On Nano $\pi^*g\beta$ - Closed Sets

B.Shanmugaraj[#1], G.Kosalai[#2]
Department of Mathematics
Sri Kaliswari College, Sivakasi,

Abstract
In this paper, new class of set is called nano $\pi^*g\beta$-closed in nano topological spaces is introduced and its properties.

Keywords: *nano π - closed set, nano πg – closed set, nano $\pi g\beta$ – closed sets, nano $\pi^*g\beta$ - closed sets.*

I .INTRODUCTION

Lellis Thivagar [2] introduced nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set , but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to binary relation based covering nano topological space.

Rajasekaran et al. [5,6,7,8] initiated the study nano πg – closed set, nano gβ – closed sets, nano πgp – closed sets, nano πgs – closed sets. In this paper , a new class of sets called nano $\pi^*g\beta$ – closed sets in nano topological spaces is introduced and its properties.

II. PRELIMINARIES

Throughout this paper (U, τ_R(X)) (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space (U, τ_R(X)), Ncl(H) and Nint (H) denote the nano closure of H and the nano interior of H respectively. We recall the following definitions which are useful in the sequel.

Definition 2.1.[3]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U.

1.The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x\in U}\{ R(x): R(x) \subseteq X\}$, Where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, $UR(X) = \cup_{x\in U}\{R(x): R(x) \cap X \neq \emptyset\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X)= U_R(X)- L_R(X)$.

Example 2.2.

Let U={ a,b,c,d}

R={ (a,a), (b,b), (c,c), (d,d), (b,d), (d,b)} be an equivalence relation on U

U\R={{a},{c},{b,d}}

Let X={a,b}⊆ U

Then $L_R(X)=\{a\}$, $U_R(X)=\{a,b,d\}$ and $B_R(X)=\{b,d\}$.

Proposition 2.3.[2]

If (U, R) is an approximation space and $X,Y \subseteq U$; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;

2. $L_R(\emptyset)= U_R(\emptyset)=\emptyset$ and $L_R(U)= U_R(U)=U$;

3. $U_R(X\cup Y)= U_R(X) \cup U_R(Y)$;

4. $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$;

5. $L_R(X\cup Y) \supseteq L_R(X) \cup L_R(Y)$;

6. $L_R(X\cap Y) \subseteq L_R(X) \cap L_R(Y)$;

7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$;

8 $U_R(X^c) =[L_R(X)]^c$ and $L_R(X^c) =[U_R(X)]^c$;

9. $U_R U_R (X) = L_R U_R (X)= U_R (X)$;

10. $L_R L_R (X) = U_R L_R (X)= L_R (X)$;

Definition 2.4.[2] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{\emptyset, U, L_R(X), U_R(X), B_R(X)\}$ where $X\subseteq U$. Then by the property 2.3, R(X) satisfies the following axioms:

1. U and ø are in $\tau_R(X)$,

2. The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$,

3. The intersection of the elemen ts of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and complement of $\tau_R(X)$ are called as closed sets.

Example 2.5.

Let $X=\{a,b\}$ be a set.

In example 2.2, $\tau_R(X)=\{\phi,\{a\},\{a,b,d\},\{b,d\},U\}$is a nano topology on U.

Therefore $(U, \tau_R(X))$ is a nano topological space.

The nano open sets are ϕ, {a}, {a,b,d}, {b,d} and U.

The nano closed sets are ϕ, {b,c,d}, {c}, {a,c} and U.

Remark 2.6.[2] If $(\tau_R(X))$ is a nano topology on U with respect to X, then the set $B=\{U, \emptyset, L_R(X), B_R(X)\}$ is the basis for $\tau_R(X)$.

Definition 2.7.[2] If $(U, \tau_R(X))$ is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by Nint (H). That is, Nint(H) is the largest nano open subset of H.

The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by Ncl(H). That is, Ncl(H) is the smallest nano closed set containing H.

Example 2.8.

In example 2.5, Let H={b,d}$\subseteq$ U

Nint(H)={b,d}

Ncl(H)={b,c,d}

Definition 2.9. A subset H of a nano topological space (U, $\tau_R(X)$) is called

1. nano semi-open [2] if H $\subseteq$ Ncl(Nint(H)).
2. nano regular-open [2] if H = Nint(Ncl(H)).
3. nano pre-open [2] if H $\subseteq$ Nint (Ncl(H)).
4. nano α-open [2] if H $\subseteq$ Nint(Ncl(Nint(H))).
5. nano β-open [4] if H $\subseteq$ Ncl(Nint(Ncl(H))).
6. nano π-open [1] if the finite union of nano regular-open sets.

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.10. A subset H of a nano topological space (U, $\tau_R(X)$) is called

1. nano g-closed [10] if Ncl(H) $\subseteq$ G, whenever H $\subseteq$ G and G is nano open.
2. nano πg-closed [5] if Ncl(H) $\subseteq$ G, whenever H $\subseteq$ G and G is nano π - open.
3. nano gβ-closed [8] if Nβcl(H) $\subseteq$ G, whenever H $\subseteq$ G and G is nano -open.
4. nano πgp-closed [6] if Npcl(H) $\subseteq$ G, whenever H $\subseteq$ G and G is nano π - open.
5. nano πgs-closed [7] if Nscl(H) $\subseteq$ G, whenever H $\subseteq$ G and G is nano π – Open
6. nano πgβ-closed [9] if Nβcl(H) $\subseteq$ G, whenever H $\subseteq$ G and G is nano π – Open

The complements of the above mentioned sets is called their respective closed sets.

III. ON NANO π*Gβ - CLOSED SETS

Definition 3.1. A subset H of a nano topological space (U, $\tau_R(X)$) is called a nano π^*gβ-closed if Nβcl(Nint(H)) $\subseteq$ G, whenever H $\subseteq$ G and G is nano π – open.

The complement of nano π^*gβ-open if H^c=U-H is nano π^*gβ-closed

Example 3.2. Let U={a, b, c, d} with U\R={{a}, {b,d}, {c}} and X={a, b}. Then the nano topology $\tau_R(X)$={∅, {a}, {a,b,d}, {b,d}, U} , {a} is nano π^*gβ-closed set and {a,b} is nano π^*gβ-closed set.

Theorem 3.3 In a space (U, $\tau_R(X)$), If H is nano gβ-closed then H is nano π^*gβ-closed.

Proof. Let H be nano gβ-closed. Let H$\subseteq$ G, G is nano π – open. Since H is nano gβ-closed. Thus Nβcl(H) $\subseteq$ G, G is nano open. Implies Nβcl(Nint(H)) $\subseteq$ Nβcl(H) $\subseteq$ G. Thereore Nβcl(Nint(H)) $\subseteq$ G. Hence H is nano π^*gβ-closed.

Theorem 3.4 In a space (U, $\tau_R(X)$), If H is nano πg-closed then H is nano π^*gβ-closed.

Proof. Let H be nano πg-closed. Let H⊆ G, G is nano π – open. Since H is nano πg-closed. Thus Ncl(H) ⊆ G, G is nano $\pi -$ open. Implies Nβcl(Nint(H)) ⊆ Ncl(H) ⊆ G. Thereore Nβcl(Nint(H)) ⊆ G. Hence H is nano π^*gβ-closed.

Theorem 3.5 In a space (U, τ_R(X)), If H is nano πgβ-closed then H is nano π^*gβ-closed.

Proof. Let H be nano πgβ-closed. Let H⊆ G, G is nano π – open. Since H is nano πgβ-closed. Thus Nβcl(H) ⊆ G, G is nano $\pi -$ open. Implies Nβcl(Nint(H)) ⊆ Nβcl(H) ⊆ G. Thereore Nβcl(Nint(H)) ⊆ G. Hence H is nano π^*gβ-closed.

Lemma 3.6 In a space (U, τ_R(X)), every nano closed set is nano π^*gβ-closed.

Proof. Let H be nano closed. We know that "every nano closed set is nano g-closed". Thus Ncl(H) ⊆ G, G is nano open. Implies Nβcl(Nint(H)) ⊆ Ncl(H) ⊆ G. Implies Nβcl(Nint(H)) ⊆ Ncl(H) ⊆ G. Thereore Nβcl(Nint(H)) ⊆ G. Hence H is nano π^*gβ-closed.

Remark 3.7 The converse of statements in lemma 3.6 are not necessarily true as seen from the following example.

Example 3.8 In example 3.2, {a} is nano π^*gβ-closed set but not nano closed.

Theorem 3.9 In a space (U, τ_R(X)), the following properties are equivalent:

1. H is H nano π - open and nano π^*gβ-closed.
2. H is nano regular-open.

Proof. (1)→(2) by (1) Nβcl(Nint(H)) ⊆ H, Since H is nano π – open and nano π^*gβ-closed. Thus Nint(Ncl(Nint(H))) ⊆ H, Since Nβcl(Nint(H)) = H ∪ Nint(Ncl(Nint(H))). As H is nano open, then H is clearly nano α–open and so H⊆Nint(Ncl(Nint(H))). Thereore Nint(Ncl(Nint(H))) ⊆ H⊆ Nint(Ncl(Nint(H))) or equivalently H= Nint(Ncl(H)), which show that H is nano regular –open.

(2)→(1). Every nano regular –open set is nano π – open and it is every nano β - closed.

Corollary 3.10 If H is nano open and nano π^*gβ-closed, then H is nano β – closed and hence nano gβ-closed.

Proof. By assumption and theorem 3.9, H is nano regular –open. Thus H is nano β – closed and hence nano gβ-closed.

Theorem 3.11 In a space (U, τ_R(X)), the union of two nano π^*gβ-closed sets is nano π^*gβ-closed.

Proof. Let H∪Q ⊆ G, then H⊆ G, Q⊆ G, where G is nano π-open. As H and Q are nano π^*gβ-closed, Nβcl(Nint(H)) ⊆ G and Nβcl(Nint(Q)) ⊆ G. Hence Nβcl(Nint(H∪Q))= Nβcl(Nint(H)) ∪ Nβcl(Nint(H)) ⊆ G.

Remark 3.12 In example 3.2, then H={a} and Q={b} is nano π^*gβ-closed sets. Clearly H∪Q={a,b} is nano π^*gβ-closed sets.

Theorem 3.13 In a space (U, τ_R(X)), the intersection of two nano π^*gβ-closed sets is nano π^*gβ-closed.

Proof. Let H∩Q ⊆ G, then H⊆ G, Q⊆ G, where G is nano π-open. As H and Q are nano π^*gβ-closed, Nβcl(Nint(H)) ⊆ G and Nβcl(Nint(Q)) ⊆ G. Hence Nβcl(Nint(H∩Q))= Nβcl(Nint(H)) ∩ Nβcl(Nint(H)) ⊆ G.

Remark 3.15 In a space (U, τ_R(X)), the union of two nano π^*gβ-closed sets but not nano π^*gβ-closed.

Remark 3.16 In example 3.2, then H={a,b} and Q={a,d} is nano π^*gβ-closed sets. Clearly H∪Q={a,b,d} is but not nano π^*gβ-closed sets.

REFERENCES

[1] Adinatha C. Upadhya, On quasi nano p-normal spaces,International Journal of Recent Scientific Research, 8(6)(2017),17748-17751.

[2] M.Lellis Thivagar and carmel Richard,on nano forms of weekly open sets ,International Journal of Mathematics and Statistics Invention ,1(1)(2013),31-37.

[3] Z.Pawlak,Rough sets, International Journal of Computer and Information Sciences,11(5)(1982),341-356.

[4] A.Revathy and G.Ilango, on nano β-open sets, Int. Jr. of Engineering, contemporary mathematics and sciences, 1(2)2015, 1-6.

[5] I.Rajasekaran and O.Nethaji, On some new subsets of nano topological space, Journal of new theory,16(2017),52-58.

[6] I.Rajasekaran T.siva Subramanian raja and O.Nethaji, On nano πgp-closed sets, journal of new theory, 19(2017), 20-26.

[7] I.Rajasekaran and O.Nethaji, On nano πgs-closed sets, journal of new theory, 19(2017), 56-62.

[8] I.Rajasekaran, M.Meharin and O.Nethaji ,On nano gβ-closed sets, International Jouranl of Mathematics and its Application,5(4-C)(2017),377-382.

[9] M.Sajan Joseph, I.Rajasekaran and O.Nethaji, on nano π gβ-closed sets, Global Journal o pure and applied mathematics. ISSN 0973-1768 volume 14, Number 1(2018), pp. 181-187

[10] K.Bhuvaneshwari and K.Mythili Gnanapriya,Nano generalizsed Closed sets,International Journal of Scientific and Research Publications,4(5)(2014),1-3.