Misspecification of Generalized Autoregressive Score Models: Monte Carlo Simulations and Applications

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Abstract

The specification and misspecification of a new class of volatility model that is robust to jumps and outliers is investigated via Monte Carlo experiment and real life examples. The class includes the Generalized Autoregressive Score (GAS) model derived from the classical Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model. The Exponential GAS (EGAS) and Asymmetric Exponential GAS (AEGAS) models form the variants of the GAS model. Using three different levels of volatility persistence and GARCH probability distributions which are Normal (N), Student-t (T) and Skewed-Student-t (SKT), with estimates of Akaike Information Criterion (AIC) and kurtosis as criteria, we obtained useful information for studying the misspecification and tail behaviour of the newly proposed volatility model. The results of the Monte Carlo experiment, the crude oil and gas prices showed that the best misspecified model for AEGAS-SKT and EGAS-T is EGAS-SKT.

Keywords - *Misspecification, Volatility Persistence, Monte Carlo, Generalized Autoregressive Score*

I. INTRODUCTION

The Generalized Autoregressive Score (GAS) class is a variant of Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model of [1]developed for capturing jumps/outliers effects in the returns series. Following [2], the classical GARCH model is not robust to capturing these abnormalities; hence the GAS class variants were proposed in [2], [3].

The driving mechanism of the GAS models and its variant is the scaled score of the likelihood function, and this makes the model class unique among other earlier proposed volatility models. It combines the ability to capture asymmetry with occasional jumps detection. The GAS model encompasses other well-known models such as the Generalized Autoregressive Conditional Heteroscedasticity (GARCH), the Autoregressive Conditional Duration (ACD), the Autoregressive Conditional Intensity (ACI), and the single source of error models. In addition, the GAS specification provides a wide range of new observation driven models. Examples include observation driven analogues of unobserved components time series models, multivariate point process models with time-varying parameters and pooling restrictions, new models for time-varying copula functions, and models for time-varying higher order moments. Based on these appealing properties of this new model, we were therefore motivated in investigating further the model class.

The literatures [4], [5] define volatility and volatility clustering in stocks, and following these definitions, several parametric volatility models have been developed. The first is the Autoregressive Conditional Heteroscedasticity (ARCH) model earlier proposed in[6] and the generalized version as GARCH model, has gained many applications in empirical financial time series literature. (see[1], [7]). These literatures have extended to studying the asymmetric behaviour and jumps in stocks and other asset prices. Different asymmetric robust volatility models have also been applied. The jump behaviour of stocks has recently been studied, and nonparametric approaches to detecting jumps have been applied (see[8]). Jump robust volatility model is introduced in[2], [3]. There, the authors proposed the Generalized Autoregressive Score (GAS) models and two variants, the Exponential GAS (EGAS) and Asymmetric Exponential GAS (AEGAS) models for predicting the conditional volatility with occasional jumps. As a result of newness of this model, there are fewer applications so far, though small sample properties have been investigated in[2], [9], [10], [11], there is need to study the property of this model class using simulation approach, with emphasis on the fitness ability and returns distributions. The fitness ability is achieved by the estimates of information criterion and the tail effects achieved by the estimates of the kurtosis.

The aim of this paper is to investigate misspecification of GAS models and its variants using Monte Carlo simulation approach. The work is extended to real life crude oil and natural gas prices. Literature has shown that these financial time series data display series of jumps over the historical years (see[12], [13]). Hence, they serve as good applicable examples in this paper.

The rest of the paper is structured as follows: Section 2 reviews literature on the volatility modelling and model misspecification. Section 3 presents the volatility models as well as misspecification testing approach. Section 4 presents the Monte Carlo experiment and results, while Section 5 renders the concluding remarks.

II. REVIEW OF LITERATURE

A framework for the construction and analysis of misspecification tests for GARCH models was developed in[14]and new asymptotically valid and locally optimal tests of asymmetry and nonlinearity were also proposed. It was argued that the asymmetry test of[15] and nonlinearity test of[16] are neither asymptotically valid (since they ignore asymptotically non-negligible estimation effects) nor locally optimal (since they ignore the recursive nature of the conditional variance structure). The framework of[14]encompasses conditional mean specification estimated by the Ordinary Least Squares (OLS), Nonlinear Least Squares (NLS) or Quasi Maximum Likelihood (QML) method, and that the GARCH misspecification tests can be asymptotically sensitive to unconsidered misspecification of the conditional mean. The Monte Carlo results indicate that the new tests are very powerful when compared with the previous tests proposed by[15], [16].

The properties of the GARCH (1,1) model and the assumptions on the parameter space under which the process is stationary was studied in depth in[17]. In particular, the ergodicity and strong stationarity for the conditional variance (squared volatility) of the process was proved. He showed under which conditions higher order moments of the GARCH (1,1) process exist and concluded that GARCH processes are heavy tailed. The impact of misspecification on the innovations in fitting GARCH (1,1) models through a Monte Carlo approach was investigated and showed that an incorrect specification of the innovations together with the reduction of the parameter space to the weak stationarity region, could give rise to a spurious Integrated GARCH (IGARCH) effect[18]. They also analysed the impact of misspecification on forecasted volatilities, showing that innovations with light tails can lead to a remarkable over-estimation of volatilities. The real size and power of the likelihood ratio and the Lagrange Multiplier (LM) misspecification tests when periodic long memory GARCH models are involved were analysed in[19]. The performance of these tests was studied by means of Monte Carlo simulations with respect to the class of generalized long memory GARCH models, and by means of a Monte Carlo analysis the real size and power of these tests were derived, evidencing their reliability apart from some special and limited cases. The test performances were however influenced by the sample length with about a thousand observations needed to obtain reliable conclusions.

On GAS modelling and its variants specification, the dynamic properties of Generalized Autoregressive Score (GAS) models were characterized by identifying the regions of the parameter space that implied stationarity and ergodicity of the corresponding nonlinear time series process[20]. They showed how these regions are affected by the choice of parameterization and scaling, which are key factors for the class of GAS models compared to other observation driven models.

As a follow-up by[11], the observation driven time series models used the scaled score of the likelihood function as the mechanism for updating the parameters over time. This approach provides a unified and consistent framework for introducing time varying parameters in a wide class of non-linear models. They developed a framework for time varying parameters which is based on the score function of the predictive model density at time t and concluded that by scaling the score function appropriately, standard observation driven models such as Generalized Autoregressive Conditional Heteroscedasticity (GARCH), Autoregressive Conditional Duration (ACD) and Autoregressive Conditional Intensity (ACI) models can be recovered.

A novel GAS model for predicting volume of shares (relative to the daily total), inspired by empirical regularities of the observed series (intra-daily periodicity pattern, residual serial dependence) was proposed in[21]. An application of the proposed GAS model to New York Stock Exchange (NYSE) ticketers confirmed the suitability of the proposed model in capturing the features of intra-daily dynamics of volume shares.

A new observation-driven time-varying parameter framework to model the financial return and realized variance jointly was proposed in[22]. The latent dynamic factor was updated by the scaled local density score as a function of past daily return and realized variance. The proposed GAS variant adapted quickly to drastic volatility changes by incorporating realized measures of volatility based on high frequency data and they demonstrated the promising performance of the proposed model by applying it to a number of equity returns, even during the 2008 financial crisis.

The consistency and asymptotic normality of the Maximum Likelihood Estimators (MLE) for a class of time series models driven by score function of the predictive likelihood was studied in[23]. They formulated primitive conditions, and asymptotic normality under correct specification and under misspecification of the GAS models.

The theoretic optimality properties of the score function of the predictive likelihood as a device to update parameters in GAS models was investigated in[24]. Their results provided a new theoretical justification for the class of GAS models, which covers the GARCH model as a special case. Their main contribution was to show that only parameter updates based on the score always reduce the local Kullback-Leibler divergence between the true conditional density and the model implied conditional density and they found out that it holds irrespective

of the severity of the model misspecification. They concluded that updates based on the score function minimized the local Kullback-Leibler divergence between the true conditional data density and the model implied conditional density.
A new class of flexible Copula models where the evolution of the dependence parameters follows a Markov-Switching Generalized Autoregressive Score (SGASC) dynamics was developed in[25]. Maximum Likelihood Estimation is consistently performed using the Inference Function for Margins (IFM) approach and a version of the Expectation-Maximisation (EM) algorithm specifically tailored to this class of models. They used their developed SGASC model to estimate the Conditional Value-at-Risk (CoVaR), which is defined as the VaR of a given asset conditional on another asset (or portfolio) being in financial distress, and the Conditional Expected Shortfall (CoES). Their empirical investigation shows that the proposed SGASC models are able to explain and predict the systemic risk contribution of several European countries. Also, they found out that the SGASC models outperformed competitors using several CoVaR back testing procedures.

III. THE GAS MODELS AND THEIR VARIANTS

The GAS model specification was derived from the classical GARCH model of [1]which is given as,

$$y_t = \varepsilon_t = z_t\sigma_t \qquad (1)$$

$$\sigma_t^2 = w + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2 \; (2)$$

where y_t is the returns time series decomposed as in (1), w, α_1 and β_1 are the parameters defined with the conditions $w > 0$, $\alpha_1 \geq 0$, $\beta_1 \geq 0$ and $\alpha_1 + \beta_1 < 1$ to ensure covariance stationarity of the model in (2). The jump volatility model as proposed in[2], [3]is given by re-writing GARCH (1,1) as,

$$\sigma_t^2 = w + \alpha_1 z_{t-1}^2\sigma_{t-1}^2 + \beta_1\sigma_{t-1}^2$$

and, $\sigma_t^2 = w + \alpha_1\left(z_{t-1}^2 - 1\right)\sigma_{t-1}^2 + \left(\alpha_1 + \beta_1\right)\sigma_{t-1}^2$,
which is finally written as,

$$\sigma_t^2 = w + \alpha_1 u_{t-1}\sigma_{t-1}^2 + \varphi_1\sigma_{t-1}^2 \qquad (3)$$

where $\varphi_1 = \alpha_1 + \beta_1$ and $u_t = z_{t-1}^2 - 1$ is proportional to the score of the conditional distribution of ε_t with respect to σ_{t-1}^2. This is Beta-GARCH model because and $\left(u_t + 1\right)/\left(v + 1\right)$ has a Beta distribution, and the innovations u_t are given as,

$$u_t = z_t^2 - 1,\; u_t \approx N\left(0,1\right); \qquad (4)$$

$$u_t = \frac{\left(v+1\right)z_t^2}{v - 2 + z_t^2} - 1,\; z_t \approx T\left(0,1,v\right); \; (5)$$

and

$$u_t = \frac{\left(v+1\right)z_t z_t^*}{\left(v-2\right)g_t\xi^{I_t}} - 1 \;\; z_t \approx skT\left(0,1,\xi,v\right); (6)$$

where

$$z_t^* = sz_t + m,\; I_t = sign\left(z_t^*\right) = I\left(z_t^* \geq 0\right) - I\left(z_t^* < 0\right),\; g_t = 1 + \frac{z_t^{*2}}{\left(v-2\right)\xi^{2I_t}},$$

$$m = \frac{\Gamma\left(\frac{v-1}{2}\right)\sqrt{v-2}}{\sqrt{\pi}\,\Gamma\left(\frac{v}{2}\right)}\left(\xi - \frac{1}{\xi}\right) \text{ and } s = \sqrt{\left(\xi^2 + \frac{1}{\xi^2} - 1\right) - m^2}$$

Now, combining (3) with (4) gives the GAS-Normal (GAS-N) model; combining (3) with (5) gives the GAS-Student-t (GAS-T) model and combining (3) with (6) gives the GAS-Skewed-Student-t (GAS-SKT) model.

The Exponential GARCH (EGARCH) and Asymmetric Exponential GARCH (AEGARCH) types of the GAS model were also considered in[3], each with the three distributional assumptions applied. The EGAS model is given as,

$$\log \sigma_t^2 = w + \alpha_1 u_{t-1} + \varphi_1 \log \sigma_{t-1}^2 \qquad (7)$$

specified without the leverage effect.[1] Now, combining (7) with (4) gives the EGAS-Normal (EGAS-N) model; combining (7) with (5) gives the EGAS-Student-t (EGAS-T) model and combining (7) with (6) gives the EGAS-Skewed-Student-t (EGAS-SKT) model.

Introducing the leverage effect into (7), we have the AEGAS model,

$$\log \sigma_t^2 = w + \alpha_1 u_{t-1} + \gamma_1 l_{t-1} + \varphi_1 \log \sigma_{t-1}^2 \qquad (8)$$

where $l_{t-1} = sign(-z_t)(u_t + 1)$ when Normal and Student-t distributions are considered, and $l_{t-1} = sign(-z_t^*)(u_t + 1)$ for the Skewed Student-t distribution. [2]

IV. MODEL MISSPECIFICATION TESTS

Each model under the distributional assumption is evaluated using[26] Information Criterion (AIC),

$$AIC = -2N^{-1} L_N \{(y_t); \hat{\psi}^{(N)}\} + 2N^{-1} \breve{\psi}; \qquad (10)$$

where $L_N\{\cdot\}$ is the maximized log-likelihood function, simplified using numerical derivatives, $\hat{\psi}^{(N)}$ is the ML estimator of the parameter vector ψ based on a sample of size *N*, and $\breve{\psi}$ gives the dimension of ψ. The excess kurtosis is then computed based on the formula,

$$k_\varepsilon = \frac{E(\varepsilon_t^4)}{\left[E(\varepsilon_t^2)\right]^2} - 3 \quad (11)$$

where $E(\varepsilon_t^2) = \dfrac{w}{1 - \alpha_1 - \beta_1}$ is the estimate of unconditional variance, and the fourth moment about the mean, $E(\varepsilon_t^4) = \dfrac{w^2(1 + \alpha_1 + \beta_1)(k_z + 3)}{\left[1 - (\alpha_1 + \beta_1)\right]\left[1 - \alpha_1^2(k_z + 2) - (\alpha_1 + \beta_1)^2\right]}$ and k_z is the excess kurtosis from the assumed GARCH distribution pro cess z_t.

V. MONTE CARLO EXPERIMENT AND RESULT DISCUSSION

Though the structural and distributional properties of classical GARCH model have been investigated theoretically and by simulations but the properties of GAS model and its variants are yet to be established. The Monte Carlo (MC) simulations experiment carried out in this work investigated both the fitness performance of the models as well as the measure of tail effect of the model residuals. Four Data Generating Processes (DGPs) considered are:

$$\text{GARCH}(1,1): \sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 (12)$$

$$\text{GAS}(1,1): \sigma_t^2 = \omega + \alpha_1 \mu_{t-1} \sigma_{t-1}^2 + (\alpha_1 + \beta_1)\sigma_{t-1}^2 \qquad (13)$$

EGAS(1,1): $log\sigma_t^2 = \omega + \alpha_1 \mu_{t-1} \sigma_{t-1}^2 + (\alpha_1 + \beta_1) log\sigma_{t-1}^2 (14)$

AEGAS(1,1): $log\sigma_t^2 = \omega + \alpha_1 \mu_{t-1} \sigma_{t-1}^2 + \gamma_1 \tau_{t-1} + (\alpha_1 + \beta_1) log\sigma_{t-1}^2 (15)$

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[1] The EGAS specification has no asymmetric parameter, unlike the classical EGARCH model of Nelson [27].

[2] Note, $E(l_t) = \dfrac{1 - \xi^2}{1 - \xi^2}$ in the three symmetric distributions, while $E(l_t) = 0$ for the Skewed Student-t distribution.

where $\tau_{t-1} = sign(-z_t)(\mu_t + 1)$. For each of the DGP in (12)-(15), a sample of 1000 time series was generated after making control for the initialization error, and each generated following Normal, Student-t and Skewed Student-t distributions. The sum $\alpha + \beta$ is referred to as the persistence of the conditional variance process. For financial return series, estimates of α and β are often in the ranges [0.02, 0.25] and [0.75,0.98], respectively with α often in the lower part of the interval and β in the upper part of the interval, such that the persistence is close but rarely exceeding 1[28]. We can then make classification into low, medium and high persistence. The parameters of the models were varied and classified in[14]as low, medium and high volatility persistence realizations as given below:

Low Persistence: $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1 = (0.04, 0.05, 0.65, 0.7, 0.01)$

Medium Persistence: $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1 = (0.04, 0.1, 0.8, 0.9, 0.01)$

High Persistence: $\omega, \alpha_1, \beta_1, \phi_1, \gamma_1 = (0.04, 0.09, 0.9, 0.99, 0.01)$ where the values of the intercept ω and asymmetric parameter γ_1 remained constant throughout and these do not affect volatility persistence. The value of $\phi_1 = \alpha_1 + \beta_1$ for the case of GAS(1,1), EGAS(1,1) and AEGAS(1,1) models.

The estimates of Akaike Information Criteria (AIC) and Excess Kurtosis from the Monte Carlo Experiments are given in table 1-3. The AIC of the DGP is denoted with single asterisk whereas the AIC of the best performed misspecified model is denoted with double asterisks. The results presented in table 1 showed that when the DGP is GAS-N, at low persistence, the misspecified model is EGAS-N while at both medium and high persistence; the misspecified model is GAS-SKT. When the DGP is both GAS-T and GAS-SKT at all persistence levels, the misspecified model is EGAS-SKT. Table 2 showed that when the DGP is EGAS-N at both low and medium persistence, the misspecified model is GAS-N while at high persistence; the misspecified model is EGAS-SKT. When the DGP is EGAS-T at all persistence levels, the misspecified model is EGAS-SKT whereas when the DGP is EGAS-SKT at all persistence levels, the misspecified model is GAS-T. In table 3, the results showed that when the DGP is AEGAS-N at low persistence, the misspecified model is EGAS-N while at both medium and high persistence; the misspecified model is AEGAS-SKT. When the DGP is both AEGAS-T and AEGAS-SKT at all persistence levels, the misspecified model is EGAS-SKT.
The results of this paper also showed that when the probability distribution of the residuals of the DGPs is normal, the probability distribution of the misspecified model will be normal since all the excess kurtosis observed under the three DGPs, at low, medium and high persistence were either negatively low or positively low and close to zero whereas when the probability distribution of the residuals of the DGPs is non-normal (skewed), the probability distribution of the residuals will be non-normal(Skewed) since the excess kurtosis observed under the three DGPs at low, medium and high persistence were positive and greater than zero.

VI. TABLE1: ESTIMATES OF AIC AND EXCESS KURTOSIS WHEN THE DGP IS GAS

Persistence	Assumed Distribution	GAS (1,1)		EGAS (1,1)		AEGAS (1,1)	
		AIC	Ex. Kurt	AIC	Ex. Kurt	AIC	Ex. Kurt
			When the DGP is GAS-N				
Low	Normal	**0.9188***	-0.1037	**0.9188****	-0.1050	0.9196	-0.1223
	Student-t	0.9209	-0.1037	0.9209	-0.1050	0.9217	-0.1224
	Skewed-t	0.9196	-0.1039	0.91967	-0.1051	0.9204	0.1225
Medium	Normal	**2.0429***	-0.1130	2.0442	-0.0986	2.0445	-0.1222
	Student-t	2.0450	-0.1127	2.0463	-0.0985	2.0466	-0.1220
	Skewed-t	**2.0435****	-0.1117	2.0447	-0.0941	2.0448	-0.1193
High	Normal	**4.6419***	-0.0823	4.6422	-0.0842	4.6436	-0.0965
	Student-t	4.6440	-0.0822	4.6443	-0.0842	4.6457	-0.0965
	Skewed-t	**4.6419****	-0.0846	4.6427	-0.0840	4.6440	-0.0982
			When the DGP is GAS-T				
Low	Normal	0.8556	1.8835	0.8770	2.8232	0.8579	2.0572
	Student-t	**0.8162***	2.8569	0.8162	2.8616	0.8183	2.8441
	Skewed-t	**0.8138****	2.8423	**0.8138****	2.8451	0.8158	2.8502
Medium	Normal	1.9687	2.6440	1.9712	2.5112	1.9713	2.6193
	Student-t	**1.9059***	2.9442	1.9051	3.0642	1.9070	3.0378
	Skewed-t	1.9034	2.9299	**1.9027****	3.0443	1.9046	3.0223
High	Normal	4.2293	2.8318	4.2350	2.7036	4.2359	2.7969
	Student-t	**4.1567***	2.9177	4.1563	2.9413	4.1583	2.9316

	Skewed-t	4.1544	2.8900	**4.1540****	2.9077	4.1560	2.9048
				When the DGP is GAS-SKT			
Low	Normal	0.8549	2.1361	0.8757	2.8379	0.8563	2.0975
	Student-t	0.8152	2.8729	0.8152	2.8779	0.8172	2.8602
	Skewed-t	**0.8136***	2.8587	**0.8136****	2.8619	0.8156	2.8648
Medium	Normal	1.9671	2.6423	1.9695	2.3060	1.9697	2.6130
	Student-t	1.9045	2.9597	1.9038	3.0833	1.9055	3.0518
	Skewed-t	**1.9029***	2.9296	**1.9022****	3.0684	1.9040	3.0406
High	Normal	4.2243	2.8137	4.2299	2.6793	4.2309	2.7637
	Student-t	4.1522	2.9233	4.1518	2.9502	4.1538	2.9349
	Skewed-t	**4.1508***	2.9017	**4.1503****	2.9230	4.1523	2.9129

VII. TABLE 2: ESTIMATES OF AIC AND EXCESS KURTOSIS WHEN THE DGP IS EGAS

Persistence	Assumed Distribution	GAS (1,1)		EGAS (1,1)		AEGAS (1,1)	
		AIC	Ex. Kurt	AIC	Ex. Kurt	AIC	Ex. Kurt
				When the DGP is EGAS-N			
Low	Normal	**3.0716****	-0.1031	**3.0715***	-0.1060	3.0724	-0.1224
	Student-t	3.0737	-0.1030	3.0736	-0.1060	3.0745	-0.1224
	Skewed-t	3.0723	-0.1031	3.0724	-0.1060	3.0732	-0.1235
Medium	Normal	**3.4071****	-0.1096	**3.4073***	-0.1039	3.4079	-0.1225
	Student-t	3.4092	-0.1092	3.4094	-0.1036	3.4101	-0.1222
	Skewed-t	3.4079	-0.1081	3.4079	-0.0992	3.4085	-0.1193
High	Normal	7.5633	-0.0312	**7.5604***	-0.0656	10.6596	
	Student-t	7.5653	-0.0309	7.5626	-0.0650	7.9513	
	Skewed-t	7.5632	-0.0326	**7.5609****	-0.0631	7.5621	-0.0815
				When the DGP is EGAS-T			
Low	Normal	3.0064	2.0375	3.0290	2.8148	3.0307	2.8140
	Student-t	2.9683	2.8509	**2.9683***	2.8567	2.9703	2.8565
	Skewed-t	2.9660	2.8720	**2.9659****	2.8397	2.9679	2.8396
Medium	Normal	3.3255	2.5689	3.3264	2.5350	3.3284	2.5430
	Student-t	3.2632	2.8334	**3.2620***	2.9804	3.2639	2.9615
	Skewed-t	3.2606	2.8154	**3.2595****	2.9545	3.2615	2.9402
High	Normal	7.1818	2.9987	7.1894	2.8448	7.1901	2.9567
	Student-t	7.1077	3.0177	**7.1071***	3.0287	7.1091	3.0260
	Skewed-t	7.1053	2.9927	**7.1046****	2.9956	7.1066	3.0010
				When the DGP is EGAS-SKT			
Low	Normal	3.0065	2.1177	3.0277	2.8290	3.0099	2.0210
	Student-t	2.9673	2.8666	2.9673	2.8728	2.9692	2.8731
	Skewed-t	**2.9659****	2.8878	**2.9657***	2.8562	2.9677	2.8563
Medium	Normal	3.3237	2.5641	3.3246	2.5281	3.3266	2.5330
	Student-t	3.2618	2.8451	3.2606	2.9962	3.2624	2.9722
	Skewed-t	**3.2601****	2.8315	**3.2590***	2.9758	3.2609	2.9558
High	Normal	7.1765	2.9816	7.1840	2.8182	7.1849	2.9215
	Student-t	7.1029	3.0230	7.1023	3.0373	7.1043	3.0278
	Skewed-t	**7.1014****	3.0043	**7.1007***	3.0109	7.1027	3.0076

VIII. TABLE 3: ESTIMATES OF AIC AND EXCESS KURTOSIS WHEN THE DGP IS AEGAS

Persistence	Assumed Distribution	GAS (1,1)		EGAS (1,1)		AEGAS (1,1)	
		AIC	Ex. Kurt	AIC	Ex. Kurt	AIC	Ex. Kurt
				When the DGP is AEGAS-N			
Low	Normal	3.0742	-0.0887	**3.0741****	-0.0923	**3.0735***	-0.1219
	Student-t	3.0763	-0.0885	3.0762	-0.0922	3.0755	-0.1219
	Skewed-t	3.0749	-0.0883	3.0750	-0.0917	3.0742	-0.1227
Medium	Normal	3.4122	-0.0908	3.4123	-0.0858	**3.4111***	-0.1207
	Student-t	3.4143	-0.0902	3.4144	-0.0854	3.4131	-0.1206
	Skewed-t	3.4131	-0.0887	3.4131	-0.0802	**3.4116****	-0.1173
High	Normal	7.5943	-0.0026	7.5913	-0.0403	**7.5905***	-0.0796
	Student-t	7.5964	-0.0020	7.5934	-0.0398	7.5926	-0.0792
	Skewed-t	7.5944	-0.0027	7.5921	-0.0359	**7.5907****	-0.0783
				When the DGP is AEGAS-T			
Low	Normal	3.0084	1.9840	3.0313	2.7744	3.0328	2.7737
	Student-t	2.9712	2.8144	2.9712	2.8210	**2.9731***	2.8206

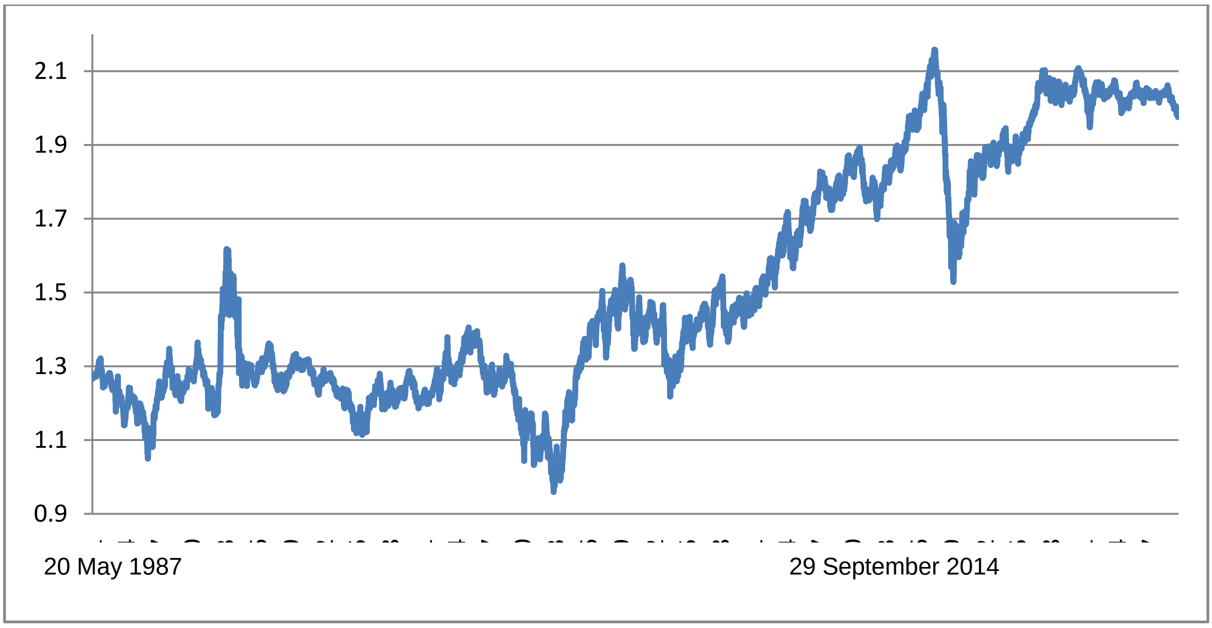
	Skewed-t	2.9690	2.8388	**2.9688****	2.8038	2.9707	2.8046
Medium	Normal	3.3335	2.5298	3.3344	2.4956	3.3360	2.5245
	Student-t	3.2718	2.7969	3.2706	2.9464	**3.2726***	2.9556
	Skewed-t	3.2693	2.7784	**3.2682****	2.9205	3.2701	2.9350
High	Normal	7.2642	2.9172	7.2728	2.9724	7.2712	2.8943
	Student-t	7.1909	2.9826	7.1904	2.9847	**7.1920***	3.0194
	Skewed-t	7.1888	2.9548	**7.1881****	2.9488	7.1896	2.9926
			When the DGP is AEGAS-SKT				
Low	Normal	3.0051	1.8743	3.0298	2.7902	3.0313	2.7880
	Student-t	2.9699	2.8313	2.9698	2.8382	2.9718	2.8363
	Skewed-t	2.9685	2.8564	**2.9683****	2.8216	**2.9703***	2.8206
Medium	Normal	3.3310	2.5266	3.3320	2.4903	3.3336	2.5154
	Student-t	3.2698	2.8101	3.2685	2.9639	3.2705	2.9673
	Skewed-t	3.2681	2.7963	**3.2670****	2.9437	**3.2689***	2.9510
High	Normal	7.2529	2.8990	7.2614	2.6980	7.2599	2.8580
	Student-t	7.1804	2.9903	7.1799	2.9965	7.1816	3.0210
	Skewed-t	7.1791	2.9700	**7.1784****	2.9685	**7.1800***	2.9990

* DGP ** Best performed misspecified model

IX. RESULTS OF CRUDE OIL AND GAS PRICES

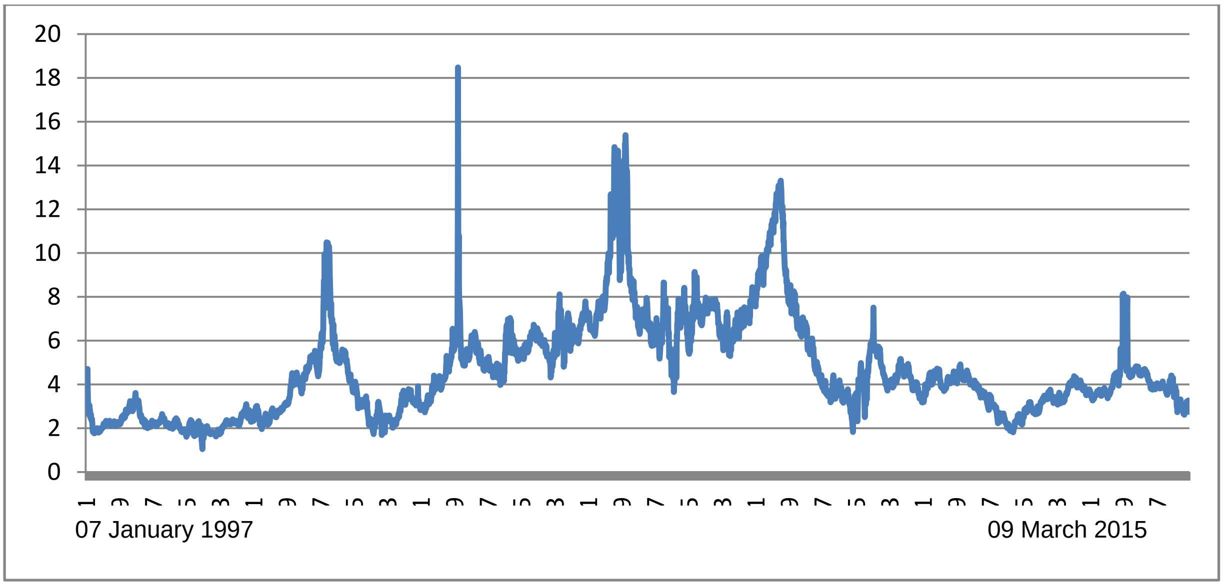
We apply both daily crude oil and Gas prices to test the effect of misspecification of volatility models. The crude oil prices are the European Brent prices (US dollars/barrel) while the gas prices are the Henry Hub Natural gas spot prices (US Dollars per Million Btu), both obtained from the website of US Energy Information Administrations (http://www.eia.gov/). The oil prices span between 20 May 1987 and 29 September 2014 while the natural gas series span between 07 January 1997 and 09 March 2015.

The plot of the crude oil prices is given in Figure 5.1. We observe stability in the prices of crude oil from 1987 to 1999 with a major spike in 1990. We observe a gradual increase in the prices of crude oil from 2000 to 2008 with the prices of crude oil getting to its peak in 2008. We also observe a fall in 2008 and a gradual increase in the prices of crude oil from 2008 to 2011 and the prices were stable from 2011 to 2015.



X. FIGURE 5.1: TIME PLOT OF CRUDE OIL PRICES (US DOLLAR/BARREL)

The plot of the natural gas prices is given in Figure 5.2. We observe major spike in the prices of natural gas in 2001, 2003, 2005, 2008, 2010 and 2014. We observe fall in prices of natural gas after each spike and stability of prices of natural gas before the next spike.



XI. FIGURE 5.2: TIME PLOT OF GAS PRICES (US DOLLAR/BTU)

The estimates of Akaike Information Criteria (AIC) and Excess Kurtosis from the model estimation using crude oil and natural gas prices are presented in table 4. The results showed that the specified model for the crude oil prices is AEGAS-SKT while the misspecified model is EGAS-SKT. The specified model for the natural gas prices is EGAS-T while the misspecified model is EGAS-SKT.

We observed positive estimates of excess kurtosis throughout Table 4 which are all greater than zero. This implies that the estimated residuals for the specified models deviate from normal distribution and they have fatter tails than the normal distribution.

XII. TABLE 4: MISSPECIFICATION TESTS FOR MODELS FOR CRUDE OIL AND NATURAL GAS PRICES

Estimated Model	Distribution Assumed	Crude Oil Prices		Natural Gas Prices	
		AIC	Ex. Kurt	AIC	Ex. Kurt
GAS	Normal	-6.6929	1.9429	-5.4666	8.7822
	T	-6.7451	2.4417	-5.5458	8.6593
	Skewed-*t*	-6.7463	2.4535	-5.5455	8.6125
EGAS	Normal	-6.6916	1.9525	-5.4147	10.506
	T	-6.7465	2.3753	**-5.5516***	8.6237
	Skewed-*t*	**-6.7476****	2.3888	**-5.5513****	8.5867
AEGAS	Normal	-6.6918	1.9381	-5.4229	11.931
	T	-6.7471	2.4110	-5.5512	8.3575
	Skewed-*t*	**-6.7482***	2.4264	-5.5509	8.3358

* Specified model ** Best performed misspecified model

XIII. CONCLUSION

This paper has investigated the misspecification of GAS models and its variants using Monte Carlo simulation approach. The work was extended to real life situation by using the daily prices of crude oil and natural gas prices. The estimation involved investigating the misspecification of GAS models and their variants assuming normal, student-t and Skewed Student-t probability distributions for the GARCH variants. Model selection performance was then investigated using information criteria and tail coefficient (kurtosis). We therefore present the results for studying the misspecification of the GAS variantsand residual tail behaviour as summarized in table 5 and table 6 respectively.

Xiv. Table 5: summary of fitness performance of the misspecified models

DGP	Best Performed Misspecified Model		
	Low Persistence	Medium Persistence	High Persistence
GAS-N	EGAS-N	GAS-SKT	GAS-SKT
GAS-T	GAS-SKT & EGAS-SKT	EGAS-SKT	EGAS-SKT
GAS-SKT	EGAS-SKT	EGAS-SKT	EGAS-SKT
EGAS-N	GAS-N	GAS-N	EGAS-SKT
EGAS-T	EGAS-SKT	EGAS-SKT	EGAS-SKT
EGAS-SKT	GAS-SKT	GAS-SKT	GAS-SKT

AEGAS-N	EGAS-N	AEGAS-SKT	AEGAS-SKT
AEGAS-T	EGAS-SKT	EGAS-SKT	EGAS-SKT
AEGAS-SKT	EGAS-SKT	EGAS-SKT	EGAS-SKT

XV. TABLE 6: Summary Of The Probability Distribution Of The Residuals (Tail Behavior)

DGP	Tail Behavior
GAS-N	Normal
GAS-T	Skewed
GAS-SKT	Skewed
EGAS-N	Normal
EGAS-T	Skewed
EGAS-SKT	Skewed
AEGAS-N	Normal
AEGAS-T	Skewed
AEGAS-SKT	Skewed

The crude oil and gas prices were used to confirm the results of the Monte Carlo experiment. The specified models for the crude oil and gas prices are AEGAS-SKT and EGAS T respectively while the misspecified model for both the crude oil and gas prices is EGAS-SKT. This result agrees with the outcome of the Monte Carlo experiment as noted in table 5, that is, the misspecification model for both AEGAS-SKT and EGAS-T is EGAS-SKT.

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