# Describe Partial Differential Equation using MATLAB with the Help of Graphs

Priyanka and AnkurBala, Shekhar
*Department of Mathematics , Govt. College, Hansi*
*Department of Mathematics , Maharaja Neempal  Singh Govt College, Bhiwani*

*Aim of this paper is to find out some partial differential equation by graphical method  using MATLAB*

**Abstract:**
　　　*The current paper deals with the Partial Differential Equation and the plotted graph of the solution of Partial Differential Equation with the help of MATLAB. Initially, there is a Partial differential Equation with two independent variables and one dependent variable. We are plotted the graph of the solution of Partial Differential Equation with the help of MATLAB and describe it. We are try to find out a better solution of Partial Differential Equation using MATLAB its give detail about MATLAB and its modules and try to find out a better  result with the help of MATLAB.*

**Key words -** *MATLAB, Partial Differential Equation, Plotting the graphs, traditional method.*

## I. INTRODUCTION

### A. Partial Differential Equation

　　　A Differential Equation involving Partial derivatives of one or more dependent variable with respect to more than one independent variable is called Partial Differential Equation.

## II. MATLAB (**MAT**RIX**LAB**ORAROTY)

　　　MATLAB is a multi-paradigm numerical computing environment and property programming language developed by mathwork.

### A. Advantages of MATLAB

　　　MATLAB has several advantages over other methods or languages: Matrix is the basic data element for MATLAB. In MATLAB a simple integer is also considered as a matrix of order 1x1. Several mathematical operations that work on arrays or matrices are built-into the MATLAB environment.

### B. Disadvantages of MATLAB

　　　In all disadvantages of MATLAB main important disadvantages is that MATLAB is so expensive it is so costly to achieve the license of MATLAB. It is so expensive for user to buy each and every module and pay for it.

### C. Why we choose MATLAB

　　　The next question front on us is that why we are choosing MATLAB for solving Partial Differential Equation. The region behind that are:-

### 1. Plotting is easy

　　　In this paper, we are plot some graphs of Partial Differential Equation the plotting of graph in MATLAB is easy for this we must have some information of Partial Differential  Equation and MATLAB also. For the plotting the graph of Partial Differential Equation, We must have the graph will be 3-D because, When take a Partial Differential Equation with one dependent variable and two independent variable, Say z as a dependent variable and x, y are independent variable .

### 2. Solution of Partial Differential Equation

　　　In this paper we re-investigate the problem of Partial Differential Equation with one dependent variable say z with respect to two independent variable say  x and y. In the $1^{st}$ method we attempt for Partial Differential Equation solution of the problem. In this method we are solving Partial Differential Equation by differentiating the

dependent variable z partially with respect to x, y, xy, yx, xx, yy. In the second method, we are trying to solve Partial Differential Equation using MATLAB. We are use MATLAB to calculate terms of series coefficients, which are compared with the manually calculated terms. Later invoke suitable expansion scheme for obtaining large number of terms in the perturbation solution.

Now we are trying to solve an example of Partial Differential Equation by a traditional method as well as using MATLAB. Which are try to solving below:-

Solution: of Partial Differential Equation $(y - x)(qy - px) = (p - q)^2$ by traditional method.

Example find the complete integral of $(y - x)(qy - px) = (p - q)^2$ where p=$\frac{\partial z}{\partial x}$ ; $q = \frac{\partial z}{\partial y}$

Solution: Let x and y be two new variable such that X = x+y and Y = xy          (*)

The given equation is $(y - x)(qy - px) = (p - q)^2$          (**)

Now, p $= \frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial z}{\partial Y}\frac{\partial z}{\partial x} = \frac{\partial z}{\partial X} + y\frac{\partial z}{\partial Y}$          (***)

[Because from (*) $\frac{\partial X}{\partial x} = 1\ and\ \frac{\partial Y}{\partial y} = y$]

And q $= \frac{\partial z}{\partial y}$, $\frac{\partial z}{\partial X}\frac{\partial X}{\partial y} + \frac{\partial z}{\partial Y}\frac{\partial Y}{\partial y} = \frac{\partial z}{\partial X} + x\frac{\partial z}{\partial Y}$          (****)

$[from\ (*)\frac{\partial X}{\partial y} = 1, \frac{\partial Y}{\partial y} = x]$

Substituting the above values of p and q in equation (**), we have

$.(y - x)\left[y\left(\frac{\partial z}{\partial X} + x\frac{\partial Z}{\partial Y}\right) - \left(\frac{\partial z}{\partial X} + y\frac{\partial z}{\partial Y}\right)\right] = \left[\left(\frac{\partial z}{\partial X} + y\frac{\partial z}{\partial Y}\right) + \left(\frac{\partial z}{\partial X} + x\frac{\partial z}{\partial Y}\right)\right]^2$

Or $(y - x)^2\frac{\partial z}{\partial X} = (y - x)^2\frac{\partial z}{\partial Y}^2$

Or $\frac{\partial z}{\partial X} = \left(\frac{\partial z}{\partial Y}\right)^2\ or\ P = Q^2$          (****)

Where P $= \frac{\partial z}{\partial X}\ and\ Q = \frac{\partial z}{\partial Y}$

Equation (****) is of the form $f(PQ) = 0$

So, solution of equation (****) is given by z = aX+bY+c          (*****)

Where $= b^2$ , obtaind by putting a for P and b for Q in (****)

So from equation (*****) , required complete integral is z=$b^2X + bY$+C

Or $z = b^2(x + y) + bxy + C$

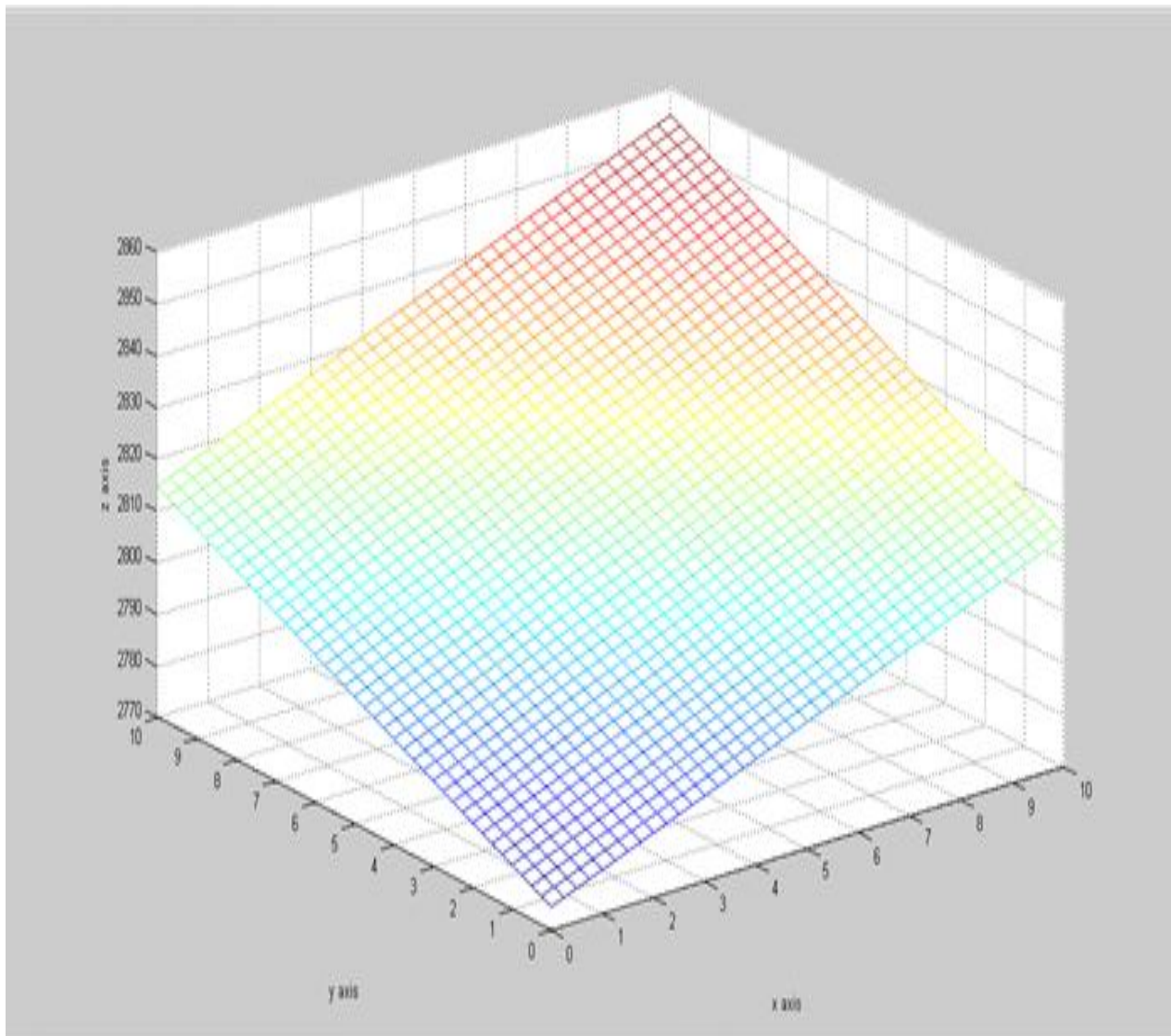This is the required complete integral.

Graphical format of the solution of Partial Differential Equation $(y - x)(qy - px) = (p - q)^2$ using MATLAB

Plotted Graph

```
>> x=0:0.25:10;
>> y=0:0.25:10;
>> b=2;
>> c=7;
>> [X,Y]=meshgrid(x,y);
>> Z=b^2.*(X+Y)+b*X*Y+c;
>> mesh(X,Y,Z)
>> xlabel('x axis');
>> ylabel('y axis');
>> zlabel('z axis')
>>
```

Graphical format of the solution of Partial Differential Equation $( y - x )( qy - px ) = ( p - q )^{2}$ using MATLAB

Plotted graph

## III. EXPLANATION

On page number  5  we are format the graph of solution of Partial Differential Equation, $( y - x )( qy - px ) = ( p - q )^2$ for this purpose we must have the range of the independent variables. In this given partial differential equation we have two independent variables say x and y. So, we are taking range of independent variable x; then we are taking range of independent variable y.  We must have to convert variable x and y into matrices x and y. By using the command:

>>[X,Y]=meshgrid(x,y);

Now here, we have 2 constant say b and c give this constant value because these constant are arbitrary. After this, convert independent variable z into matrix z by applying operation of matrix. Which are describe below. After this we have plot the graph by the command:

>>mesh(X,Y,Z)

After this give the name of axis by applying the command:

>>xlabel('x axis');

>>ylabel('y axis');

>>zlabel('z axis')

We will plot the graph given partial differential equation using above commands.

All these commands are given below:-

>>x=0:0.25:10;

>>y=0:0.25:10;

>>[X,Y]=meshgrid(x,y);

>>b=2;

>>c=7;

>>Z= b.^2(X+Y)+b*X*Y+c;

>>mesh(X,Y,Z)

>>xlabel('x axis');

>>ylabel('y axis');

>>zlabel('z axis')

Now we are try to describe the graphical format of the solution of Partial Differential Equation and we are try to difference between geometrical representation of the solution of Partial Differential Equation and the solution of Partial Differential Equation which we are get from traditional paper pencil method.

For good result we are reduces the range of independent variable from "0 to 10" to "0 to 7"
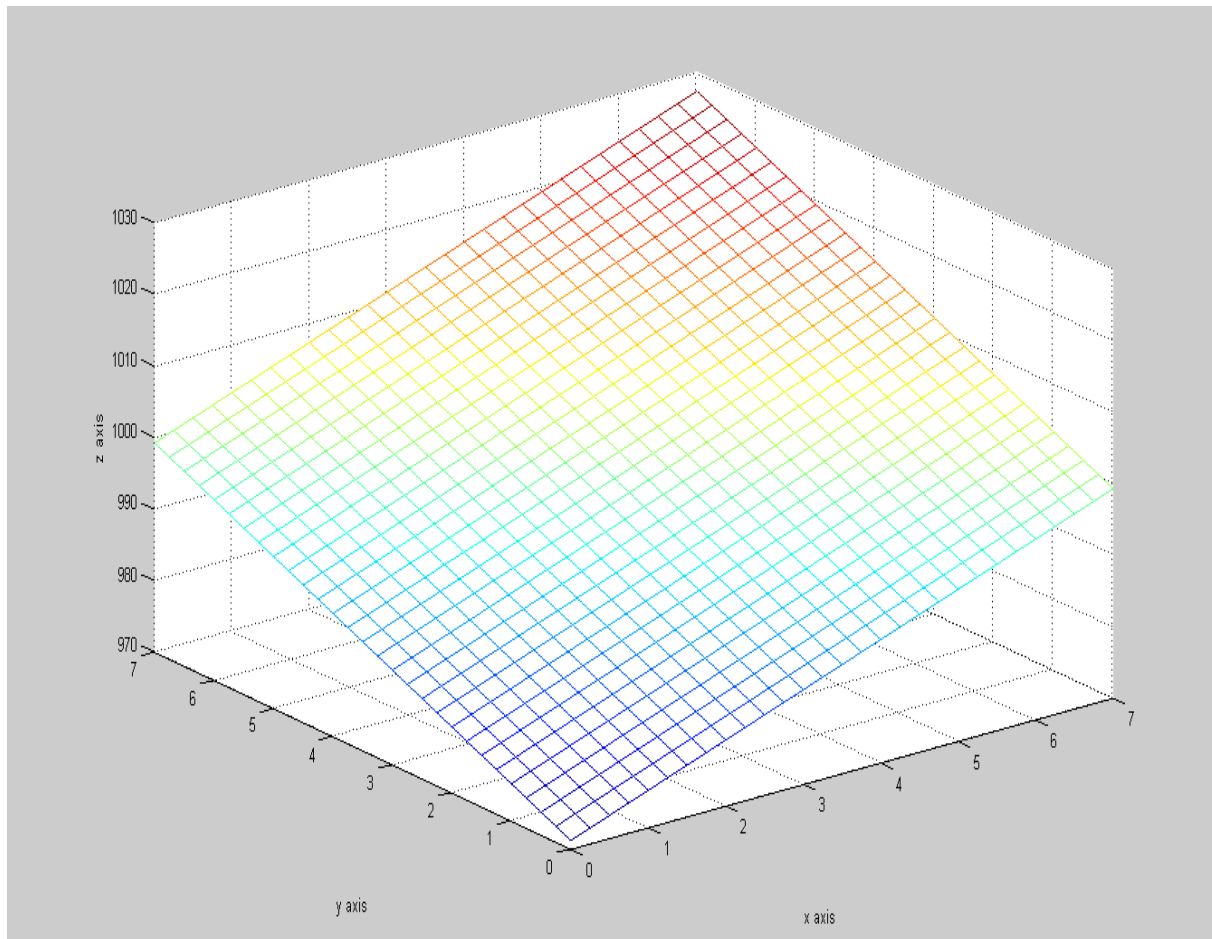
On the page number 5 above graph of the solution of Partial Differential Equation we are taking the range of independent variable x and y is "from 0 to 7" this means that we are plotting the graph between the range "0 to 7" and also the range of dependent variable z is automatically described by MATLAB corresponding to x-axis and y-axis that is

Started from 970 and 980, 990, 1000, 1010, 1020, 1030 these range are corresponding to x-axis and y-axis. Here the graph is started from the z-axis of point 998 which is corresponding to x-axis and y-axis in this graph range of independent variable's variable x and y that is from 0 to 7 the space between 0 to 1 is also divided in four parts by the distance 0.25 in both ranges of the axis that is x-axis and y-axis so the graph which describe above is also 4 squares of size 0.25x0.25 in a square of size 1x1. This square of size 1x1 is made the corresponding z-axis on the point 998 which is made corresponding to x-axis and y-axis.

The first square of size 1x1 is made by the points (0,6,994), (0,7,998), (1,6,998) and (1,6,1002) like this the next point of size 1x1 is made by the points (1,6,998), (1,7,1002), (2,6,1002) and (2,7,1006) corresponding to the z-axis with respect to x-axis and y-axis of the complete range of independent variable x and y that is "from 0 to 7".So, There is next square of size 1x1 is made by the points (2,6,1002), (2,7,1006), (3,6,1006) and (3,7,1010)And like this we getting a sequence of squares and here last square of size 1x1 corresponding to z-axis is (6,6,1018), (6,7,1022), (7,6,1022) and (7,7,1026).This sequence of squares is getting with respect to z-axis corresponding to x-axis and y-axis.

Also in the above graph we are observe that the next sequence of squares of size 1x1 are made by the points (0,5,978), (0,6,982), (1,5,982) and (1,6,986) like this the next point of size 1x1 is made by the points (1,5,982), (1,6,986), (2,5,990) and (2,6,994) corresponding to z-axis with respect to x-axis and y-axis of the complete range of independent variable x and y that is "from 0 to 7".

So, there is next square of size 1x1 is made by the points (2,5,986), (2,6,990), (3,5,990) and (3,6,994) And like this we getting a sequence of squares and here last square of size 1x1 corresponding to z-axis is (6,5,1000), (6,6,1004), (7,5,1004) and (7,6,1008). This sequence of squares is getting with respect to z-axis corresponding to x-axis and y-axis.

The next sequence of square of size 1x1 are made by the points (0,4,968), (0,5,972), (1,4,972) and (1,5,976) this sequence is end with the square of size 1x1 which are made by the points (6,4,990), (6,5,994), (7,4,994) and (7,5,998)

When we are observed the graph we notice there this graph have a variation from starting. So, this graph is starting from middle of z-axis with respect to x-axis and y-axis when we are grow with x-axis and also with y-axis the graph will be extended with the values of x-axis is increasing and also with y-axis values on increasing. When we try to observe this variation we are getting a variation from starting the graph. (0,0,12), (0.25,0.25,11.8), (0.5,0.5,11.6), (0.75,0.75,11.4), (1,1,11.2), (1.25,1.25,11), (1.50,1.50,11) these points are gives the rotation of graph downward from left hand side.Also the points (1.75,1.75,11), (2,2,11.2), (2.25,2.25,11.4), (2.5,2.5,11.6), (2.75,2.75,11.8),(3,3,12) are gives the rotation of graph variation upward from left hand side. Also the point (3.75,3.75,12), (4,4,11.8), (4.25,4.25,11.6), (4.5,4.5,11.4), (4.75,4.75,11.2), (5,5,11), (5.25,5.25,11) these points are gives the rotation of graph downward from right hand side. Also the points (5.75,5.75,11), (6,6,11.2), (6.25,6.25,11.4), (6.5,6.5,11.6), (6.75,6.75,11.8), (7,7,12) are gives the rotation of graph variation upward from right hand side.

When we observe the graph from left hand side to right hand side we get the graph is starting from middle of the value of z-axis and grow downward with respect to x-axis and y-axis and also it is grow upward with respect to x-axis and y-axis parallel. When it is grow downward and upward parallel with respect to x-axis and y-axis on pick of the height in both upward and downward it is changed from downward to upward and upward to downward. We are getting a rhombus type graph here.

From this graph we are observed that the geometrical format of the solution of given Partial Differential Equation is have 28 vertical lines with respect to x-axis and 28 horizontal lines with respect to y-axis.the 1st line is made on y-axis where the value of x-axis on y-axis is '0' and values of y-axis is changed from 0 to 7 and here the value of z is changed from 970 to 1030. In the next line of the point '1' on the x-axis here also the point of x-axis is constant and points of y-axis is changed from 0 to 7, here also the points of z-axis is starting from 970 to 1030 In between these two lines of the points of x-axis that is 0 and 1 here we have also other lines in the distance of 0.25 and 0.25.

Also 2nd line is made on y-axis where the value of x-axis on y-axis is '1' and values of y-axis are changed from 0 to 7 and here the value of z is changed from 970 to 1030. In the process of 970, 980, 990, 1000, 1010, 1020 and 1030. In between these two lines of the points of x-axis that is 0 and 1 here we have also other lines in the distance of 0.25 and 0.25.

Like this in this graph we getting 8 like as we are also getting different lines and 7 sequence of squares and in between these 8 lines we have 28 lines of sixe 0.25 with respect to y-axis like as we are also getting 8 different lines and 7 sequence of squares and in between these 8 lines we have 28 lines of sixe 0.25 with respect to x-axis.

These 28 lines of x-axis 28 lines of y-axis are cross each on that points these points are the points of Equations which are made by the points of x-axis, y-axis and z-axis.

Here we try to more clarify the above graph by using some other commands which are given below:-
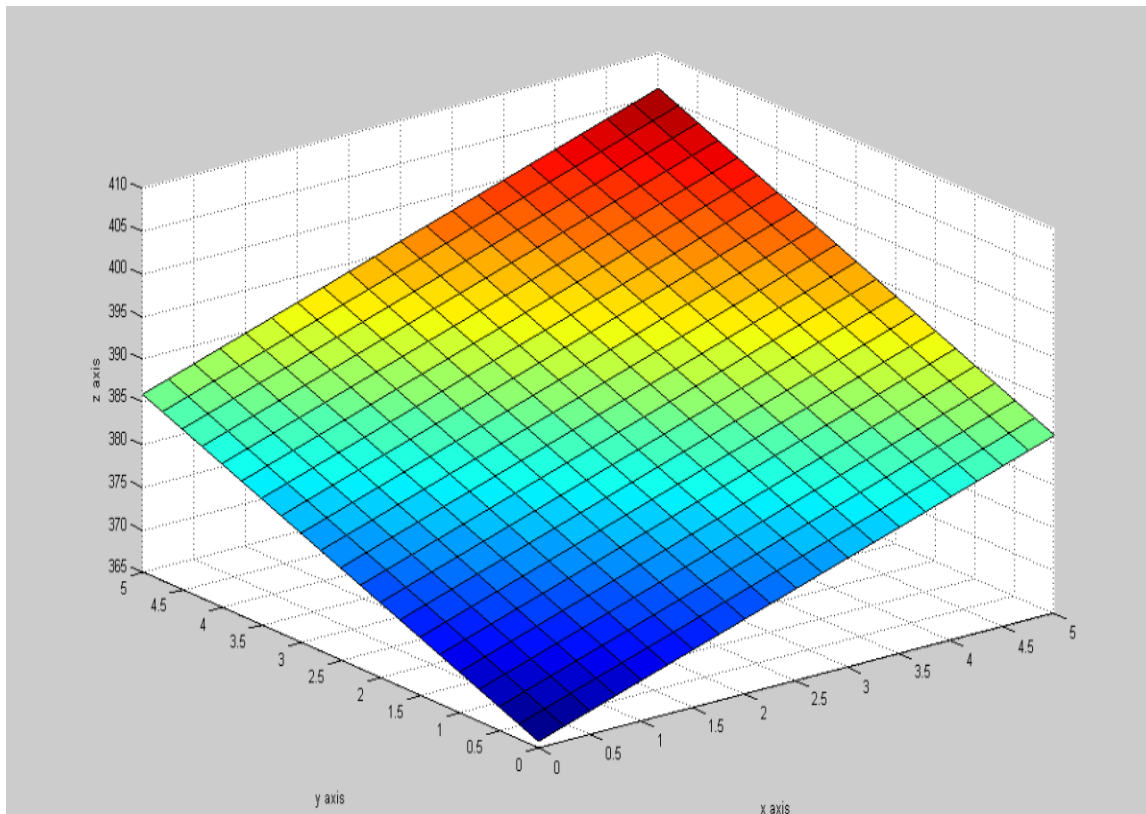
>>x=0:0.25:5;

>>y=0:0.25:5;

>>[X,Y]=meshgrid(x,y);

>>b=2;

>>c=7;

>>Z= b.^2(X+Y)+b*X*Y+c;

>>surf(X,Y,Z)

>>xlabel('x axis');

>>ylabel('y axis');

>>zlabel('z axis')

The graph after using these commands are given below:-

```
>> x=0:0.25:5;
>> y=0:0.25:5;
>> [X,Y]=meshgrid(x,y);
>> b=2;
>> c=7;
>> Z=b.^2.*(X+Y)+b*X*Y+c;
>> surf(X,Y,Z)
>> xlabel('x axis');
>> ylabel('y axis');
>> zlabel('z axis')
>> |
```

## IV. CONCLUSION

The graph of the solution of the partial differential equation is plotted by the use of MATLAB. MATLAB give us a very clear and good graph of the solution of the given partial differential equation that is $.z = -e^{-y} sinx + c3(y) + c2(y)$, we are solving Partial Differential Equation by differentiating the dependent variable z partially with respect to x, y, xy, yx, xx, yy.

In the second method, we are trying to solve Partial Differential Equation using MATLAB. We are use MATLAB to calculate terms of series coefficients, which are compared with the manually calculated terms. Later invoke suitable expansion scheme for obtaining large number of terms in the perturbation solution and getting the good result.

## REFERENCES

The work reported in this thesis is mainly based on the following papers.

[1]     Birajdar Gunwant, stability of non-linear fractional partial differential equations, Aurangabad     (M.S.) INDIA.
[2]     A.Freidman, Partial Differential Equations, Halt, Rineheart and Winston inc., New-york,(1969).
[3]     Agan Kumar Tyagi :MATLAB and SIMUNINk for engineerings.(2011).
[4]     Alllen Downay:Physical Modeling in MATLAB.(2008).
[5]     Andrei D. Polyanin, Valentin F. Zaitsev: Handbook of Partial differential Equations, Second Edition.(2016).
[6]     Andrew Knight:Basic of MATLAB and Beyond.(1999).
[7]     Stormy Attaway :Matlab: A Practical introduction to programming and problem solving. (2013)
[8]     T. Amarnath, Narosa publication house: An elementary course in Partial Differential Equations second addition.75-83 (2003).
[9]     Taan ElAli, Mohammad A. Karim: Continuous signals and systems with MATLAB.
[10]    Thomas Hillen, I. E. Leonard, Henry Van Rossel,:Partial Differential equations.(2014).
[11]    Tichy, J.A. and bourgin, P., "The effect of inertia in lubrication flow including intrence and initial conditions", AMSE Journal of applied mechanics, 107(4), (1986).
[12]    Timothy A. Davis, Kermit Sigmon :MATLAB primer.(2015).
[13]    Tobin A. Driscoll: learning MATLAB (2009).
[14]    Tomas roubick.: Non-linear Partial Differential Equations with Applications.(2013).
[15]    Victor hanner, Tatyana Belozerova, Mikhail Khenner: Ordinary and partial differential Equations.(2013).
[16]    W.F. Ames, "non-linear Partial Differential Equations in engineering", Academic press, 1972.
[17]    Wang, C.Y., "Three dimensional flow due to a stretching flat", The physics of fluids, (1984).