On nano π* g* - Closed Sets

B.Shanmugaraj[#1], S.Meena Devi[#2]
Department of Mathematics
Sri Kaliswari College, Sivakasi,

Abstract
*In this paper, we introduce nano π*g*-closed sets in nano topological spaces and investigate some of their properties.*

Keywords: *nano π - closed set, nano g*- closed set, nano πg – closed set, nano π*g – closed set and nano π*g*-closed sets.*

I .INTRODUCTION

Lellis Thivagar [11] introduced a nano topological space with respect to a subset X of an universe which is defined in terms of lower approximation and upper approximation and boundary region. The classical nano topological space is based on an equivalence relation on a set, but in some situation, equivalence relations are nor suitable for coping with granularity, instead the classical nano topology is extend to general binary relation based covering nano topological space. Bhuvaneswari [4] introduced and investigated nano g-closed set in nano topological spaces. P.Jeyalakshmi [5] introduced and investigated nano π*g – closed sets in nano topological spaces. In this paper we introduce nano π - closed set, nano g*- closed set, nano πg – closed set, nano π*g – closed set and nano π*g*- closed sets.

II. PRELIMINARIES

Throughout this paper (U, $\tau_R(X)$) (or X) represent nano topological spaces on which no separation axioms are assumed unless otherwise mentioned. For a subset H of a space (U, $\tau_R(X)$), Ncl(H) and Nint (H) denote the nano closure of H and the nano interior of H respectively.

Definition 2.1.[8]

Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X ⊆ U.

1.The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$. That is, $L_R(X) = \cup_{x\in U}\{ R(x): R(x) \subseteq X\}$, Where R(x) denotes the equivalence class determined by x.

2. The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$. That is, UR(X) = $\cup_{x\in U}\{R(x): R(x) \cap X \neq \emptyset\}$.

3. The boundary region of X with respect to R is the set of all objects, which can be classified neither as X nor as not - X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X)= U_R(X)- L_R(X)$.

Example 2.2.

Let U={ a,b,c,d}

R={ (a,a), (b,b), (c,c), (d,d), (b,d), (d,b)} be an equivalence relation on U

U\R={{a},{c},{b,d}}

Let X={a,b}⊆ U

Then $L_R(X)$={a}, $U_R(X)$={a,b,d} and $B_R(X)$={b,d}.

Proposition 2.3.[10]

If (U, R) is an approximation space and X,Y ⊆ U; then

1. $L_R(X) \subseteq X \subseteq U_R(X)$;

2. $L_R(\emptyset)= U_R(\emptyset)=\emptyset$ and $L_R(U)= U_R(U)=U$;

3. $U_R(X\cup Y)= U_R(X) \cup U_R(Y)$;

4. $U_R(X\cap Y) \subseteq U_R(X) \cap U_R(Y)$;

5. $L_R(X\cup Y) \supseteq L_R(X) \cup L_R(Y)$;

6. $L_R(X\cap Y) \subseteq L_R(X) \cap L_R(Y)$;

7. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$, whenever $X \subseteq Y$;

8 $U_R(X^c) =[L_R(X)]^c$ and $L_R(X^c) =[U_R(X)]^c$;

9. $U_R\, U_R\, (X) = L_R\, U_R\, (X)= U_R\, (X)$;

10. $L_R\, L_R\, (X) = U_R\, L_R\, (X)= L_R\, (X)$;

Definition 2.4.[10] Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{\emptyset, U, L_R(X), U_R(X), B_R(X) \}$ where $X\subseteq U$. Then by the property 2.3, R(X) satisfies the following axioms:

1. U and ø are in $\tau_R(X)$,

2. The union of the elements of any subcollection of $\tau_R(X)$ is in $\tau_R(X)$,

3. The intersection of the elemen ts of any finite subcollection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ is a topology on U called the nano topology on U with respect to X. We call (U, $\tau_R(X)$) as the nano topological space. The elements of $\tau_R(X)$ are called as nano open sets and complement of $\tau_R(X)$ are called as closed sets.

Example 2.5.

Let X={a,b} be a set.

In example 2.2, $\tau_R(X)$={ϕ,{a},{a,b,d},{b,d},U}is a nano topology on U.

Therefore (U, $\tau_R(X)$) is a nano topological space.

The nano open sets are ϕ, {a}, {a,b,d}, {b,d} and U.

The nano closed sets are ϕ, {b,c,d}, {c}, {a,c} and U.

Remark 2.6.[10] If ($\tau_R(X)$) is a nano topology on U with respect to X, then the set B={U, ø, $L_R(X)$, $B_R(X)$} is the basis for τ_R (X).

Definition 2.7.[10] If (U, $\tau_R(X)$) is a nano topological space with respect to X and if $H \subseteq U$, then the nano interior of H is defined as the union of all nano open subsets of H and it is denoted by Nint (H). That is, Nint(H) is the largest nano open subset of H.

The nano closure of H is defined as the intersection of all nano closed sets containing H and it is denoted by Ncl(H). That is, Ncl(H) is the smallest nano closed set containing H.

Example 2.8.

In example 2.5, Let H={b,d}$\subseteq$ U

Nint(H)={b,d}

Ncl(H)={b,c,d}

Definition 2.9. A subset H of a nano topological space (U, τ_R(X)) is called

1. nano regular-open[10] if H = Nint(Ncl(H)).
2. nano pre-open[10] if H $\subseteq$ Nint (Ncl(H)).
3. nano α-open [7] if H $\subseteq$ Nint(Ncl(Nint(H))).
4. nano π-open[1] if the finite union of nano regular-open sets.

The complements of the above mentioned sets is called their respective closed sets.

Definition 2.10. A subset H of a nano topological space (U, τ_R(X)) is called

1. nano g-closed [3] if Ncl(H) $\subseteq$ G, whenever H $\subseteq$ G and G is nano open.
2. nano wg-closed[6] if Ncl(Nint(H)) $\subseteq$ G, whenever H $\subseteq$ G and G is nano open.
3. nano πg-closed [9] if Ncl(H) $\subseteq$ G, whenever H $\subseteq$ G and G is nano π -open.
4. nano π^*g-closed [5] if Ncl(Nint(H)) $\subseteq$ G, whenever H $\subseteq$ G and G is nano π – open.
5. nano g^*-closed if Ncl(H) $\subseteq$ G, whenever H $\subseteq$ G and G is nano g-open

III. ON NANO π^*g^*- CLOSED SETS

Definition 3.1. A subset H of space (U, τ_R(X)) is called nano π^*g^*- closed if Ncl(Nint(H)) $\subseteq$ G, whenever H $\subseteq$ G and G is nano πg-open.

The complement of nano π^*g^*- open is nano π^*g^*- closed.

Example 3.2. Let U={ a,b,c,d } with U\R={ {a},{c},{b,d}} and X={ a,b}.

Then nano topology τ_R(X)={ ø, {a},{a,b,d},{b,d},U}.

Let G={b,c,d} be nano πg-open.

Let H={b,d} $\subseteq$ G

Ncl(Nint(H))={b,c,d} $\subseteq$ G

Hence {b,d} is nano π^*g^*- closed.

Remark 3.3. In nano topological space, the concepts of nano π-open sets and nano πg- open sets are independent as seen from the following example.

Example 3.4. In example 3.2,

H={ b,c,d } is nano πg- open.

But H={ b,c,d } is not nano π- open.

Therefore every nano πg-open set need not be nano π- open.

Let G={ a,b,d} be nano π-open.

Let H= { a,b,d} ⊆ G

Ncl(H)={ a,b,c,d } ⊈ {a,b,d}=G

Therefore Ncl(H) ⊈G

Hence {a,b,d} is not nano πg- closed.

Hence {c} is not nano πg- open.

Therefore every nano π-open set need not be nano πg- open.

Theorem 3.5. In a space (U, τ_R(X)), every nano closed is nano π^*g^*- closed.

Proof. Suppose that H is nano closed in U. Let H ⊆ G where G is nano πg-open. We know that "every nano closed set is nano g-closed set". Since H is nano g- closed, Ncl(H) ⊆ G and Ncl(Nint(H)) ⊆ Ncl(H) ⊆ G. Hence Ncl(Nint(H)) ⊆ G, where G is nano πg-open. Thus H is nano π^*g^*-closed.

Remark 3.6. The converse of the Theorem 3.3 need not be true in general as shown in the following example.

Example 3.7. In example 3.2, observe that {b,d} is nano π^*g^*- closed set but not nano closed.

Clearly {b,d} not nano closed.

Theorem 3.8. In a space (U, τ_R(X)), every nano g-closed is nano π^*g^*- closed.

Proof. Suppose that H is nano g-closed in U. Let H ⊆ G where G is nano πg-open. Since H is nano g- closed, Ncl(H) ⊆ G and Ncl(Nint(H)) ⊆ Ncl(H) ⊆ G. Hence Ncl(Nint(H)) ⊆ G, where G is nano πg-open. Thus H is nano π^*g^*-closed.

Remark 3.9. The converse of the Theorem 3.6 need not be true in general as shown in the following example.

Example 3.10. In example 3.2, observe that {b,d} is nano π^*g^*- closed set **.**

Let G={b,d} be nano open.

Let H={b,d} ⊆ {b,d}

Ncl(H)={b,c,d} ⊈ G

Hence {b,d} is not nano g-closed.

Therefore every nano π^*g^*- closed set need not be nano g-closed.

Theorem 3.11. In a space (U, τ_R(X)), every nano α-closed is nano π^*g^*- closed.

Proof. Suppose that H is nano α-closed in U. Let H ⊆ G where G is nano πg-open.Since H is nano α- closed, Ncl(Nint(Ncl(H))) ⊆ H ⊆ G and Ncl(Nint(H)) ⊆ Ncl(Nint(Ncl(H))) ⊆ G. Hence Ncl(Nint(H)) ⊆ G, where G is nano πg-open. Thus H is nano π^*g^*-closed.

Remark 3.12. The converse of the Theorem 3.9 need not be true in general as shown in the following example.

Example 3.13. In example 3.2, observe that {b,d} is nano $\pi^{*}g^{*}$- closed set **.**

Let H={b,d}

Nint(Ncl(Nint(H))) ={b,d} ⊆ H

Hence {b,d} is nano α - open.

Therefore {a,c} is nano α – closed.

Hence {a,c}⊈ H

Therefore nano $\pi^{*}g^{*}$- closed set need not be nano α – closed.

Theorem 3.14. In a space (U, τ_R(X)), every nano πg-closed is nano $\pi^{*}g^{*}$- closed.

Proof. Suppose that H is nano πg-closed in U. Let H ⊆ G where G is nano πg-open. Since H is nano πg- closed, Ncl(H) ⊆ G and Ncl(Nint(H)) ⊆ Ncl(H) ⊆ G. Hence Ncl(Nint(H)) ⊆ G, where G is nano πg-open. Thus H is nano $\pi^{*}g^{*}$-closed.

Remark 3.15. The converse of the Theorem 3.12 need not be true in general as shown in the following example.

Example 3.16. In example 3.2,

Let G={U} be nano πg-open.

Let H={a,b} ⊆ G

Ncl(Nint(H))={a,c} ⊆ G

Hence {a,b} is nano $\pi^{*}g^{*}$- closed.

Let G={a,b,d} be nano π-open.

Ncl(H)={a,b,c,d} ⊈ G

Hence {a,b} is not nano πg- closed.

Thus every nano $\pi^{*}g^{*}$- closed set need not be nano πg- closed.

Theorem 3.17. In a space (U, τ_R(X)), every nano wg-closed is nano $\pi^{*}g^{*}$- closed.

Proof. Suppose that H is nano wg-closed in U. Let H ⊆ G where G is nano πg-open. By definition of nano wg-closed and the fact that every nano regular open is nano πg-open . Since H is nano wg- closed, Ncl(Nint(H)) ⊆ G, where G is nano π g-open. Thus H is nano $\pi^{*}g^{*}$-closed.

Remark 3.18. The converse of the Theorem 3.15 need not be true in general as shown in the following example.

Example 3.19. In example 3.2, observe that {b,d} is nano $\pi^{*}g^{*}$- closed set.

Let G={b,d} be nano open

Let H={b,d} ⊆ G

Ncl(Nint(H))={b,c,d} ⊈ G

H={b,d} is not wg- closed.

Therefore every nano $\pi^{*}g^{*}$- closed need not be nano wg- closed.

Theorem 3.20. In a space (U, $\tau_R(X)$), every nano pre-closed is nano $\pi^* g^*$- closed.

Proof. Suppose that H is nano pre-closed in U. Let H ⊆ G where G is nano πg-open. Since H is nano pre- closed, Ncl(Nint(H)) ⊆ H ⊆ G. Hence Ncl(Nint(H)) ⊆ G, where G is nano πg-open. Thus H is nano $\pi^* g^*$-closed.

Remark 3.21. The converse of the Theorem 3.18 need not be true in general as shown in the following example.

Example 3.22. In example 3.2, observe that {b,d} is nano $\pi^* g^*$- closed.

Let H={b,d}

Nint(Ncl(H))={b,c,d}

H ⊆ Ncl(Nint(H))

Thus the set {b,d} is nano pre open.

Hence {a,c} is nano pre-closed

Therefore every nano $\pi^* g^*$- closed need not be nano pre- closed.

Remark 3.23. In a space (U, $\tau_R(X)$), the union of two nano $\pi^* g^*$- closed set is $\pi^* g^*$- closed.

Example 3.24. In example 3.2

Let G={U} be be nano πg-open.

Let S={a,b} ⊆ G be nano $\pi^* g^*$- closed.

Let T={a,b,c} ⊆ G be nano $\pi^* g^*$- closed.

S∪T={a,b,c}

Now, Ncl(Nint(S∪T))={a,c} ⊆ G

Hence {a,b,c} is nano $\pi^* g^*$- closed.

Thus the union of two nano $\pi^* g^*$- closed set is $\pi^* g^*$- closed.

Remark 3.25. In a space (U, $\tau_R(X)$), the intersection of two nano $\pi^* g^*$- closed set is $\pi^* g^*$- closed.

Example 3.26. In example 3.2

Let G={U} be be nano πg-open.

Let S={a,b} ⊆ G be nano $\pi^* g^*$- closed.

Let T={a,b,c} ⊆ G be nano $\pi^* g^*$- closed.

S∩T={a,b} ⊆ G

Ncl(Nint(S∩T))={a,c} ⊆ G

Hence {a,b} is nano $\pi^* g^*$- closed.

Thus the intersection of two nano $\pi^* g^*$- closed set is $\pi^* g^*$- closed.

Remark 3.27. The following example prove that nano $\pi^* g^*$-closed and nano semi-closed are independent of each other.

Example 3.28. In example 3.2,

We have {a,b,c} is nano $\pi^{*}g^{*}$-closed .

Let H={a,b,c}

Ncl(Nint(H))={a,c}

H $\nsubseteq$ Ncl(Nint(H))

Hence {a,b,c} is not nano semi-closed.

Theorem 3.29. In space $(U,\tau_R(X))$, if H is nano $\pi^{*}g^{*}$-closed and H $\subseteq$ K $\subseteq$ Ncl(Nint(H)) then K is also nano $\pi^{*}g^{*}$-closed.

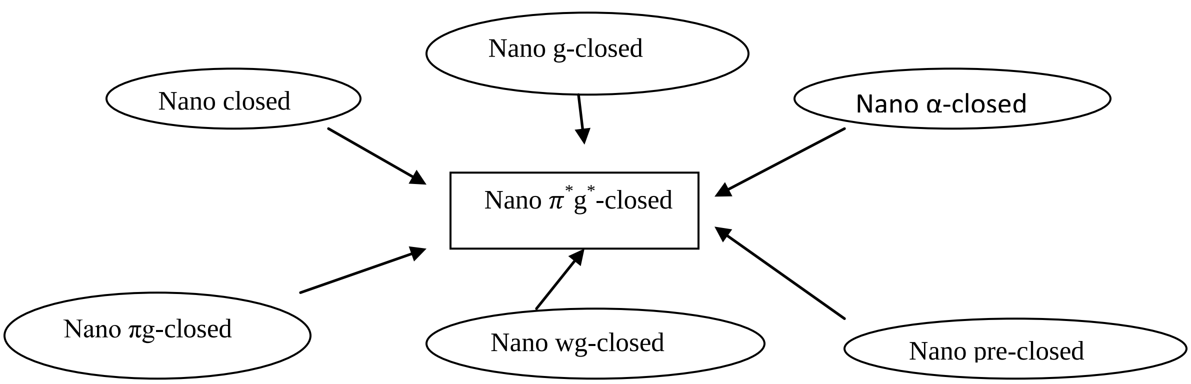
Proof. Let K $\subseteq$ G where G is nano πg-open. Then HCK implies H$\subseteq$ G and G is nano πg-open. Since H is nano $\pi^{*}g^{*}$-closed, Ncl(Nint(H)) $\subseteq$ G. Using hypothesis, Ncl(Nint(k)) $\subseteq$ G. Thus K is nano $\pi^{*}g^{*}$-closed

Theorem 3.30. In space $(U, \tau_R(X))$, if H is both nano regular open and nano $\pi^{*}g^{*}$-closed then it is nano clopen.

Proof. Since H is nano regularopen, H is open and H=Nint(H). H is nano $\pi^{*}g^{*}$-closed implies Ncl(Nint(H)) $\subseteq$ H. Ncl(H)= Ncl(Nint(H)) $\subseteq$ H implies Ncl(H)=H. Hence H is nano clopen.

Remark 3.31.

We obtain definition, theorems and examples follows from the implications.



REFERENCES

[1] Adinatha C. Upadhya, On quasi nano p-normal spaces,International Journal of Recent Scientific Research, 8(6)(2017),17748-17751.
[2] K.Bhuvaneshwari and A.Ezhilarasi, On nano semi-generalised and Nano semi-generalised closed sets in nano topological spaces,International Journal of Math-ematics and Computer Applications Research, 4(3)(2014),117-124.
[3] K.Bhuvaneshwari and K.Mythili Gnanapriya,Nano generalizsed Closed sets,International Journal of Scientific and Research Publications,4(5)(2014),1-3.
[4] K.Bhuvaneshwari and K.Mythili Gnanapriya,On nano generalized Pre closed sets and nano pre generalized closed sets in nano topological spaces,International Journal of Innovative Research in Science,Engineering and Technology,3(10)(2014),16825-16829.
[5] P.Jeyalakshmi, On nano $\pi^{*}g^{*}$-closed sets,Global Journal of pure and Applied Mathematics. ISSN 0973-1768 volume 14, 4(2018),pp.611-618 ,Research India publication.
[6] N.Nagaveni and M.Bhuvaneswari,on nano weekly g-closed topological spaces,International Journal of Innovative Research in sets,International Journal of pure and Applied Mathematics 7(103(2016),129-137.
[7] C.R.Parvathy and S.Praveena, On nano g-pre regular closed set in nano topological spaces, ISOR Journal of Mathematics (IOSR-JM), 13(2)(2017),56-60.
[8] Z.Pawlak,Rough sets, International Journal of Computer and Information Sciences,11(5)(1982),341-356.
[9] I.Rajasekaran and O.Nethaji ,On some new subsets of nano topological space, Journal of new theory,16(2017),52-58.
[10] M.Lellis Thivagar and carmel Richard,on nano forms of weekly open sets ,International Journal of Mathematics and Statistics Invention ,1(1)(2013),31-37.