=$v_1$and u_2 is adjacent to v_2) or (u_2=$v_2$and u_1 is adjacent to v_1) .It is denoted by $G_1 \times G_2$.

Definition: 2.4

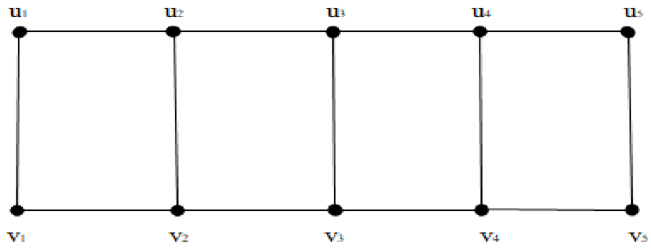
The Corona of two graphs G_1 and G_2 is the graph G=$G_1 \odot G_2$ formed by taking one copy of G_1 and $|(G_1)|$ copies of G_2 where the i^{th} vertex of G_1 is adjacent to every vertex in the i^{th} copy of G_2.

Definition: 2.5

The product graph $P_2 \times P_n$is called a ladder and it is denoted by L_n.

Example:

Ladder graph of L_5 is given below

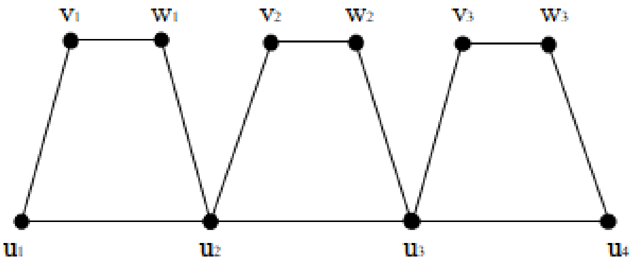


Definition: 2.6

A Quadrilateral Snake Q_n is obtained from path $u_1, u_2, \ldots\ldots, u_n$ by joining u_i and u_{i+1} to two new vertices v_i and w_i respectively and then joining $v_i$andw_i. That is every edge of a path is replaced by a cycle C_4.

Example:

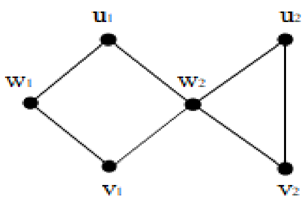
Quadrilateral graph of Q_4 is given below



Definition: 2.7

The Fish graph is the graph with 6 vertices and 7 edges.

Example:



III. MAIN RESULTS

Theorem: 3.1

Any path P_n is a cube root cube mean labeling graph.

Proof:

Let $G = P_n$ be the graph with vertices $u_1, u_2, \ldots\ldots, u_n$ and the edges $e_1, e_2, \ldots\ldots, e_q$

Define the function $f: V(P_n) \to \{1,2, \ldots\ldots\ldots, q+1\}$ as follows.

$f(u_i) = i \quad , 1 \leq i \leq n$

And the induced edge labeling function $f^*: E(G) \to N$ defined by

$$f^*(e = uv) = \left\lceil \sqrt[3]{\frac{f(u)^3 + f(v)^3}{2}} \right\rceil$$

Now $f^*(u_i u_{i+1}) = \left\lfloor \sqrt[3]{\frac{f(u_i)^3 + f(u_{i+1})^3}{2}} \right\rfloor$

$$= \left\lfloor \sqrt[3]{\frac{i^3 + (i+1)^3}{2}} \right\rfloor$$

$$= \left\lfloor \sqrt[3]{\frac{2i^3 + 3i^2 + 3i + 1}{2}} \right\rfloor$$

Clearly $f^*(u_{n-1} u_n) = \left\lfloor \sqrt[3]{\frac{2n^3 - 3n^2 + 3n - 1}{2}} \right\rfloor$

Hence the edge labels are distinct.

Thus Path graph admits a Cube root cube mean labeling.

Example: 3.2

The Cube Root Cube Mean labelingof P_5 is given below

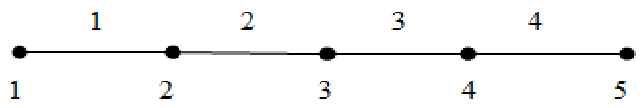


Figure: 1

Theorem: 3.3

Any Comb graph is Cube Root Cube Mean labeling graphs.

Proof:

Let G be a comb graph.

Let $u_1, u_2, \ldots\ldots, u_n$ be the vertices of comb and the edges are$e_1, e_2, \ldots\ldots, e_q$.

Let P_n be the path $u_1, u_2, \ldots\ldots, u_n$ in G and join a vertex v_i to u_i for $1 \leq i \leq n$.

Define the function $f: V(P_n \odot k_1) \rightarrow \{1,2, \ldots\ldots, q+1\}$ as follows

$f(u_i) = 2i - 1$; $1 \leq i \leq n$;

$f(v_i) = 2i$; $1 \leq i \leq n$;

Then the induced edge labeling function $f^*: E(G) \rightarrow N$ defined by

$f^*(e = u_i u_{i+1}) = 2i$; $1 \leq i \leq n$;

$f^*(e = u_i v_i) = 2i - 1$; $1 \leq i \leq n$; are distinct.

Hence the comb graphs are Cube root cube mean labeling graphs.

Example: 3.4

The Cube Root Cube Mean labeling of$P_7 \odot k_1$ is given below

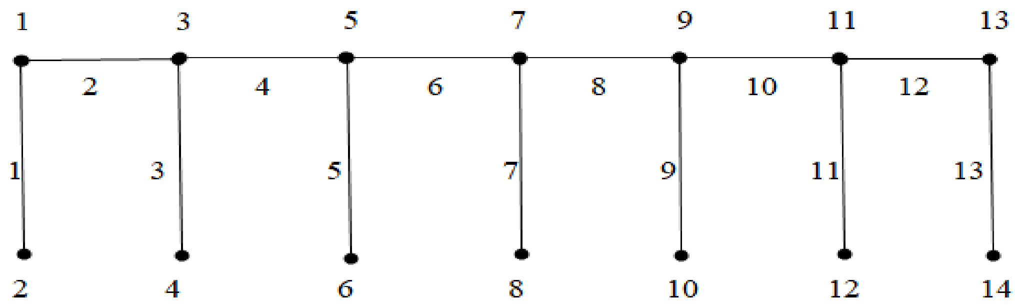


Figure: 2

Theorem: 3.5

The ladder L_n is a Cube Root Cube Mean graph.

Proof:

Let G be a Ladder graph.

Let $\{u_1, u_2, \ldots\ldots, u_n, v_1, v_2, \ldots\ldots, v_n\}$ be the vertices of ladder.

Define a function $f: V(L_n) \to \{1,2,\ldots\ldots, q+1\}$ as

$f(u_1) = 1$

$$f(u_i) = \begin{cases} 3i-1 & ; i \text{ is even} \\ 3i-3 & ; i \text{ is odd} \end{cases}; \quad 1 \le i \le n$$

$$f(v_i) = \begin{cases} 3i-3 & ; i \text{ is even} \\ 3i-1 & ; i \text{ is odd} \end{cases}; \quad 1 \le i \le n$$

Then we got the edge labels

$$f^*(u_i u_{i+1}) = \begin{cases} 3i & ; i \text{ is odd} \\ 3i-1 & ; i \text{ is even} \end{cases}; \quad 1 \le i \le n-1$$

$$f^*(v_i v_{i+1}) = \begin{cases} 3i-1 & ; i \text{ is odd} \\ 3i & ; i \text{ is even} \end{cases}; \quad 1 \le i \le n-1$$

$f^*(u_i v_i) = 3i-2$; $1 \le i \le n$are distinct.

Hence L_n is a Cube Root Cube Mean labeling graph.

Example: 3.6

A Cube Root Cube Mean labeling of L_6.

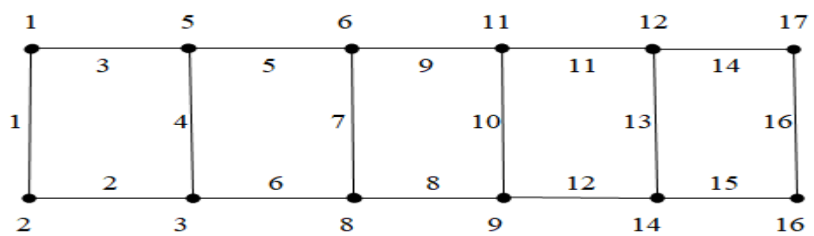


Figure: 3

Theorem: 3.7

Quadrilateral snake Q_n is Cube Root Cube Mean graph.

Proof:

Let G be a Quadrilateral snake Q_n.

Define a function $f: V(Q_n) \rightarrow \{1,2,\ldots\ldots, q+1\}$ by

$f(u_i) = 4i - 3\,;\quad 1 \leq i \leq n$

$f(v_i) = 4i - 2\,;\quad 1 \leq i \leq n-1$

$f(w_i) = 4i - 1\,;\quad 1 \leq i \leq n-1$

Then the induced edge function

$f^*: E(G) \rightarrow N$ defined by

$f^*(u_i v_i) = 4i - 3\,;\quad 1 \leq i \leq n-1$

$f^*(u_i u_{i+1}) = 4i - 1\,;\quad 1 \leq i \leq n-1$

$f^*(u_{i+1} w_i) = 4i\,;\quad 1 \leq i \leq n-1$

$f^*(u_i w_i) = 4i - 2\,;\quad 1 \leq i \leq n-1$are distinct.

Hence the Quadrilateral snake Q_n is Cube Root Cube Mean Labeling graphs.

Example: 3.8

A Cube Root Cube Mean labeling of Q_5.

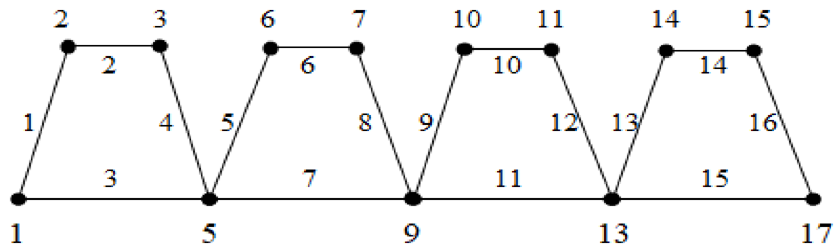


Figure: 4

Theorem: 3.9

Fish graph is a Cube Root Cube Mean graph.

Proof:

Let G be Fish graph.

Let $\{u_1, u_2, v_1, v_2, w_1, w_2\}$ be the vertices of G.

The Fish graph consists of n vertices and n+1 edges.

Define $f: V(G) \rightarrow \{1,2,\ldots\ldots, q+1\}$ by

$f(u_i) = 2i + 3\,; 1 \leq i \leq 2$

$f(v_i) = 6i - 4\,; 1 \leq i \leq 2$

$f(w_i) = 2i - 1\,; 1 \leq i \leq 2$

Then we find the edge labels

$f^{*}(e) = i$;are distinct.

Hence Fish graph is a Cube Root Cube Mean graph.

Example: 3.10

The Cube Root Cube Mean labeling of Fish graph is given below.

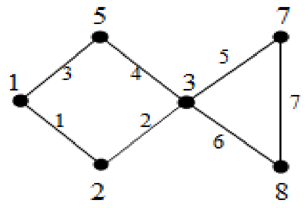


Figure: 5

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