Isomorphism of Groups of Operators on Hilbert Space

Prabhat Kumar[#1], Amar Nath Kumar[*2], Lalit Kumar Sharan[#3]
[#1] Research Scholar,P.G. Departmen of Mathematics,V.K.S.University,Ara,Bojpur,Bihar,India
[#2] Assistant Professor, Departmen of Mathematics,J.N.College.L.N.M.University,Darbhanga,Bihar,India
[#3] Retired Professor and Head ,P.G. Departmen of Mathematics,V.K.S.University,Ara,Bojpur,Bihar,India

Abstract

In this paper we have established two theorems by using the role of identity operator analogous to that for identity element e of a group. Using the property of isomorphism efforts have also been made to establish a result according to that the order of an element of a group is equal to the order of the image of that group. It has also been established that the f image of an identity operator of a domain group is an identity element of co-domain group. The same kind of result is also established for the additive inverse of a group considered. It has also been observed a relation between homomorphism and abelian group. In fact this result is an analogous result imposing a stronger condition on the homomorphism we have established a set of necessary and sufficient conditions for the group to be an abelian. Efforts have also been made by establishing a result that the inverse of an isomorphism is again an isomorphism. We have also observed by establishing a result that the product of two isomorphism is isomorphism.

Keywords - *Linear Space, Linear Transformation, Operator, Non singular Transformation, Norm, Normed Linear Space, Cauchy Sequence, Complete Metric Space, Banach Space, Inner Product, Hilbert Space, Continuous Linear Transformation, Operator on a Hilbert Space, Group. Homomorphism, Isomorphism.*

I. INTRODUCTION

Sincere effort has been made in order to study **isomorphism of groups of operators**. It is a well known fact that a mapping from a group to another group satisfying a set of special conditions is called an isomorphism. A good number of authors have given a collection of works on groups and isomorphism. As our groups considered in this work are not groups of numbers (real or complex) rather our groups are of nonsingular operators on a Hilbert space H. So we have made a stories to establish some analogous results for our groups of operators in place of the results already established for the groups of numbers e.t.c. As a result of which the Technique used for is a bit different from that for the groups of numbers..

In course of doing so to give a completion to this paper we first of all have exhibited a few examples. Through these examples we have shown that mapping defined in a suitable form becomes isomorphism. While doing so we have observed that under the same suitable mapping even when the groups were changed from additive group to multiplicative group or from multiplicative group to additive group even then the results followed. Going through the examples two to five the picture of our saying may be more and more clear.

II. PRELIMINARIES AND DEFINITIONS

In this section we are giving below a collection of all essential preliminaries ,definition and results by the notions of which we shall establish results in subsequent section..

Linear Space:—The symbol **K** will stand for either the set **R** of all real numbers and the set **C** of all complex numbers.

A structure of linear space on a set **E** is defined by two maps.

a. $(x,y) \rightarrow x + y$ of **E**×**E** into **E** and is called vector addition.

b. $(a,x) \rightarrow ax$ of **K**×**E** into **E** and is called scalar multiplication.

The above two maps are assumed to satisfy the following conditions.

(i) $x + y = y + x$, for every x,y in **E**. (ii) $x + (y + z) = (x + y) + z$ for every x,y,z in **E**. (iii)There exists an element 0 in **E** such that $x + 0 = 0 + x = x$, for every x in **E**. (iv) For every element x in **E** there exists an element denoted by -x in **E**, such that $x +(-x) =(-x)+ x = 0$, for every x in **E**.(v) $a(x + y) = ax + ay$, for every a in **K** and all x,y in **E**.(vi) $(a + b) x = ax + bx$, for every a,b in **K** and all x in **E**. (vii) $(ab)x = a(bx)$, for every a,b in **K** and all x in **E** (viii) $1\cdot x = x$, for every x in **E**.
Whenever all the above axioms are satisfied, we say that **E** is a linear space (or a vector space) over **K**.

Now if **K** be the set of all real numbers then **E** is a real linear space and similarly if **K** be the set of all complex numbers then **E** is called a complex linear space.Here every element of **E** is called a vector and every element in **K** is called a scalar.The zero vector '**O**' is unique and called the zero element or the origin.

LINEAR SUBSPACE:— Let **E** be a linear space (over a field **K**).A non empty subset **F** of **E** is called a linear subspace (or simply subspace) of **E** if **F** itself forms a vector space over **K** with respect to the addition and scalar multiplication defined on **E**.

Zero Space:—A linear space may consists solely of the vector O with scalar multiplication defined by $\alpha\cdot 0 = 0$, *for every* α.We call this linear space as zero space and we always denote it by{0}.

Linear Transformation:—Let **E** and **E**$'$ be any two linear spaces (over the field **K**). A mapping **T : E → E**$'$ is called a linear transformation if the following conditions are satisfied.

(i)T(u+v)=T(u)+T(v),for every u,v are in **E**.(ii)T(αu)=αT(u)for every u ∊ **E** and α is in **K.**
Here the conditions (i) and (ii) can be together expressed as,T(αu+βv)=αT(u)+βT(v),for every u,v in **E** and α,β in **K**.

A linear transformation is also called a linear mapping.If there be no chance of confusion then we shall write T_x in place T(x).Now it is easy to see that,If T:**E→E**$'$ be a linear transformation of a linear space **E** in to a linear space **E**$'$ then T preserves the origin and negatives.

For since,T(0) = T(0·0) = 0·T(0) = 0. Also,T(-x)=T(-1·x)=(-1)·T(x)=-T(x),for every x in **E**.

Linear Operator:—Let **E** be a linear space (over the field **K**). then a mapping T:E→**E**$'$ from a linear space **E** in to **E** itself is called a linear operator on **E**, if it satisfies the following conditions. T(αu+βv)=αT(u)+βT(v),*for every u,v in **E** and α,β in **K**.*

Thus T is a linear operator on a linear space **E** if T is a linear transformation from **E** in to **E** itself.

Zero Transformation:—Let **E** and **E**$'$ be any two linear spaces (over the field **K**).

Let T:**E→E**$'$ be a mapping from **E** in to **E**$'$. Now, if T is defined as T(u)=0(zero vector of **E**$'$)

for every u in **E**, then for u,v ∊ **E**; α,β ∊ K. we have au + βv ∊ **E.** But then, T(αu + βv) = 0.
= α·0+β·0=αT(u)+βT(v),Hence,T in this situation is a linear transformation from **E** in to **E**$'$and

T is called a zero transformation.If there be no chance of confusion then whenever T is a zero transformation we shall denote it by 0 (zero).

Negative of a Linear Transformation:—Let **E** and **E**$'$ be any two linear spaces (over the field **K**).Let T:**E→E**$'$ be a linear transformation from **E** into **E**$'$.Then the negative of T denoted by –T is defined by (-T)(u) = -[T(u)] for every u ∊ E and T(u) ∊ **E**$'$**.** Since, T(u) ∊ **E**$'$ ⇒ -T(u) ∊ **E**$'$.Thus, –T is also a function from **E** into **E**$'$**.** Now, let α,β ∊ **K** and u,v ∊ **E.**

Then,αu+βv∊**E**Also,(-T)(αu+βv)=-[T(αu+βv)]=-[αT(u)+βT(v)][T is a linear transformation] = α[-T(u)] + β[-T(v)]Thus, –T is a linear transformation from **E** in to **E**$'$.

Hence, –T is called the negative of the linear transformation T.

The Inverse of a Linear Transformation:-Let **E** and **E**$'$ be any two linear spaces (over the field K). Let T:**E→E**$'$ be a linear transformation from **E** in to **E**$'$.

When T is onto and one-one then a function of the form of T^{-1}:**E**$'$**→E**,a function from **E**$'$ in to**E**, is read as the inverse of T.

The Sum of Two Linear Transformations:—LetT_1,T_2 be two linear transformations from linear space **E** into linear space **E**$'$ then T_1+T_2 is also linear transformation from E into**E**$'$.

As on the same ground the sum of a finite number of linear transformations is again a linear transformatio. Similarly αT_1 is also a linear transformation

The Product of Two Linear Transformations:—For any two linear transformations T_1 T_2 on **E** we define $(T_1T_2)(x)=T_1(T_2(x))$, x ∊ v. We call T_1T_2 as the product of T_1 and T_2. The product T_1T_2 is also a linear transformation because $(T_1T_2)(\alpha x+\beta y)=T_1(T_2(\alpha x+\beta y))$ $=T_1(\alpha T_2(x)+\beta T_2(y))$ $=\alpha T_1(T_2(x))+\beta T_1(T_2(y)=\alpha(T_1T_2)(x)+\beta(T_1T_2)(y)$∀α,β are scalars, x,y∊**E**.

(we refer to JHA, K.K (1) page 168-169)

Non Singular Linear Transformation:—Let T be a linear transformation on a linear space **E** then T is called invertible or non singular if T is one-one and onto otherwise T is called singular.Thus if T is a non singular linear transformation on a linear space **E** then the

transformation T:**E**→**E** is one-one and onto.Hence its inverse,T^{-1}:**E**→**E** exists such that, T(x) = y ⇔ x = T^{-1}(y) ,Also, if I is the identity function on **E.** Then, TT^{-1} = I = $T^{-1}T$ Further,
(i) T^{-1} is also a linear transformation on **E**.(ii) A linear transformation on a finite dimensional vector space is **E** is non singular if and only if it is one to one.(For proof of (i) and (ii) we refer to Jha, K.K (1) p-172) (iii) A linear transformation T on a linear space **E** is one-one if and only if T is onto.

Norm:—Let **E** be a linear space over a field **K**.Then by a norm on **E** we understand a map f:**E**→$R^+$from **E** intoR^+ (the set of non negative real numbers)Which satisfied the following conditions. (i)f(x)=0⇔x=0.(ii)f(λx)=|λ|f(x),∀λ∈K,x∈E. f(x+y)≤f(x)+f(y) ∀ x,y∈E Now if there be no chance of confusion in writing || || for f and ||x|| for f(x) then the all above three conditions take the form.(i) ||x|| = 0 ⇔ x = 0. (ii) |λx|| = |λ| ||x|| ∀ λ ∈ K, x ∈ E. (iii) ||x+y|| ≤ ||x|| + ||y|| ∀ x,y ∈ E.

Normed Linear Space:—By a normed linear space we understand a linear space E together with a norm || || defined on it.At times a normed linear space is also called a normed vector space or a normed space.

Metric (or Distance Function):—Let M be a non empty set then a real valued function d defined on M×M is called a distance function (or metric function or simply metric on M) if the following conditions are satisfied.(i) d(x,y) ≥ 0.(ii) d(x,y) = 0 ⇔ x = y.(iii)d(x,y)=d(y,x). (iv) d(x,z) ≤ d(x,y) + d(y,z).Here the condition (iii) is known as the condition of symmetry and condition (iv) is known as triangle inequality. Also, d(x,y) due to symmetry does not depend on the order of the elements.

Metric Space:—The system (or the pair) (M,d) containing a non empty set M and a metric d defined on it is called a metric space. The elements of M are called the points of the metric space (M,d). If there is no chance of confusion, we denote the metric space (M,d) by the symbol M which is used for the underlying set of points. One should always keep in mind that a metric space is not merely a non empty set.
(For the definition of metric and metric space, we refer to simmons, G.F (1)(p-51)).
Now we can verify that the normed linear space N is a metric space with respect to the metric d defined by d(x,y) = ||x-y||.(For verification we refer to Jha, k.k(1)p-181).

Cauchy Sequence:—A sequence (x_n) of points of a metric space (M,d) is said to be a cauchy sequence,if for each ∈>0,There exists a natural number n_t Such that,m,n∈N and m,n≥n_t⟹d(x_m,x_n) < t , That is, if $\lim_{m,x\to\infty} d(x_m,x_n)=0$, It is worth much to note that.

(1) Every convergent sequence is a cauchy sequence.
(2) Cauchy sequence is not necessarily convergent.(We refer to simmons, G.F(1) p-71).

Complete Metric Space:—A metric space (M,d) is said to be complete if every cachy sequence in (M,d) is convergent in (M,d).(We refer to Simmons, G.F(1) p-71).

Banach Space:—A normed linear space N is said to be a Banach space if it is complete as a metric space.

Inner Product (or Scalar Product): —Let E be a linear space over a field **K**.By an inner product (or scalar proudct) on **E** we mean a map.(x,y)→(x/y) of **E**×**E** into **K**.
Satisfying the following conditions namely. (i) (x/x) ≥ 0 ∀ x in **E**.(ii) (x/x) = 0 ⇔ x = 0.(iii) (x/y) = $(\overline{y/x})$.(iv) (λx + μ y/z) = λ(x/z) + μ(y/z) ∀ λ,μ ∈ **K**, x,y,z ∈ **E.**

How ever if **K** = R (the set of real numbers),Then, the condition (iii) takes the for (x/y)=(y/x). Also, from conditions (iii) and (iv) we have (x/λy+μz)=($\overline{\lambda y + \mu z/x}$)= $\overline{\lambda(y/x) + \mu(z/x)}$

=$\bar{\lambda}(\overline{y/x})+\bar{\mu}(\overline{z/x})$= $\bar{\lambda}$(x/y)+$\bar{\mu}$(x/z) (We refer to Simmons, G.F (1) (p-245))

It is worthy much to note that an inner product space **E** is a normed linear space with respect to the norm defined interm of an inner product given by ||x|| = +$\sqrt{x/x}$

(For verification we refer to Jha,K.K (1) p-270)

There is no chance of confusion then (x/y) is read as the inner product of x with y (or equivalently the dot product of x with y or the dot product of x and y.

Hilbert Space:—A Banach space is said to be a Hilbert space if its norm is or can be defined by means of an inner product.

Continuous Linear Transformation (Continuous Linear Operator): —Let **E** and **E**' be normed linear spaces over the same field **K**. Let T:**E**→**E**' be a linear transformation from **E** in to **E**'.Then T is said to be continuous if it is continuous as a mapping of the metric space **E**

into the metric space **E**ʹ where the metric is defined in terms of norm.Hence,T is continuous ⇔ $x_n \to x$ in **E**⟹T(x_n)→T(x) in **E**ʹ.

It is worthy much to note that.(1).For any normed linear space **E** the identity transformation I:**E**→**E** defined by I(x) = x for every x ∊ **E** ,is a continuous linear transformation Infact $x_n \to x$ in **E**⟹Ix_n→I_x in **E**.(2).For normed linear spaces **E** and **E**ʹ the zero transformation O:**E**→**E**ʹ denoted by O(x)=O∊**E**for every x ∊ **E** is a continuous linear transformation.In fact x_n→x in E⟹O(x_n)=O→O(x)=O

Operator on a Hilbert Space H:—By an operator on a Hilbert space H we shall mean a continuous linear transformation from H into self.

Binary Operation:—Let **E** be a non empty set then by a binary operation on **E** we mean a mapping. O:**E**×**E**→**E** from **E**×**E** into **E** given by, O:(x,y)→xOy∊E for all x,y∊E and(x,y)∊E×E. That is the closure property is satisfied.

Group:—Let G be a non empty set and 'O'be a binary operation defined on G.Then the pair (G,O) is said to be a group if and only if the following conditions are satisfied :(i) As 'O' is a binary operation so the closure property is automatically satisfied.(ii) The binary operation 'O' is associative That is,(a O b) O c = a O (b O c) for all a,b,c ∊ G.(iii) Existence of an identity elements. For every element a ∊ G ∃ an element e in G.such that, aOe= eOa = a (iv) Existence of an inverse elements.For every a ∊ G ∃ an elements a^{-1}, read as the inverse of a,such that, a O a^{-1} = a^{-1} O a = e.How ever if an additinal condition of commutative is satisfied. ie. a O b = b O a ∀ a,b ∊ G.

Then, (G,O) is called an abelian group or a commutative if there be no chance of confusion then in place of writing (G,O) we simply write G.

Homomorphism:—Let (G,O) and (Gʹ,Oʹ) be two groups then a mapping f:G→Gʹis called a homomorphism if f(a O b) = f(a) Oʹ f(b). for a,b ∊ G when there is no scope of confusion we shall use the same symbol ' . ' in place of 'O' and ' Oʹ '.Hence the above definition at once takes the form f:G→Gʹ is a homomorphism if f(ab) = f(a)f(b)

That is f preserves the composition in groups.

Isomorphism:—f:G→Gʹ is an isomorphism If (i) f preserves the composition in groups (ii) f is one-one.(iii) f is onto.Clearly every isomorphism is a homomorphism or an isomorphism is a special case of homomorphism .

Also if,f : G → Gʹ is homomorphism then we say that f(G) is the homomorphic image of G in Gʹ.Also Gʹ is called homomorphic image of G if f is onto homomorphism and whenever f : G → Gʹ is isomorphism then f(G) is called an isomorphic image of G in Gʹ.

In this situation we also say that G is isomorphic to Gʹ or G is equivalent to Gʹ.

Kernel Of Homomorphism :—Let G and Gʹ be two groups of nonsingular operators on a Hilbert space. Also let f:G→Gʹ be a homomorphism then Kernel of f(denoted by Kerf) is defined by Kerf = { T_i ∊ G : f(T_i) = Iʹ }. Where Iʹ is the identity operater of Gʹ thus if I be the identity operator of G. Then, f(I) = Iʹ.

III.RESULTS

In this section we establish some of the results using the definitions given in above section of this paper.

Theorem (4.3, I):—Let G and Gʹ be any two multiplicative groups of nonsingular operators on a Hilbert space H.Also,f:G→Gʹ be an isomorphism.Then, prove that If I be an identity element of G then f(I) is the identity element of Gʹ.

Proof:—Let I be the identity element of G and I′ be the identity element of G′.Let T_i be an element of G for all i.Then, $f(T_i) \in G'$ for all i.Then, $I'f(T_i) = f(T_i)$[As I′ is the identity element of G′.] = $f(IT_i) = f(I)f(T_i)$ [since f is isomorphism]Thus, $I'f(T_i) = f(I)f(T_i)$

This implies that, $I' = f(I) \in G'$ [by right cancelation law] Therefore f(I) is the identity element of G′.

Theorem (4.3, II):—Let $f : G \rightarrow G'$ be an isomorphism from a multiplicative group G to a multiplicative group G′ Then, $f(T_i^{-1}) = [f(T_i)]^{-1}$Or equivalently the f image of the inverse of an element T_i of G is the inverse of the f-image of T_i.

Proof: —Let I be the identity element of G. And I′ be the identity element of G′. Hence, $I' = f(I)$.Also, for $T_i \in G \Rightarrow T_i^{-1} \in G$ and G is a multiplicative group.Hence,$T_iT_i^{-1} = I$

Since$I' = f(I) = f(T_iT_i^{-1}) = f(T_i)f(T_i^{-1})$ [Since f is isomorphism ⇒ f preserves composition] i.e.$I' = f(T_i)f(T_i^{-1})$ Hence $f(T_i^{-1})$ is the inverse of $f(T_i)$ or equivalently $f(T_i^{-1}) = [f(T_i)]^{-1}$.

Theorem (4.3, III):—Let G and G′ be two multiplicative groups of nonsingular operators on a Hilbert space H.Let $f:G \rightarrow G'$ be an isomorphism of G onto G′.Then the order of an element T_i of G is equal to the order of its image $f(T_i)$ in G′.

Proof: —Let I and I′ be the identity element of G and G′ respectively.Also,$f:G \rightarrow G'$ is an isomorphism from G onto G′.Thus, f is one-one, onto and preserves composition of G and G′ (which is the same).Now,let $O(T_i)=n$ and $Of(T_i)=m$ Then,$T_i^n = I \Rightarrow f(T_i^n)=f(I) \Rightarrow f(T_i,T_i,T_i,\ldots n$ times$)=f(I) \Rightarrow f(T_i)f(T_i)f(T_i)\ldots\ldots n$ times$)=f(I) \Rightarrow [f(T_i)]^n = f(I) \Rightarrow Of(T_i) \leq n \Rightarrow m \leq n$ .....(4.11) Further since $Of(T_i) = m \Rightarrow [f(T_i)]^m = f(I). \Rightarrow f(T_i)f(T_i)f(T_i)\ldots\ldots m$ times$) = f(I) \Rightarrow f(T_iT_iT_i\ldots\ldots\ldots m$ times$) = f(I) \Rightarrow f(T_i^m) = f(I) \Rightarrow T_i^m = I$ [since f is one-one also]$\Rightarrow O(T_i) \leq m$ But, $O(T_i) = n$,Hence, $n \leq m$ …………(4.12) Hence, from [(4.11)and(4.12)]. $m = n$

That is, the order of an element T_i of G is equal to the order of its image $f(T_i)$ in G′.

We now want to make an attempt to observe that whether theorems (4.3,I) and (4.3,II) can be established for additive groups also.

Theorem (4.3, IV):—Let G and G′ be two additive groups of nonsingular operators on a Hilbert space H.Also let $f : G \rightarrow G'$ be an isomorphism from G onto G′.Then, if I be an identity element of G.Then, f(I) is the identity element of G′.

Proof:—Let I and I′ be the identity elements of G and G′ respectively.To prove that, $I' = f(I)$

For this, let $T_i \in G$ be an arbitry for all i.Then,$I' + f(T_i)=f(T_i) = f(I+T_i) = f(I)+f(T_i)$.

Thus, by right cancellation law.$I' = f(I) \in G'$.Hence f(I) is the identity element of G′.

Theorem (4.3, V):—Let $f:G \rightarrow G'$ be an isomorphism from G onto G′.Where G and G′are additive groups of nonsingular operators on a Hilbert space H.Then f image of the additive inverse of $T_i \in G$ is the inverse of f image of T_i.That is,$f(-T_i) = -f(T_i)$

Proof :—Let I and I′ be identity elements of G and G′ respectively .Then clearly$I' = f(I)$

Let, $T_i \in G$ be an arbitrary element for all i.Also, by assumption G is an additive group.

Thus, for $T_i \in G \Rightarrow -T_i \in G$.Also,$T_i + (-T_i) = (-T_i) + T_i = I$,Now since, $I' = f(I) = f(T_i + (-T_i))$

Thus, $I'=f(T_i)+f(-T_i)$ [since f preserves the composition] ,It means that $f(-T_i)$ is the additive inverse of $f(T_i)$ in G′.Hence, $f(-T_i) = -f(T_i)$.

Theorem (4.3, VI):—Let G be a group and $f:G \rightarrow G$ be a homomorphism. sauch that,$f(T_i)=T_i^{-1}$. Show that G is abelian.

Proof:—Let $T_i,T_j \in G$ be any elements.Then,$T_iT_j = (T_j^{-1}T_i^{-1})^{-1} = f(T_j^{-1}T_i^{-1})$ [since $f(T_i) = T_i^{-1}$] $f(T_j^{-1})f(T_i^{-1})$ [since f is homomorphism so it preserves composition]$= T_jT_i$ That is, we have

$T_iT_j = T_jT_i$ for $T_i, T_j \in G$ be arbitrary elements,Thus, G is abelian.We now establish a theorem on the necessary and sufficient conditions.

Theorem (4.3, VII):—Let G be a group then show that the mapping f:G→G from G onto G given by $f(T_i)=T_i^{-1}$ is an isomorphism if and only if G is abelian. T_i being any element of G.

Proof:—Since,f is isomorphism ⟹ f is one-one,But then, $f(T_i) = f(T_j) \Rightarrow T_i = T_j$

Also,$f(T_iT_j)=(T_iT_j)^{-1}=T_j^{-1}T_i^{-1}$ ....(4.13) Again,f is isomorphism ⟹ f preserves composition. Hence, $f(T_iT_j) = f(T_i)f(T_j) = T_i^{-1}T_j^{-1}$ ....(4.14) Thus, from (4.13) and (4.14), we have $T_j^{-1}T_i^{-1} = T_i^{-1}T_j^{-1}$ for $T_i^{-1}, T_j^{-1} \in G$ as $T_i, T_j \in G$. ⟹ G is abelian.

Conversly :—Let G be abelian. ⟹ $T_iT_j = T_jT_i$ for $T_i, T_j \in G$.Also,$f(T_i) = f(T_j) \Rightarrow T_i^{-1} = T_j^{-1} \Rightarrow T_i = T_j$ ⟹ f is one-one.Also, for each $T_i^{-1} \in G$ ∃ T_i in G.such that, $f(T_i) = T_i^{-1}$Thus, f is onto also.Finally $f(T_iT_j) = (T_iT_j)^{-1} = T_j^{-1}T_i^{-1} = T_i^{-1}T_j^{-1}$ [since G is abelian] = $f(T_i)f(T_j)$

That is, $f(T_iT_j) = f(T_i)f(T_j)$.That is f preserves composition in G.Thus f is isomorphism.

Theorem (4.3, VIII):—Let G be any group. We now define a mapping f : G → G given by $f(T_i) = T_jT_iT_j^{-1}$,Then we prove that f is an isomorphism of G onto itself for $T_i \in G$ ∀ i.

Proof:—since $f(T_1)=f(T_2) \Rightarrow T_jT_1T_j^{-1}=T_jT_2T_j^{-1} \Rightarrow T_1T_j^{-1} = T_2T_j^{-1}$ [by left cancellation law] $\Rightarrow T_1 = T_2$ [by right cancellation law].Hence,$f(T_1) = f(T_2) \Rightarrow T_1 = T_2$Thus, f is one-one.

Also, for each $T_i \in G$ ∃ $T_jT_iT_j^{-1}$ in G such that, $f(T_i) = T_jT_iT_j^{-1}$Thus f is onto also.

Finally we see that,$f(T_1T_2) = T_j(T_1T_2)T_j^{-1}= T_j(IT_1T_2)T_j^{-1} = T_j(T_1I)T_2T_j^{-1}= T_j(T_1T_j^{-1}T_j)T_2T_j^{-1} = (T_jT_1T_j^{-1})(T_jT_2T_j^{-1}) = f(T_1)f(T_2)$That is, $f(T_1T_2)=f(T_1)f(T_2)$.Hence, f preserves the composition in G.Hence, f is isomorphism from G onto G.Thus, the theorem is established.

IV.CONCLUSIONS

In this research paper we have applied our mind to study isomorphism of groups of non singular operators. Here we have given some of the suitable examples of mappings which form isomorphism. A few theorems are established keeping the role of identity operators in centre. This has also been observed for additive inverse operator of a group also.

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