

# Graphical Solution of Partial Differential Equation using MATLAB

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## Abstract

In this paper, we try to find out a better solution of Partial Differential Equation using MATLAB its give detail about MATLAB and its modules, plotting the graphs of solution of Partial Differential Equations, also we try to solve and simulate the Partial Differential Equations using MATLAB. We will describe the process of Partial Differential and its solution in graphical format. Also we are plotting the results of the Partial Differential Equations and thus compare and show that better results obtained by MATLAB.

**Key words** - MATLAB, Partial Differential Equation, Plotting the graphs, traditional method.

## I. INTRODUCTION

### A. Differential Equation

An Equation involving derivatives of one or more dependent variable with respect to one or more independent variables is called Differential Equation.

### B. Partial Differential Equation

A Differential Equation involving Partial derivatives of one or more dependent variable with respect to more than one independent variable is called Partial Differential Equation.

## II. MATLAB (MATRIXLABORATORY)

MATLAB is a multi-paradigm numerical computing environment and property programming language developed by mathwork.

### A. Basic Equations

Partial Differential Equations are one of an important subjects and it is a foundation of many other aspects of applied science and engineering. It concerns with study of Differential Equations. From the early days of calculus, the Partial Differential Equation has been a great area of theoretical research and particular applications and it continues to be so in our days.

$$\frac{d}{dx}(e^{ax}) = ae^{ax} \quad (1.1.1)$$

### B. Partial Differential Equation

A Differential Equation involving Partial derivatives of one or more dependent variable with respect to more than one independent variable is called Partial Differential Equation. We will consider the case of two independent variable and one dependent variable, usually take x and y as independent variable and z as dependent variable.

For example:-

$$\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = z + xy \quad (1.2.1)$$

### C. Order of Partial Differential Equation

The order of the highest Partial derivative is occurring in the Partial Differential Equation is called the order of a Partial Differential Equation.

**D. Degree of Partial Differential Equation**

The degree of Partial Differential Equation is the degree of highest order derivative which occurs in it after the Equation has been rationalized ,i.e., Made free from radicals and fractions as far as derivatives are concerned the Equation;

**E. Solution of Partial Differential Equation**

In this paper we re-investigate the problem of Partial Differential Equation with one dependent variable say z with respect to two independent variable say x and y. In the 1<sup>st</sup> method we attempt for Partial Differential Equation solution of the problem. In this method we are solving Partial Differential Equation by differentiating the dependent variable z partially with respect to x, y, xy, yx, xx, yy. In the second method, we are trying to solve Partial Differential Equation using MATLAB. We are use MATLAB to calculate terms of series coefficients, which are compared with the manually calculated terms. Later invoke suitable expansion scheme for obtaining large number of terms in the perturbation solution.

Now we are trying to solve an example of Partial Differential Equation by a traditional method as well as using MATLAB. Which are try to solving below:-

Example: Find the complete integral of  $z^2 = pqxy$

Solution: The given partial differential equation is  $z^2 = pqxy$  (#)

Where  $p = \frac{\partial z}{\partial x}$  ;  $q = \frac{\partial z}{\partial y}$

Rewrite the given differential equation as

$$\frac{xy}{z^2} \cdot \frac{\partial z}{\partial x} \cdot \frac{\partial z}{\partial y} = 1.$$

$$\text{Or } \left(\frac{x}{z} \frac{\partial z}{\partial x}\right) \cdot \left(\frac{y}{z} \frac{\partial z}{\partial y}\right) = 1 \tag{##}$$

Put  $\left(\frac{1}{x}\right) dx = dX$  ;  $\left(\frac{1}{y}\right) dy = dY$  and  $\left(\frac{1}{z}\right) dz = dZ$

So that  $\log x = X$  ;  $\log y = Y$  and  $\log z = Z$  (###)

Using (###) in (##) , we get

$$\left(\frac{\partial Z}{\partial X}\right) \left(\frac{\partial Z}{\partial Y}\right) = 1 \text{ Or } PQ = 1 \text{ (####)}$$

Where  $P = \frac{\partial Z}{\partial X}$  ,  $Q = \frac{\partial Z}{\partial Y}$

Then , solution of equation (####) is

$$Z = aX + bY + C'$$

Where  $ab = 1$  so that  $b = \frac{1}{a}$

So  $\log z = a \log x + \frac{1}{a} \log y + \log c$  ;

Taking  $C' = \log c$  and using (###)

Or  $z = x^a y^{1/a} c$  ; a and c are arbitrary constant.

This is the required complete integral.

Explanation

On the page number 3 we are solve Partial Differential Equation by traditional method. In this method we are trying to solving the given Partial Differential Equation. Here the given Partial Differential Equation is solving and finds the complete integration. First of all rewrite the given Differential Equation and convert it as.

$PQ=1$  then this becomes a famous result. So, by using of that result gives a complete integral directly watch in last page. The given partial differential equation with two independent variables say  $x$ ,  $y$  and one dependent variable say  $z$ . After applying all these operations we are getting the value of  $z$ . This is the solution of given Partial Differential Equation by traditional method.

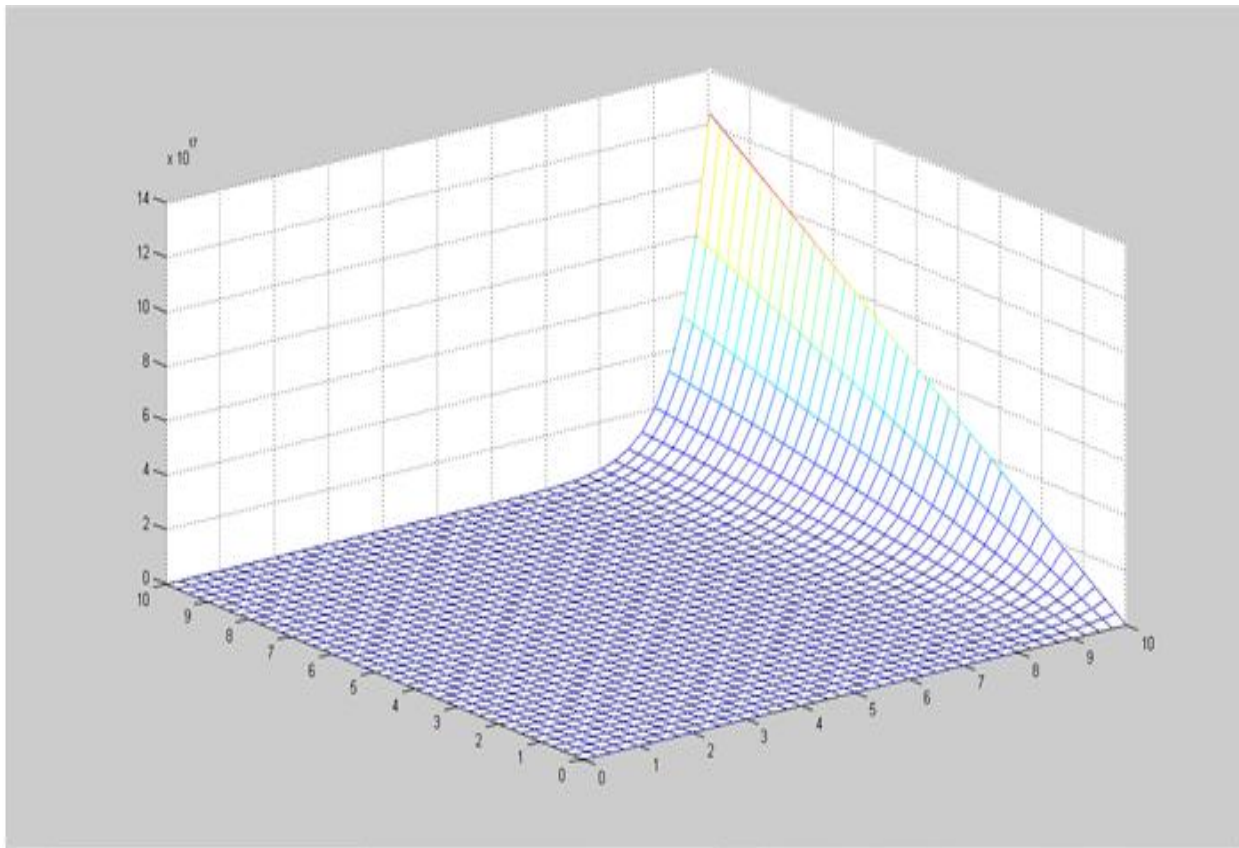
Graphical format of Partial Differential Equation  $z^2 = pqxy$  using MATLAB

Plotted graph

```
>> x=0:0.25:10;  
>> y=0:0.25:10;  
>> [X, Y] = meshgrid(x, y);  
>> a=17;  
>> c=21;  
>> Z=X.^a.*Y.^c/a.^c;  
>> mesh(X, Y, Z)  
>>
```

Graphical format of Partial Differential Equation  $z^2 = pqxy$  using MATLAB

Plotted Graph



## Explanation

On the page 5 we are format the graph of solution Partial Differential Equation, for this purpose we must have the range of the independent variables. In this given Partial Differential Equation we have 2 independent variable says x and y. So,

We are taking range of independent variable x. Then we are taking range of independent variable y, we must have to convert variables x and y into matrices x and y. By using the command:

We must have to convert variable x and y into matrices x and y by using the command.

```
>>[X,Y]=meshgrid(x,y);
```

Now here, we have 2 constant say b and c give this constant value because these constant are arbitrary. After this, convert independent variable z into matrix z by applying operation of matrix. Which are describing below.

After this we have plot the graph by the command:

```
>>mesh(X,Y,Z)
```

After this give the name of axis by applying the command

```
>>xlabel('x axis');
```

```
>>ylabel('y axis');
```

```
>>zlabel('z axis')
```

We will plot the graph of given partial differential equation.

All these commands are given below:-

```
>>x=0:0.25:10;
```

```
>>y=0:0.25:10;
```

```
>>[X,Y]=meshgrid(x,y);
```

```
>>a=17;
```

```
>>c=21;
```

```
>>Z= X.^a.*Y.^1/a.*c;
```

```
>>mesh(X,Y,Z)
```

```
>>xlabel('x axis');
```

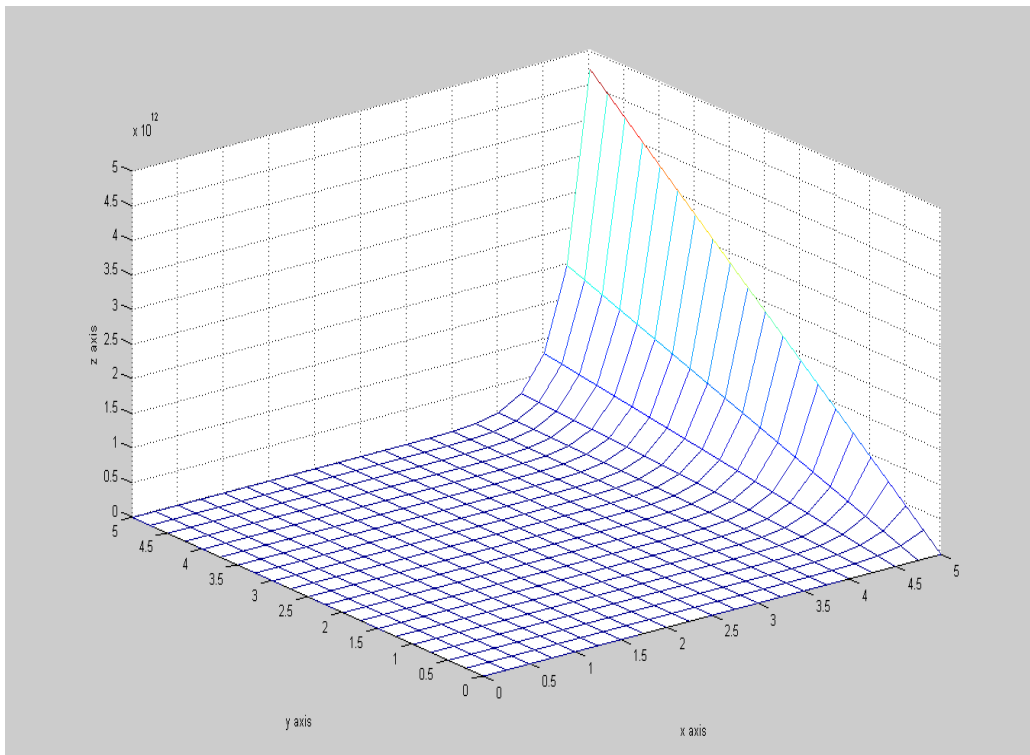
```
>>ylabel('y axis');
```

```
>>zlabel('z axis')
```

Now we are try to describe the graphical format of the solution of Partial Differential Equation and we are try to difference between geometrical representation of the solution of Partial Differential Equation and the solution of Partial Differential Equation which we are get from traditional paper pencil method.

For good result we are reduces the range of independent variable from “0 to 10” to “0 to 5”

```
>> x=0:0.25:5;  
>> y=0:0.25:5;  
>> [X,Y]=meshgrid(x,y);  
>> a=17;  
>> c=21;  
>> Z=X.^a.*Y.^1/a.*c;  
>> mesh(X,Y,Z)  
>> xlabel('x axis');  
>> ylabel('y axis');  
>> zlabel('z axis')  
>>
```



The above graph is very easy to observe. This graph is starting from corresponding to z-axis with respect to x-axis and y-axis. In the above graph of the solution of Partial Differential Equation we are taking the range of independent variable x and y is “from 0 to 5” this means that we are plotting the graph between the range “0 to 5” and also the range of dependent variable z is automatically described by MATLAB corresponding to x-axis and y-axis that is Started from “0” and 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5 these range are corresponding to x-axis and y-axis.

Here the graph is started from the z-axis of point 0 which is corresponding to x-axis and y-axis in this graph range of independent variable’s variable x and y that is from 0 to 5 the space between 0 to 1 is also divided in four parts by the distance 0.25 in both ranges of the axis that is x-axis and y-axis so the graph which describe above is also 4 squares of size 0.25x0.25 in a square of size 1x1 this square of size 1x1 is made the corresponding z-axis on the point 0 which is made corresponding to x-axis and y-axis. When we are study the graph vertically corresponding to z-axis with respect to x-axis and y-axis.

The first square of size  $1 \times 1$  is made by the points  $(0,5,0)$ ,  $(1,5,0)$ ,  $(0,4,0)$  and  $(1,4,0)$  like this the next point of size  $1 \times 1$  is made by the points  $(0,4,0)$ ,  $(1,4,0)$ ,  $(0,3,12)$  and  $(1,3,0)$  corresponding to the point 0 of z-axis with respect to x-axis and y-axis of the complete range of independent variable x and y that is “from 0 to 5”.

So, there is next square of size  $1 \times 1$  is made by the points  $(0,3,0)$ ,  $(1,3,0)$ ,  $(0,2,0)$  and  $(1,2,0)$  and like this we getting a sequence of squares and here last square of size  $1 \times 1$  corresponding to point 0 of z-axis is  $(0,1,0)$ ,  $(1,1,0)$ ,  $(0,0,0)$  and  $(1,0,0)$ . This sequence of squares is getting with respect to z-axis corresponding to x-axis and y-axis. Also in the above graph we are observe that the next sequence of squares of size  $1 \times 1$  are made by the points  $(1,5,0)$ ,  $(2,5,0)$ ,  $(1,4,0)$  and  $(2,4,0)$  like this the next point of size  $1 \times 1$  is made by the points  $(1,4,0)$ ,  $(2,4,0)$ ,  $(1,3,0)$  and  $(2,3,0)$  corresponding to the point 0 of z-axis with respect to x-axis and y-axis of the complete range of independent variable x and y that is “from 0 to 5”.

So, there is next square of size  $1 \times 1$  is made by the points  $(1,3,0)$ ,  $(2,3,0)$ ,  $(1,2,0)$  and  $(2,2,0)$  and like this we getting a sequence of squares and here last square of size  $1 \times 1$  corresponding to point 0 of z-axis is  $(1,1,0)$ ,  $(2,1,0)$ ,  $(1,0,0)$  and  $(2,0,0)$ . This sequence of squares is getting corresponding to z-axis with respect to x-axis and y-axis. The next sequence of square of size  $1 \times 1$  are made by the points  $(2,5,0)$ ,  $(3,5,0)$ ,  $(2,4,0)$  and  $(3,4,0)$  this sequence is end with the square of size  $1 \times 1$  which are made by the points  $(2,1,0)$ ,  $(3,1,0)$ ,  $(2,0,0)$  and  $(3,0,0)$ .

The next sequence of square of size  $1 \times 1$  are made by the points  $(3,5,0)$ ,  $(4,5,0)$ ,  $(3,4,0)$  and  $(4,4,0)$  this sequence is end with the square of size  $1 \times 1$  which are made by the points  $(3,1,0)$ ,  $(4,1,0)$ ,  $(3,0,0)$  and  $(4,0,0)$ . The next sequence of squares is important for us because after this sequence of squares we are observing a variation in the graph of the solution of Partial Differential Equation given above. When the graph is vary with x-axis on the points 4, 4.25, 4.50, 4.75 and 5 with respect to y-axis on the points 0, 0.25, 0.5, 0.75, 1 we are getting a small variation with respect to z-axis from the point 0 to 0.75 also we are go towards to the points of y-axis 1, 1.25, 1.5, 1.75, 2 on x-axis on same point 4, 4.25, 4.5, 4.75 and 5 we are getting again variation with respect to z-axis from the point 0.75 to 1.65. Again we are go towards to the points of y-axis 2, 2.25, 2.5, 2.75 and 3 on x-axis on same point 4, 4.25, 4.5, 4.75 and 5 we are getting again variation with respect to z-axis from the point 1.65 to 3.125. Again we are go towards to the points of y-axis 3, 3.25, 3.5, 3.75, 4 on x-axis on same point 4, 4.25, 4.5, 4.75 and 5 we are getting again variation with respect to z-axis from the point 3.125 to 3.5. And again we are going towards to the points of y-axis 4, 4.25, 4.5, 4.75, 5 on x-axis on same point 4, 4.25, 4.5, 4.75 and 5 we are getting again variation with respect to z-axis from the point 3.5 to 4.75

In this graph we are also observed that graph is behavior is constant on x-axis and y-axis of the value from 0 to 4 with difference 0.25 on y-axis and 0 to 4 on x-axis with difference 0.25 also graph is variate from the value of 0 to 5 on y-axis and 4 to 5 on x-axis with respect to z-axis on the points 0 to 4.75.

From this graph we are observed that the geometrical format of the solution of given Partial Differential Equation is have 20 vertical lines with respect to x-axis and 20 horizontal lines with respect to y-axis. the 1<sup>st</sup> line is made on y-axis where the value of x-axis on y-axis is ‘0’ and values of y-axis is changed from 0 to 5 and here the value of z is 0. In the next line of the point ‘1’ on the x-axis here also the point of x-axis is constant and points of y-axis is changed from 0 to 5, here also the points of z-axis is starting from 0 and it is going to upward from 0 to 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5. Also 2<sup>nd</sup> line is made on y-axis where the value of x-axis on y-axis is ‘1’ and values of y-axis is changed from 0 to 5 and here the value of z is changed from 0 to 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5. In the next line of the point ‘2’ on the x-axis here also the point of x-axis is constant and points of y-axis is changed from 0 to 5, here also the points of z-axis is starting from 0 and it is going to toward from 0 to 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5 and 5 In between these two lines of the points of x-axis that is 0 and 1 here we have also other lines in the distance of 0.25 and 0.25.

Like this in this graph we getting 6 like as we are also getting different lines and 5 sequence of squares and in between these 6 lines we have 20 lines of sixe 0.25 with respect to y-axis like as we are also getting 6 different lines and 5 sequence of squares and in between these 6 lines we have 20 lines of sixe 0.25 with respect to x-axis. These 20 lines of x-axis 20 lines of y-axis are cross each on that points these points are the points of Equations which are made by the points of x-axis, y-axis and z-axis.

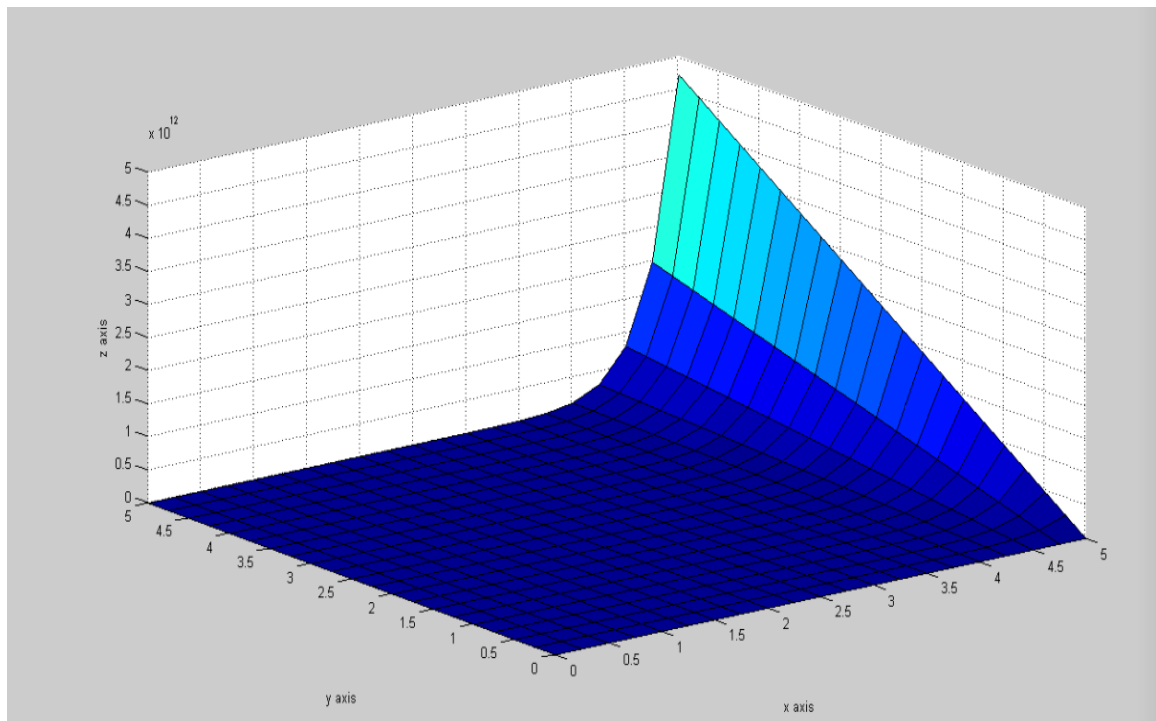
Here we try to more clarify the above graph by using some other commands which are given below:-

```
>>x=0:0.25:5;
```

```
>>y=0:0.25:5;  
>>[X,Y]=meshgrid(x,y);  
>>a=17;  
>>c=21;  
>>Z= X.^a.*Y.^1/a.*c;  
>>surf(X,Y,Z)  
>>xlabel('x axis');  
>>ylabel('y axis');  
>>zlabel('z axis')
```

The graph after using these commands are given below:-

```
>> x=0:0.25:5;  
>> y=0:0.25:5;  
>> [X, Y]=meshgrid(x, y);  
>> a=17;  
>> c=21;  
>> Z=X.^a.*Y.^1/a.*c;  
>> surf(X, Y, Z)  
>> xlabel('x axis');  
>> ylabel('y axis');  
>> zlabel('z axis')  
>> |
```





### III. CONCLUSION

The graph of the solution of the partial differential equation is plotted by the use of MATLAB. MATLAB give us a very clear and good graph of the solution of the given partial differential equation that is  $z = -e^{-y} \sin x + c_3(y) + c_2(y)$ , we are solving Partial Differential Equation by differentiating the dependent variable  $z$  partially with respect to  $x$ ,  $y$ ,  $xy$ ,  $yx$ ,  $xx$ ,  $yy$ .

In the second method, we are trying to solve Partial Differential Equation using MATLAB. We are use MATLAB to calculate terms of series coefficients, which are compared with the manually calculated terms. Later invoke suitable expansion scheme for obtaining large number of terms in the perturbation solution and getting the good result.

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