Contra $g^{\#}\psi$ - Continuous Functions and almost Contra $g^{\#}\psi$ – Continuous Functions in Topological Spaces

S. Deepika [# 1], N. Balamani [*2]

[1] M.Sc Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women University, Coimbatore, Tamilnadu, India

[2] Department of Mathematics, Avinashilingam Institute for Home Science and Higher Education for Women University, Coimbatore, Tamilnadu, India

Abstract:

The aim of this paper is to introduce and study a new generalization of contra continuous functions called contra $g^{\#}\psi$ - continuous functions and almost contra $g^{\#}\psi$ - continuous functions in topological spaces.

Keywords - *contra continuous function, almost contra continuous function, $g^{\#}\psi$ - continuous function, $g^{\#}\psi$ - closed sets and $g^{\#}\psi$ - open sets.*

I. INTRODUCTION

Singal M.K and Singal A.R, [11] introduced almost continuous mappings in topological spaces. Levine [10] introduced the concept of continuous functions in topological spaces. Dontchev [4] introduced the notion of contra continuous functions in topological spaces. Ekici [5] introduced almost contra continuous functions in topological spaces. Kanimozhi et al [8] introduced $g^{\#}\psi$ - continuous functions in topological spaces. Deepika and Balamani [3] introduced totally $g^{\#}\psi$ - continuous functions and $g^{\#}\psi$ - totally continuous functions in topological spaces. In this paper we introduce and study a new type of contra continuous functions called contra $g^{\#}\psi$ - continuous functions and almost contra $g^{\#}\psi$ - continuous functions in topological spaces. Also we obtain the relations between these functions.

II. PRELIMINARIES

DEFINITION 2.1:

Let (X,τ) *be a* topological space. A subset A of a topological space (X,τ) is called

1. **regular open** [12] if A = int (cl(A)).
2. **semi open** [9] if A ⊆ cl(int(A)).
3. **generalized closed** [10] if cl (A) ⊆ U whenever A ⊆ U and U is open in (X,τ).
4. **semi - generalized closed** [2] if scl(A) ⊆ U whenever A ⊆ U and U is semi open in (X,τ).
5. **ψ - closed** [13] if scl(A) ⊆ U whenever A ⊆ U and U is sg - open in (X,τ).
6. **$g^{\#}\psi$ - closed** [7] if ψcl(A) ⊆ U whenever A ⊆ U and U is ψ - open in (X,τ).
7. **$g^{\#}\psi$ - clopen** [8] if it is both $g^{\#}\psi$ - open and $g^{\#}\psi$ - closed in (X,τ).

***RESULT 2.2*:**

1. Every **closed (open)** subset in (X,τ) is $g^{\#}\psi$ **- closed (**$g^{\#}\psi$ **- open)**.
2. Every **clopen** subset in (X,τ) is $g^{\#}\psi$ **- clopen**.
3. Every **regular open (regular closed)** subset in (X,τ) is **open (closed).**

DEFINITION 2.3:

Let (X,τ) and (Y,σ) be two topological spaces. A function f: $(X,\tau) \to (Y,\sigma)$ is called

1. **Almost continuous** [11] if $f^{-1}(V)$ is closed in (X,τ) for every regular closed set V of (Y,σ).
2. **continuous** [10] if $f^{-1}(V)$ is closed in (X,τ) for every closed set V of (Y,σ).
3. **Completely continuous** [1] if $f^{-1}(V)$ is regular open in (X,τ) for every open set V of (Y,σ).
4. **Totally continuous** [6] if $f^{-1}(V)$ is clopen in (X,τ) for every open subset V of (Y,σ).

5. **Almost contra continuous** [5] if f^{-1} (V) is closed in (X, τ) for every regular-open set V of (Y, σ).
6. **Contra continuous** [4] if f^{-1}(V) is a closed set of (X,τ) for every open set V of (Y,σ).
7. **$g^{\#}\psi$ - continuous** [8] if f^{-1}(V) is $g^{\#}\psi$ - closed in (X,τ) for every closed set V of (Y,σ).
8. **Totally $g^{\#}\psi$ - continuous** [3] if f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ) for every open set V of (Y,σ).
9. **$g^{\#}\psi$ - totally continuous** [3] if f^{-1}(V) is clopen in (X,τ) for every $g^{\#}\psi$ - open set V of (Y,σ).

III. CONTRA $g^{\#}\psi$ - CONTINUOUS FUNCTIONS

Definition 3.1: A function f**:** (X,τ) → (Y,σ) is called **contra $g^{\#}\psi$ - continuous** if f^{-1}(v) is $g^{\#}\psi$ - open in (X,τ) for every closed set V of (Y,σ).

Example 3.2: Let X = Y = {a,b,c} , τ = {ϕ,{a},X} and σ = {ϕ,{a,b}Y}. Let f:(X,τ) →(Y,σ) be a function defined by f(a) = c, f(b) = b, f(c) =a. Then f is contra $g^{\#}\psi$ - continuous.

Theorem 3.3: A function f**:** (X,τ) → (Y,σ) is contra $g^{\#}\psi$ - continuous if and only if f^{-1}(U) is $g^{\#}\psi$ - closed in (X,τ) for every open set U of (Y,σ).

Proof: Let f**:** (X,τ) → (Y,σ) be contra $g^{\#}\psi$ - continuous and U be any open set in (Y,σ). Then Y-U is closed in (Y,σ). Since f is contra $g^{\#}\psi$ - continuous. f^{-1} (Y-U) = X- f^{-1}(U) is $g^{\#}\psi$ - open in (X,τ) which implies that f^{-1}(U) is $g^{\#}\psi$ - closed in (X,τ).

Conversely assume that V is any closed set in (Y,σ).Then Y-V is open in (Y,σ). By assumption f^{-1}(Y-V) = X - f^{-1}(V) is $g^{\#}\psi$ - closed in (X,τ) which implies that f^{-1}(V) is $g^{\#}\psi$ - open in (X,τ). Hence f is contra $g^{\#}\psi$ - continuous.

Proposition 3.4: Every contra continuous function is a contra $g^{\#}\psi$ - continuous function but not conversely.

Proof: Let V be any open set in (Y,σ). Since f is a contra continuous function, f^{-1}(V) is closed in (X,τ). By result 2.2 f^{-1}(V) is $g^{\#}\psi$ - closed in (X,τ). Hence f is contra $g^{\#}\psi$ - continuous.

Example 3.5: Let X = Y = {a,b,c} , τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a},Y}. Let f : (X,τ)→(Y,σ) be a function defined by f(a) = b, f(b) = a, f(c) =c. Then f is contra $g^{\#}\psi$ - continuous but not contra continuous, since for the open set {a} in (Y,σ), f^{-1}{a}={b} is $g^{\#}\psi$ - closed but not closed in (X,τ).

Proposition 3.6: Every totally continuous function is a contra $g^{\#}\psi$ - continuous function but not conversely.

Proof: Let V be any open set in (Y,σ). Since f is totally continuous, f^{-1}(V) is clopen in (X,τ). By result 2.2 , f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ) which implies that f^{-1}(V) is $g^{\#}\psi$ - closed in (X,τ). Hence f is contra $g^{\#}\psi$ - continuous.

Example 3.7: Let X = Y = {a,b,c}, τ= {ϕ,{a},{a,b}, X}and σ = {ϕ,{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined f(a) = c, f(b) = a, f(c) = b. Then f is contra $g^{\#}\psi$ - continuous but not totally continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b}) = {b,c} is $g^{\#}\psi$ - closed but not clopen in (X,τ).

Proposition 3.8: Every totally $g^{\#}\psi$ - continuous function is a contra $g^{\#}\psi$ - continuous function but not conversely.

Proof: Let V be any closed set in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous, f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ) which implies that f^{-1}(V) is $g^{\#}\psi$ - open in (X,τ). Hence f is a contra $g^{\#}\psi$ **-** continuous function.

Example 3.9: Let X = Y = {a,b,c} , τ = {ϕ,{a,b}, X} and σ = {ϕ,{a},Y}. Let f : (X,τ) →(Y,σ) be a function defined by f(a) = b, f(b) = c, f(c) =a. Then f is contra $g^{\#}\psi$ - continuous but not totally $g^{\#}\psi$ - continuous, since for the closed set {b,c} in (Y,σ), f^{-1}(b,c) = {a,b} is $g^{\#}\psi$ - open but not $g^{\#}\psi$ - closed in (X,τ).

Proposition 3.10: Every $g^{\#}\psi$ - totally continuous function is a contra $g^{\#}\psi$ - continuous function but not conversely.

Proof: Let V be any open set in (Y,σ). By result 2.2, V is $g^{\#}\psi$ - open in (Y,σ). Since f is $g^{\#}\psi$ - totally continuous, f^{-1}(V) is clopen in (X,τ). By result 2.2, f^{-1}(V) is $g^{\#}\psi$ - clopen in (X,τ) which implies that f^{-1}(V) is $g^{\#}\psi$ - closed in (X,τ). Hence f is contra $g^{\#}\psi$ - continuous.

Example 3.11: Let X = Y = {a,b,c}, τ = {ϕ, {a},{a,b},{a,c}, X} and σ = {ϕ, {a},{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined f(a) = c, f(b) = b, f(c) = a. Then f is contra $g^{\#}\psi$ - continuous but not $g^{\#}\psi$ - totally continuous, since for the $g^{\#}\psi$ - open set {a} in (Y,σ), f^{-1}({a }) ={c} is $g^{\#}\psi$ - closed but not clopen in (X,τ).

Remark 3.12: Contra $g^{\#}\psi$ **-** continuous function is independent from $g^{\#}\psi$ **-** continuous function as seen from the following examples.

Example 3.13: Let X = Y = {a,b,c} ,τ = {ϕ,{a},{a,b}, X} and σ = {ϕ,{a,b},Y}. Let f: (X,τ)→(Y,σ) be a function defined by f(a) = c, f(b) = a, f(c) =b. Then f is contra $g^{\#}\psi$ - continuous but not $g^{\#}\psi$ - continuous, since for the closed set {c} in (Y,σ), f^{-1}({c}) = {a} is $g^{\#}\psi$ - open but not $g^{\#}\psi$ - closed in (X,τ).

Example 3.14: Let X = Y = {a,b,c} , τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a},Y}. Let f :(X,τ)→(Y,σ) be the identity function. Then f is $g^{\#}\psi$ - continuous but not contra $g^{\#}\psi$ - continuous, since for the closed set {b,c} in (Y,σ), f^{-1}({b,c}) = {b,c} is $g^{\#}\psi$ - closed but not $g^{\#}\psi$ - open in (X,τ).

Proposition 3.15: If f**:** (X,τ) → (Y,σ) is a contra $g^{\#}\psi$ - continuous function and g:(Y,σ)→(Z,η) is a continuous function , then g o f : (X,τ) →(Z, η) is a contra $g^{\#}\psi$ - continuous function.

Proof: Let V be any closed set in (Z, η). Since g is continuous. g^{-1}(V) is closed in (Y,σ). Since f is contra $g^{\#}\psi$ - continuous. $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.16: If f**:** (X,τ) → (Y,σ) is a totally $g^{\#}\psi$ - continuous function and g:(Y,σ)→(Z,η) is a continuous function , then g o f : (X,τ) →(Z, η) is a contra $g^{\#}\psi$ - continuous function.

Proof: Let V be any closed set in (Z, η). Since g is continuous. g^{-1}(V) is closed in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous. $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - clopen in (X,τ). By result 2.2 $(gof)^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.17: If f : (X,τ) → (Y,σ) is a totally continuous function and g: (Y,σ)→(Z, η) is a continuous function , then g o f : (X,τ) →(Z, η) is a contra $g^{\#}\psi$ - continuous function .

Proof: Let V be any closed set in (Z, η). Since g is continuous. g^{-1}(V) is closed in (Y,σ). Since f is totally continuous. $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is clopen in (X,τ). By result 2.2 $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.18: If f **:** (X,τ) → (Y,σ) is a $g^{\#}\psi$ - totally continuous function and g:(Y,σ)→(Z,η) is a continuous function , then g o f : (X,τ) →(Z, η) is a contra $g^{\#}\psi$ - continuous function .

Proof: Let V be any closed set in (Z, η). Since g is continuous. g^{-1}(V) is closed in (Y,σ). By result 2.2 g^{-1}(V) is $g^{\#}\psi$ - closed in (Y,σ). Since f is $g^{\#}\psi$ - totally continuous. $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{1}(V))$ is clopen in (X,τ). By result 2.2 $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ)which implies that $(g \text{ o } f)^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.19: If f**:** (X,τ) → (Y,σ) is contra $g^{\#}\psi$ - continuous and g: (Y,σ)→(Z, η) is totally continuous, then g o f : (X,τ) →(Z, η) is a contra $g^{\#}\psi$ - continuous function.

Proof: Let V be any closed set in (Z, η). Since g is totally continuous. g^{-1}(V) is clopen in (Y,σ) which implies that g^{-1}(V) is closed in (Y,σ) . Since f is contra $g^{\#}\psi$ - continuous. $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.20: If f**:** (X,τ) → (Y,σ) is contra $g^{\#}\psi$ - continuous and g: (Y,σ)→(Z, η) is $g^{\#}\psi$ - totally continuous , then g o f : (X,τ) →(Z, η) is a contra $g^{\#}\psi$ - continuous function.

Proof: Let V be any closed set in (Z, η). By result 2.2 V is $g^{\#}\psi$ - closed in (Z, η). Since g is $g^{\#}\psi$ - totally continuous, g^{-1}(V) is clopen in (Y,σ) which implies that g^{-1}(V) is closed in (Y,σ). Since f is contra $g^{\#}\psi$ - continuous. $(g \text{ o } f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - open in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.21: If f**:** (X,τ) → (Y,σ) is a contra continuous function and g: (Y,σ)→(Z, η) is a continuous function, then g o f : (X,τ) →(Z, η) is a contra $g^{\#}\psi$ - continuous function.

Proof: Let V be any open set in (Z, η). Since g is continuous. $g^{-1}(V)$ is open in (Y,σ). Since f is contra continuous. $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is closed in (X,τ). By result 2.2 $(g \circ f)^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 3.22: If f:(X,τ)→(Y,σ) is a totally $g^{\#}\psi$ - continuous function and g:(Y,σ)→(Z, η) is a completely continuous function, then g o f :(X,τ) →(Z,η) is a contra $g^{\#}\psi$ - continuous function.

Proof: Let V be any open set in (Z, η). Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y,σ). By result 2.2 $g^{-1}(V)$ is open in (Y,σ). Since f is a totally $g^{\#}\psi$ - continuous function, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $(g \circ f)^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ) . Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Remark 3.23: The composition of two contra $g^{\#}\psi$ - continuous functions need not be a contra $g^{\#}\psi$ - continuous function as seen from the following example

Example 3.24: Let X = Y = Z ={a,b,c} , τ = {ϕ,{a},{a,b},X} and σ = {ϕ,{a,b},Y} and η = {ϕ,{a},Z}. Let f**:** (X,τ) → (Y,σ) be a function defined by f(a) = c, f(b) = b ,f(c) =a and g:(Y,σ)→(Z, η) be a function defined by g(a) = c, g(b) = b ,g(c) = a. Then the functions f and g are contra $g^{\#}\psi$ - continuous but their composition g o f : (X,τ) →(Z, η) is not contra $g^{\#}\psi$ - continuous. Since for the closed set {b,c}in (Z, η) , $(g \circ f)^{-1}$ ({b,c}) = {b,c} is not $g^{\#}\psi$ - open in (X,τ).

IV. ALMOST CONTRA $g^{\#}\psi$ - CONTINUOUS FUNCTIONS

Definition 4.1: A function f: (X,τ) →(Y,σ) is called **almost contra $g^{\#}\psi$ - continuous** if $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ) for every regular open set V of (Y,σ).

Example 4.2: Let X = Y = {a,b,c}, τ = { ϕ ,{a},{a,b},X} and σ = {ϕ ,{a},{b},{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a)=c, f(b)=b, f(c)=a. Then f is almost contra $g^{\#}\psi$ - continuous.

Theorem 4.3: A function f: (X,τ) →(Y,σ) is almost contra $g^{\#}\psi$ - continuous if and only if the inverse image of every regular open subset of (Y,σ) is $g^{\#}\psi$ - closed in (X,τ) .

Proof: Let f : (X,τ) →(Y,σ) be almost contra $g^{\#}\psi$ - continuous. Let V be any regular open set in (Y,σ). Then Y-V is regular closed in (Y,σ).Since f is almost contra $g^{\#}\psi$ - continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is $g^{\#}\psi$ - open in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ).

Conversely, assume that U is any regular closed set in (Y,σ). Then Y-U is regular open in (Y,σ). By assumption, $f^{-1}(Y - U) = X - f^{-1}(U)$ is $g^{\#}\psi$ - closed in (X,τ) which implies that $f^{-1}(U)$ is $g^{\#}\psi$ - open in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Proposition 4.4: Every contra continuous function is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ). By result 2.2, V is open in (Y,σ). Since f is contra continuous, $f^{-1}(V)$ is closed in (X,τ). By result 2.2, $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Example 4.5: Let X = Y = {a,b,c},τ = {ϕ, {a},{b},{a,b},X} and σ = {ϕ,{a},{b},{a,b},{a,c},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a)= b, f(b) = a, f(c) = c. Then f is almost contra $g^{\#}\psi$ - continuous but not contra continuous, since for the open set {a} in (Y,σ), f^{-1}({a }) = {b} is not closed in (X,τ).

Proposition 4.6: Every totally continuous function is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ). By result 2.2, V is open in (Y,σ). Since f is totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 2.2, $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Example 4.7: Let X = Y = {a,b,c},τ = {ϕ,{a},X} and σ = {ϕ,{a},{b},{a,b},Y}. Let f:(X,τ)→(Y,σ) be a function defined by f(a) = c, f(b) =a , f(c)= b. Then f is almost contra $g^{\#}\psi$ - continuous but not totally continuous, since for the open set {a,b} in (Y,σ), f^{-1}({a,b }) = {b,c} is not clopen in (X,τ).

Proposition 4.8: Every totally $g^{\#}\psi$ - continuous function is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ). By result 2.2, V is open in (Y,σ). Since f is totally $g^{\#}\psi$ - continuous, $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Example 4.9: Let X = Y = {a,b,c},τ = {ϕ, {a},{a,b},{a,c},X} and σ = {ϕ,{a},{b},{a,b},Y}. Let $f:(X,\tau)\to(Y,\sigma)$ be a function defined by f(a) = c, f(b) =a , f(c)= b. Then f is almost contra $g^{\#}\psi$ - continuous but not totally $g^{\#}\psi$ - continuous, since for the open set {a,b} in (Y,σ), $f^{-1}(\{a,b\})$ = {b,c} is not $g^{\#}\psi$ - clopen in (X,τ).

Proposition 4.10: Every $g^{\#}\psi$ - totally continuous function is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ). By result 2.2, V is open in (Y,σ) which implies that V is $g^{\#}\psi$ - open in (Y,σ). Since f is $g^{\#}\psi$ - totally continuous, $f^{-1}(V)$ is clopen in (X,τ). By result 2.2, $f^{-1}(V)$ is $g^{\#}\psi$ - clopen in (X,τ) which implies that $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is almost contra $g^{\#}\psi$ - continuous.

Example 4.11: Let X = Y = {a,b,c},τ = {ϕ, {a},{a,b},X} and σ = {ϕ,{a},{b},{a,b},Y}. Let $f:(X,\tau)\to(Y,\sigma)$ be function defined by f(a) = c, f(b) =b , f(c)= a. Then f is almost contra $g^{\#}\psi$ - continuous but not $g^{\#}\psi$ - totally continuous, since for the $g^{\#}\psi$ - open set {a} in (Y,σ), $f^{-1}(\{a\})$ = {c} is not clopen in (X,τ).

Proposition 4.12: If a function $f: (X,\tau)\to(Y,\sigma)$ is contra $g^{\#}\psi$ - continuous, then it is a almost contra $g^{\#}\psi$ - continuous function but not conversely.

Proof: Let V be any regular open set in (Y,σ). By result 2.2, V is open in (Y,σ). Since f is contra $g^{\#}\psi$ - continuous, $f^{-1}(V)$ is $g^{\#}\psi$ - closed in (X,τ). Hence f is a almost contra $g^{\#}\psi$ - continuous function.

Example 4.13: Let X = Y = {a,b,c}, τ = {ϕ, {a},{b},{a,b},X} and σ = {ϕ,{a},{b},{a,b},{a,c},x}. Let $f: (X,\tau)\to(Y,\sigma)$ be a function defined by f(a)=b, f(b)=a, f(c)=c. Then f is almost contra $g^{\#}\psi$ - continuous but not contra $g^{\#}\psi$ - continuous, since for the open set {a,b} in (Y,σ), $f^{-1}(\{a,b\})$ = {a,b} is not $g^{\#}\psi$ - closed in (X,τ).

Proposition 4.14: If $f:(X,\tau)\to(Y,\sigma)$ is a almost contra $g^{\#}\psi$ - continuous function and $g:(Y,\sigma)\to(Z,\eta)$ is a completely continuous function, then $g \circ f :(X,\tau)\to(Z,\eta)$ is a almost contra $g^{\#}\psi$ - continuous function.

Proof: Let V be any regular open set in (Z,η). By result 2.2, V is open in (Z,η). Since g is a completely continuous function, $g^{-1}(V)$ is regular open in (Y,σ). Since f is almost contra $g^{\#}\psi$ - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a almost contra $g^{\#}\psi$ - continuous function.

Proposition 4.15: If $f:(X,\tau)\to(Y,\sigma)$ is a almost contra $g^{\#}\psi$ - continuous function and $g:(Y,\sigma)\to(Z,\eta)$ is a completely continuous function, then $g \circ f :(X,\tau)\to(Z,\eta)$ is a contra $g^{\#}\psi$ - continuous function.

Proof: Let V be any open set in (Z,η). Since g is completely continuous, $g^{-1}(V)$ is regular open in (Y,σ). Since f is almost contra $g^{\#}\psi$ - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a contra $g^{\#}\psi$ - continuous function.

Proposition 4.16: If $f:(X,\tau)\to(Y,\sigma)$ is a contra $g^{\#}\psi$ - continuous function and $g:(Y,\sigma)\to(Z,\eta)$ is a almost continuous function, then $g \circ f :(X,\tau)\to(Z,\eta)$ is a almost contra $g^{\#}\psi$ - continuous function.

Proof: Let V be any regular open set in (Z,η). Since g is almost continuous, $g^{-1}(V)$ is open in (Y,σ). Since f is contra $g^{\#}\psi$ - continuous, $(g \circ f)^{-1}(V) = f^{-1}(g^{-1}(V))$ is $g^{\#}\psi$ - closed in (X,τ). Hence g o f is a almost contra $g^{\#}\psi$ - continuous function.

REFERENCES

1. Arya, S.P. and Gupta, R. "On strongly continuous mappings", kyungpook Math. J., vol .14, pp. 131-143, 1974.
2. Bhattacharyya, P. and Lahiri, B.K. "Semi-generalized closed sets in topology", Indian J. Math., vol. 29, pp. 376-382, 1987.
3. Deepika, S and Balamani, N. "Totally $g^{\#}\psi$ - Continuous Functions and $g^{\#}\psi$ - Totally Continuous Functions in Topological Spaces", International Journal of Scientific Research in Mathematical and Statistical Sciences. (submitted) ,2019.
4. Dontchev, J. "contra continuous functions and strongly S-closed spaces", Internat. J. Math. & Math. Sci., vol. 19, pp. 303 - 310, 1996.
5. Ekici.E "Almost contra pre-continuous functions", Bull. Malaysian Math.Sci. Soc., vol. 27, pp. 53-65, 2004.
6. Jain, R.C. "Role of regular open sets in general topology", Ph.D. thesis, Meerut University, Meerut, India, 1980.
7. Kanimozhi, K. and Balamani, N. "$g^{\#}\psi$ - closed sets in topological spaces" Imperial Journal of Interdisciplinary Research (IJIR) – vol. 3, pp. 1931-1935, 2017.

8. Kanimozhi, K. and Balamani, N. "$g^{\#}\psi$ - closed sets in topological spaces", M.Sc Thesis , Avinashilingam Institute for Home Science and Higher Education for Women University, Coimbatore , India, 2017.
9. Levine, N. "Semi open sets and semi continuity in topological spaces", Amer. Math. Monthly, vol .70, pp. 36 - 41, 1963.
10. Levine, N. "Generalized closed sets in topology", Rend. Circ. Math. Palermo, vol. 19, pp. 89 - 96, 1970.
11. Singal M.K and Singal A.R, "Almost continuous mappings", Yokohama Math J., vol.16, pp. 63-73, 1968.
12. Stone, M. "Application of the theory of Boolean rings to general topology", Trans. Amer. Math. Soc., vol. 41, pp. 374 - 481, 1937.
13. Veera kumar, M.K.R.S."Between semi-closed sets and semi-pre closed sets", Rend. Istit. Mat. Univ. Trieste, (ITALY), XXXXII, pp. 25-41, 2000.