

Alternative Method to Solve Partial Differential Equation using MATLAB

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Abstract

In this paper, we re-present the solution of Partial Differential Equation using MATLAB give detail about MATLAB and its modules, plotting the graphs of solution of Partial Differential Equations, also we try to solve and simulate the Partial Differential Equations using MATLAB. We will describe the process of Partial Differential and its solution in graphical format. Also we are plotting the results of all these Partial Differential Equations mentioned above, and thus compare and show that better results obtained by MATLAB.

Key words - MATLAB, Partial Differential Equation, Plotting the graphs, traditional method.

I. INTRODUCTION

A. Differential Equation

An Equation involving derivatives of one or more dependent variable with respect to one or more independent variables is called Differential Equation.

B. Partial Differential Equation

A Differential Equation involving Partial derivatives of one or more dependent variable with respect to more than one independent variable is called Partial Differential Equation.

II. MATLAB (MATrixLABoratory)

MATLAB is a multi-paradigm numerical computing environment and property programming language developed by mathwork.

A. Basic Equations

Partial Differential Equations are one of an important subjects and it is a foundation of many other aspects of applied science and engineering. It concerns with study of Differential Equations. From the early days of calculus, the Partial Differential Equation has been a great area of theoretical research and particular applications and it continues to be so in our days.

$$\frac{d}{dx}(e^{ax}) = a e^{ax} \quad (1.1.1)$$

B. Partial Differential Equation

A Differential Equation involving Partial derivatives of one or more dependent variable with respect to more than one independent variable is called Partial Differential Equation. We will consider the case of two independent variable and one dependent variable, usually take x and y as independent variable and z as dependent variable.

For example:-

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy \quad (1.2.1)$$

C. Order of Partial Differential Equation

The order of the highest Partial derivative is occurring in the Partial Differential Equation is called the order of a Partial Differential Equation.

D. Degree of Partial Differential Equation

The degree of Partial Differential Equation is the degree of highest order derivative which occurs in it after the Equation has been rationalized ,i.e., Made free from radicals and fractions as far as derivatives are concerned the Equation;

E. Solution of Partial Differential Equation

In this paper we re-investigate the problem of Partial Differential Equation with one dependent variable say z with respect to two independent variable say x and y. In the 1st method we attempt for Partial Differential Equation solution of the problem. In this method we are solving Partial Differential Equation by differentiating the dependent variable z partially with respect to x, y, xy, yx, xx, yy. In the second method, we are trying to solve Partial Differential Equation using MATLAB. We are use MATLAB to calculate terms of series coefficients, which are compared with the manually calculated terms. Later invoke suitable expansion scheme for obtaining large number of terms in the perturbation solution.

Now we are trying to solve an example of Partial Differential Equation by a traditional paper pencil method as well as using MATLAB. Which are try to solving below:-

Solution of partial differential equation by traditional method of $\frac{\partial^2 z}{\partial x \partial y} = e^{-y} \cos x$

Example : we have to find the complete integral of $\frac{\partial^2 z}{\partial x \partial y} = e^{-y} \cos x$

Solution: the given partial differential equation is $\frac{\partial^2 z}{\partial x \partial y} = e^{-y} \cos x$

$$\frac{\partial}{\partial x} \left(\frac{\partial z}{\partial y} \right) = e^{-y} \cos y (*)$$

Integrating equation (*) with respect to x keeping y as a constant, we have

$$\frac{\partial z}{\partial x} = e^{-y} \sin x + c1(y) \tag{**}$$

Where c1(y) is arbitrary function of y is a constant of integration

Again integrating equation (**) with respect to y, we get

$$.z = -e^{-y} \sin x + \int c1(y)dy + c2(y)$$

$$.z = -e^{-y} \sin x + c3(y) + c2(y)$$

Where $c3(y) = \int c1(y)dy$

$$.z = -e^{-y} \sin x + c3(y) + c2(y)$$

This is required complete integral.

III. EXPLANATION

On the page number 20 we are solved a Partial Differential Equation that is $\frac{\partial^2 z}{\partial x \partial y} = e^{-y} \cos x$ by traditional paper pencil method. In this method we are try to solve the given Partial Differential Equation by direct integration. So, First of all we are integrating the given Partial Differential Equation with respect to x treating y as a constant.

We are integrating it with respect to y by treating x as a constant. The given partial differential equation with two independent variable say x, y and one dependent variable say z.

After applying all these operations we are getting the value of z. This is the solution of given Partial Differential Equation by traditional paper pencil method. The value of dependent variable z is the solution of our given Partial Differential Equation. In the next page we are try to plot a graph of this solution of Partial Differential Equation by using MATLAB.

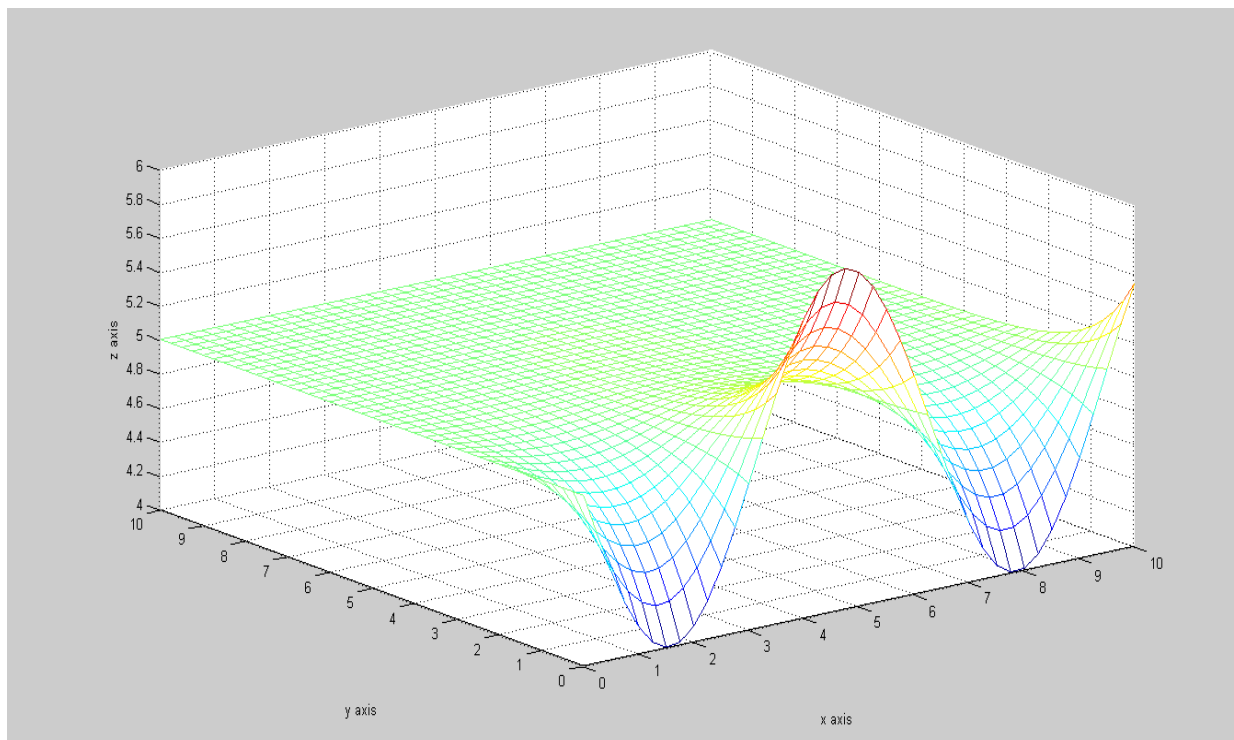
Graphical format of the solution of the Partial Differential Equation $\frac{\partial^2 z}{\partial x \partial y} = e^{-y} \cos x$ using MATLAB

Plotted Graph

```

>> x=0:0.25:10;
>> y=0:0.25:10;
>> [X,Y]=meshgrid(x,y);
>> c3=3;
>> c2=2;
>> e=2.718;
>> Z=-e.^-Y.*sin(X)+c3+c2;
>> mesh(X,Y,Z)
>> xlabel('x axis');
>> ylabel('y axis');
>> zlabel('z axis')
>> |
    
```

Plotted graph



IV. EXPLANATION

On page number 23 we format the graph of solution of given Partial Differential Equation, for this purpose we must have the range of the independent variables. In this given Partial Differential Equation we have 2 independent variable says x and y. So,

First of all we are taking range of independent variable x;

We are taking the range of independent variable y;

After this we are converted variable x and y into matrices x and y by using the command `>>[X,Y]=meshgrid(x,y);`

Now here, we have 2 constant say c3 and c2 give this constant value because these constant are arbitrary.

Also the value of e= 2.718;

After this, convert independent variable z into matrix z by applying operation of matrix

which are describing below. After this we have plot the graph by the command:

```
>>mesh(X,Y,Z)
```

After this we are give the name of axis by applying the commands.

```
>>xlabel('x axis');
```

```
>>ylabel('y axis');
```

```
>>zlabel('z axis')
```

We will plot the graph of the given partial differential equation.

All these commands are given below:-

```
>>x=0:0.25:10;
```

```
>>y=0:0.25:10;
```

```
>>[X,Y]=meshgrid(x,y);
```

```
>>c3=3;
```

```
>>c2=2;
```

```
>>e=2.718;
```

```
>>Z= -e.^-Y.*sin(X)+c3+c2;
```

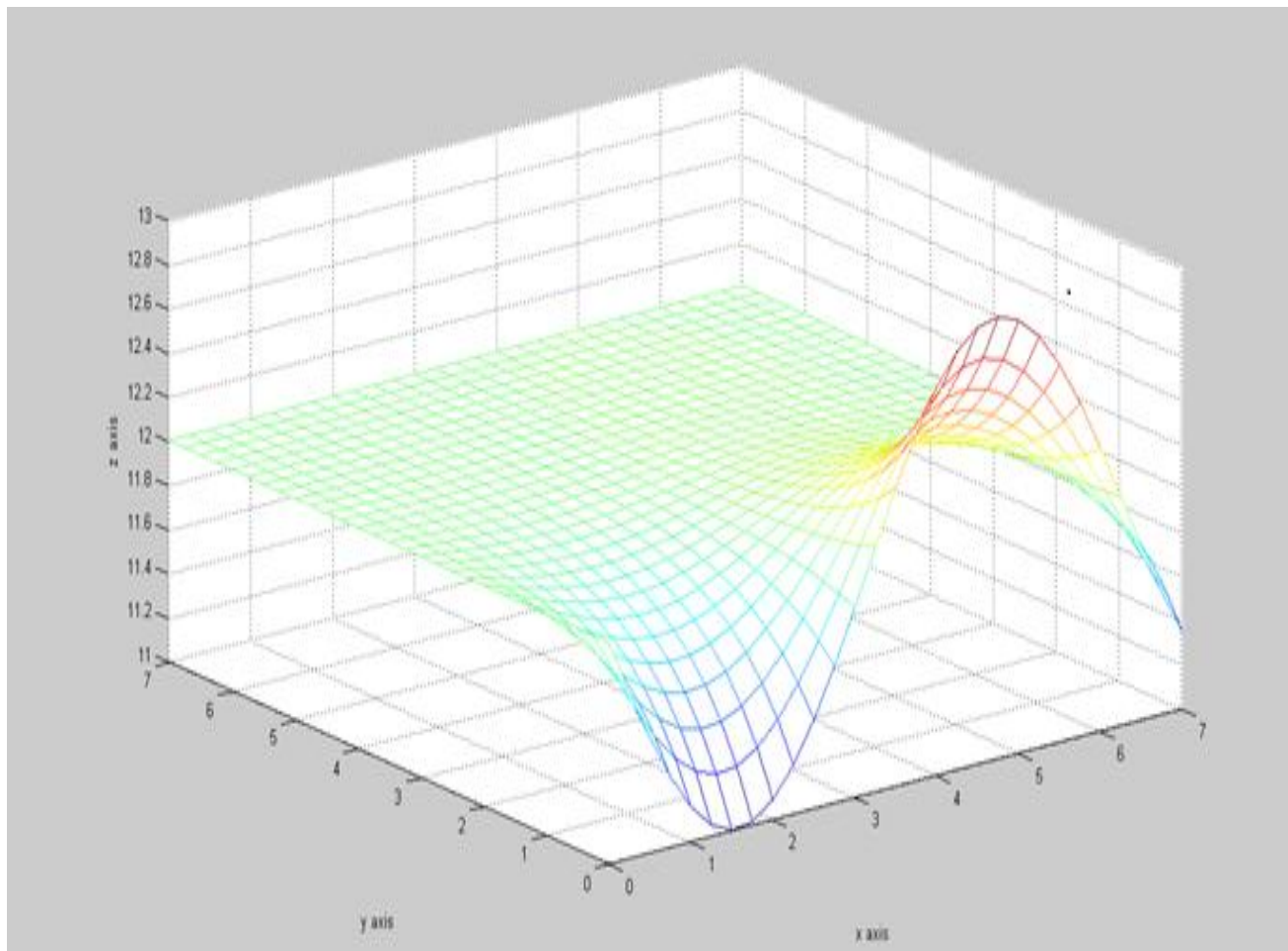
```
>>mesh(X,Y,Z)
```

```
>>xlabel('x axis');
```

```
>>ylabel('y axis');
```

```
>>zlabel('z axis')
```

For good result we are reduces the range of independent variable from “0 to 10” to “0 to 7”



On page number 26 we are format the graph of the solution of Partial Differential Equation we are taking the range of independent variable x and y is “from 0 to 7” this means that we are plotting the graph between the range “0 to 7” and also the range of dependent variable z is automatically described by MATLAB corresponding to x -axis and y -axis that is

Started from 11 and 11.2, 11.4, 11.6, 11.8, 12, 12.2, 12.4, 12.6, 12.8 these range are corresponding to x -axis and y -axis.

Here the graph is started from the z -axis of point 12 which is corresponding to x -axis and y -axis in this graph range of independent variable's variable x and y that is from 0 to 7 the space between 0 to 1 is also divided in four parts by the distance 0.25 in both ranges of the axis that is x -axis and y -axis so the graph which describe above is also 4 squares of size 0.25×0.25 in a square of size 1×1 this square of size 1×1 is made the corresponding z -axis on the point 12 which is made corresponding to x -axis and y -axis.

The first square of size 1×1 is made by the points $(0,7,12)$, $(1,7,12)$, $(0,6,12)$ and $(1,6,12)$ like this the next point of size 1×1 is made by the points $(1,7,12)$, $(2,7,12)$, $(1,6,12)$ and $(2,6,12)$ corresponding to the point 12 of z -axis corresponding to x -axis and y -axis of the complete range of independent variable x and y that is “from 0 to 7”.

So, There is next square of size 1×1 is made by the points $(2,7,12)$, $(3,7,12)$, $(2,6,12)$ and $(3,6,12)$

And like this we getting a sequence of squares and here last square of size 1x1 corresponding to point 12 of z-axis is (6,7,12), (7,7,12), (6,6,12) and (7,6,12).

This sequence of squares is getting with respect to z-axis corresponding to x-axis and y-axis. Also in the above graph we are observe that the next sequence of squares of size 1x1 are made by the points (0,6,12), (1,6,12), (0,5,12) and (1,5,12) like this the next point of size 1x1 is made by the points (1,6,12), (2,6,12), (1,5,12) and (2,5,12) corresponding to the point 12 of z-axis corresponding to x-axis and y-axis of the complete range of independent variable x and y that is “from 0 to 7”.

So, there is next square of size 1x1 is made by the points (2,6,12), (3,6,12), (2,5,12) and (3,5,12)

And like this we getting a sequence of squares and here last square of size 1x1 corresponding to point 12 of z-axis is (6,6,12), (7,6,12), (6,5,12) and (7,5,12).

This sequence of squares is getting with respect to z-axis corresponding to x-axis and y-axis. The next sequence of square of size 1x1 are made by the points (0,5,12), (1,5,12), (0,4,12) and (1,4,12) this sequence is end with the square of size 1x1 which are made by the points (6,5,12), (7,5,12), (6,4,12) and (7,4,12)

This sequence of squares is important for us because after this sequence of squares we are observing a variation in the graph of the solution of Partial Differential Equation given above. After this we are getting a variation one part of the graph going in downward from the left hand side and one other going to upward from the right hand side. When we try to observe this variation we are getting the points (0,0,12), (0.25,0.25,11.8), (0.5,0.5,11.6), (0.75,0.75,11.4), (1,1,11.2), (1.25,1.25,11), (1.50,1.50,11) these points are gives the rotation of graph downward from left hand side.

Also the points (1.75,1.75,11), (2,2,11.2), (2.25,2.25,11.4), (2.5,2.5,11.6), (2.75,2.75,11.8), (3,3,12) are gives the rotation of graph variation upward from left hand side. Also the point (3.75,3.75,12), (4,4,11.8), (4.25,4.25,11.6), (4.5,4.5,11.4), (4.75,4.75,11.2), (5,5,11),

(5.25,5.25,11) these points are gives the rotation of graph downward from right hand side.

Also the points (5.75,5.75,11), (6,6,11.2), (6.25,6.25,11.4), (6.5,6.5,11.6), (6.75,6.75,11.8),

(7,7,12) are gives the rotation of graph variation upward from right hand side.

In this graph we are also observed that graph is behavior is constant on x-axis and y-axis of the value from 7 to 4 with difference 0.25 on y-axis and 0 to 7 on x-axis with difference 0.25 also graph is variety from the value of 4 to 0 on y-axis and 0 to 7 on x-axis. The graph is rotate from upward to downward from the value 0 to 1.75 on x-axis and 0 to 4 on y-axis with respect to

z-axis after this the graph is vary from downward to upward from the point 1.75 to 5.25 on x-axis and 0 to 4 on y-axis with respect to z-axis. After this the graph is again going in downward from the point 4.75 to 7 on x-axis and from 0 to 4 on y-axis with respect to z-axis.

From this graph we are observed that the geometrical format of the solution of given Partial Differential Equation is have 28 vertical lines with respect to x-axis and 28 horizontal lines with respect to y-axis. The 1st line is made on y-axis where the value of x-axis on y-axis is ‘0’ and values of y-axis are changed from 0 to 7 and here the value of z is 12. In the next line of the point ‘1’ on the x-axis here also the point of x-axis is constant and points of y-axis is changed from 0 to 7, Here also the points of z-axis is starting from 12 and it is going to downward from 12 to 11.8, 11.6, 11.4, 11.2 and 11. This is deepest height of this graph in downward rotation. In between these two lines of the points of x-axis that is 0 and 1 here we have also other lines in the distance of 0.25 and 0.25.

Also 2nd line is made on y-axis where the value of x-axis on y-axis is ‘1’ and values of y-axis are changed from 0 to 7 and here the value of z is changed from 12 to 11 in the process of 12, 11.8, 11.6, 11.4, 11.2 and 11. In the next line of the point ‘2’ on the x-axis here also the point of x-axis is constant and points of y-axis is changed from 0 to 7,

Here also the points of z-axis is starting from 12 and it is going to downward from 12 to 11.8, 11.6, 11.4. In between these two lines of the points of x-axis that is 0 and 1 here we have also other lines in the distance of 0.25 and 0.25.

Like this in this graph we getting 8 like as we are also getting different lines and 7 sequence of squares and in between these 8 lines we have 28 lines of sixe 0.25 with respect to y-axis like as we are also getting 8 different lines and 7 sequence of squares and in between these 8 lines we have 28 lines of sixe 0.25 with respect to x-axis.

These 28 lines of x-axis 28 lines of y-axis are cross each on that points these points are the points of Equations which are made by the points of x-axis, y-axis and z-axis.

Here we try to more clarify the above graph by using some other commands which are given below:-

```
>>x=0:0.25:5;
>>y=0:0.25:5;
>>[X,Y]=meshgrid(x,y);
>>c3=3;
>>c2=2;
>>e=2.718;
>>Z= -e.^-Y.*sin(X)+c3+c2;
>>surf(X,Y,Z)
>>xlabel('x axis');
>>ylabel('y axis');
>>zlabel('z axis')
```

V. CONCLUSION

The graph of the solution of the partial differential equation is plotted by the use of MATLAB. MATLAB give us a very clear and good graph of the solution of the given partial differential equation that is $z = -e^{-y} \sin x + c_3(y) + c_2(y)$, we are solving Partial Differential Equation by differentiating the dependent variable z partially with respect to x, y, xy, yx, xx, yy.

In the second method, we are trying to solve Partial Differential Equation using MATLAB. We are use MATLAB to calculate terms of series coefficients, which are compared with the manually calculated terms. Later invoke suitable expansion scheme for obtaining large number of terms in the perturbation solution and getting the good result.

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