On (4, k) - Regular Fuzzy Graphs

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Abstract

In this paper, we define d_4-degree and total d_4- degree of vertex in fuzzy graphs. Further we study (4, k)-regularity and totally (4, k)- regularity of fuzzy graphs and the relation between (4,K)- regularity and totally (4, k)-regularity.

Keywords: *Regular fuzzy graphs, total degree, totally regular fuzzy graph, d_4 degree of a vertex in graphs.*

I. INTRODUCTION

In 1965, Lofti A.Zadeh[3] introduced the concept of a fuzzy subset of a set as a method for representing the phenomena of uncertainty in real life situation. Azriel Rosenfeld introduced fuzzy graphs in 1975[3], which is growing fast and has numerous applications in various fields. Nagoor Gani and Radha [2] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. Alison Northup [1] studied some properties on (2, k)- regular graphs in her bachelor thesis. N.R Santhi Maheswari and C. Sekar introduced d_2 of a vertex in graphs [4] and (3, k)-regular fuzzy graphs[5] are introduced S. Meena Devi and M. Andal. In this paper, we define d_4- degree of a vertex in fuzzy graphs and total d_4-degree total d_4- degree of vertex in fuzzy graphs and introduce (4, k) - regular fuzzy graphs, totally (4, k)-regular fuzzy graphs.

We present some known definitions and results for a ready reference to go through the work presented in this paper.

II. PRELIMINARIES

Definition 2.1: For a given graph G, the d_4-degree of a vertex v in G, denoted by $d_4(v)$ means number of vertices at a distance two away from v.

Definition 2.2: A graph G is said to be (4, k)- regular(d_4-regular) if $d_4(v)=k$, for all v in G.

We observe that (4, k)- regular and d_4-regular graphs are same.

Definition 2.3: A fuzzy graph denoted by G:(σ,μ) on graph G^*:(V, E) is a pair of functions(σ,μ) where σ : V→[0,1] is a fuzzy subset of a non empty set V and μ : V×V→[0,1] is a symmetric fuzzy relation on σ such that for all u, v in V the relation μ (u, v)= μ (uv) ≤ σ(u) Λ σ(v) is satisfied.

A fuzzy graph G is complete if μ (u, v)= μ (uv) =σ(u) Λ σ(v) for all u, v ЄV, where uv denotes the edge between u and v.

Definition 2.4: Let G:(σ,μ) be a fuzzy graph. The degree of a vertex u is $d_G(u)= \sum_{u\neq v} \mu(uv)$ for uvЄ E and $\mu(uv) = 0$, for uv not in E; this is equivalent to $d_G(u)= \sum_{uv \in E} \mu(uv)$.

Definition 2.5: The strength of connectedness between two vertices u and v is μ^∞ (u,v)= sup{ μ^K (u, v)/ k= 1,2,3…..} where μ^K (u, v)= sup{ $\mu(u,u_1) \Lambda \mu(u_1u_2) \Lambda \dots\dots\dots\dots\dots \Lambda \mu(u_{k-1}\ v)$ / u,u_1,u_2 ,….. u_{k-1}, v} is a path connecting u and v of length k}.

Definition 2.6: Let G:(σ,μ) be a fuzzy graph on G^*:(V,E). If d(v)= k for all v ЄV, then G is said to be regular fuzzy graph of degree k.

If each vertex of G has not the same degree k, then G is said to be irregular fuzzy graph.

Definition 2.7: Let G:(σ,μ) be a fuzzy graph on G^*:(V, E). The total degree of a vertex u is defined as td(u)= $\sum\mu(u,v)+\sigma(u) = d(u) + \sigma(u)$, uv∈E.

If each vertex of G has the same total degree k, then G is said to be totally regular fuzzy graph of degree K or k-totally regular fuzzy graph.

If each vertex of G has not the same total degree k, then G is said to be totally irregular fuzzy graph.

Definition 2.8: Snark graph is a graph in which every vertex has three neighbours.

Example 2.9:

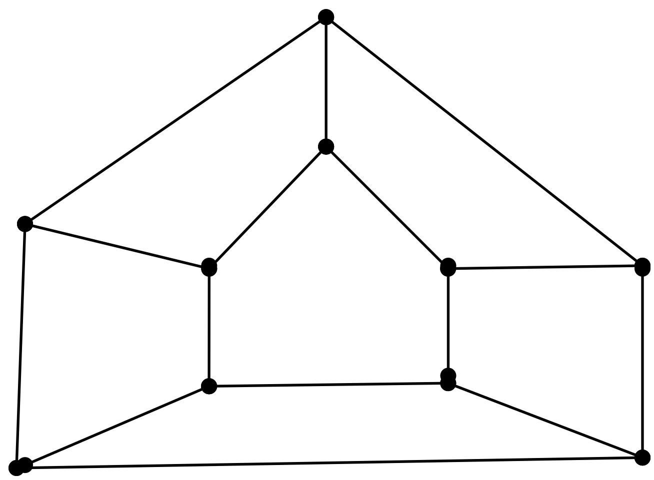


Figure 1

III. d_4-DEGREE AND TOTAL d_4-DEGREE OF A VERTEX IN FUZZY GRAPHS

Definition 3.1: Let G:(σ,μ) be a fuzzy graph .The d_2-degree of a vertex u in G is $d_4(u)= \sum\mu^4(uv)$, where $\sum\mu^4(uv)$ =sup{$\mu(uu_1)\wedge\mu(u_1u_2)\wedge\mu(u_2u_3)\wedge\mu(u_3v)$: u, u_1, u_2, u_3, v is the shortest path connecting u and v of length 4}. Also $\mu(uv) = 0$, for uv not in E.

The minimum d_4-degree of G is $\delta_4(G)= \wedge\{d_4(v):v\in V\}$.

The maximum d_4-degree of G is $\Delta_4(G)= \vee\{d_4(v):v\in V\}$.

Example 3.2: Consider G^* : (V, E), where V={ u_1, u_2, u_3, u_4, u_5} and E={ u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_1}. Define G :(σ,μ) by $\sigma(u_1)$= 0.3, $\sigma(u_2)$= 0.4, $\sigma(u_3)$= 0.5, $\sigma(u_4)$= 0.6, $\sigma(u_5)$= 0.7, and $\mu(u_1u_2)$= 0.3, $\mu(u_2u_3)$= 0.4, $\mu(u_3u_4)$= 0.3, $\mu(u_4u_5)$= 0.2, $\mu(u_5u_1)$= 0.1.

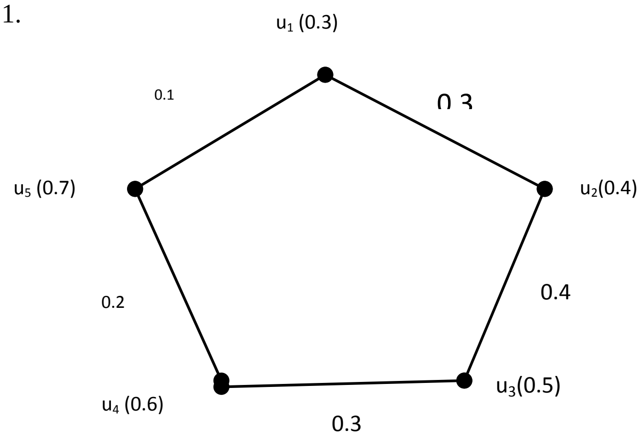


Figure 2

Here, $d_4(u_1) = \{ 0.3 \Lambda 0.4 \Lambda 0.3 \Lambda 0.2\} + \{ 0.1 \Lambda 0.2 \Lambda 0.3 \Lambda 0.4\}$

$= 0.2 + 0.1 = 0.3.$

$d_4(u_2) = \{ 0.4 \Lambda 0.3 \Lambda 0.2 \Lambda 0.1\} + \{ 0.3 \Lambda 0.1 \Lambda 0.2 \Lambda 0.3\}$

$= 0.1 + 0.1 = 0.2.$

$d_4(u_3) = \{ 0.3 \Lambda 0.2 \Lambda 0.1 \Lambda 0.3\} + \{ 0.4 \Lambda 0.3 \Lambda 0.1 \Lambda 0.2\}$

$= 0.1 + 0.1 = 0.2.$

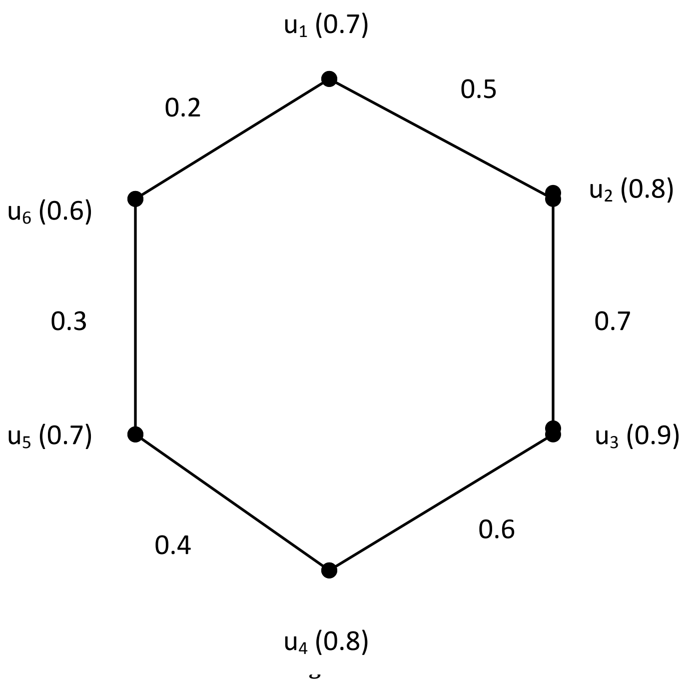
$d_4(u_4) = \{ 0.2 \Lambda 0.1 \Lambda 0.3 \Lambda 0.4\} + \{ 0.3 \Lambda 0.4 \Lambda 0.3 \Lambda 0.1\}$

$= 0.1 + 0.1 = 0.2.$

$d_4(u_5) = \{ 0.1 \Lambda 0.4 \Lambda 0.4 \Lambda 0.3\} + \{ 0.2 \Lambda 0.3 \Lambda 0.4 \Lambda 0.3\}$

$= 0.1 + 0.2 = 0.3.$

Example 3.3: Consider G^* : (V, E), where V={ $u_1, u_2, u_3, u_4, u_5, u_6$} and E={ $u_1u_2, u_2 u_3, u_3u_4, u_4 u_5, u_5 u_6, u_6 u_1$ }. Define G :(σ, μ) by $\sigma(u_1)$= 0.7, $\sigma(u_2)$= 0.8, $\sigma(u_3)$= 0.9, $\sigma(u_4)$= 0.8, $\sigma(u_5)$= 0.7, $\sigma(u_6)$= 0.6 and $\mu(u_1u_2)$= 0.5, $\mu(u_2u_3)$= 0.7, $\mu(u_3u_4)$= 0.6, $\mu(u_4u_5)$= 0.4, $\mu(u_5u_6)$= 0.3, $\mu(u_6u_1)$= 0.2.



$d_4(u_1) = \sup\{ 0.5 \Lambda 0.7 \Lambda 0.6 \Lambda 0.4, 0.2 \Lambda 0.3 \Lambda 0.4 \Lambda 0.6\}$

$= \sup \{0.4, 0.2\} \quad = 0.4$

Here, $d_4(u_2) = \sup\{ 0.7 \Lambda 0.6 \Lambda 0.4 \Lambda 0.3, 0.5 \Lambda 0.2 \Lambda 0.3 \Lambda 0.4\}$

= sup {0.3, 0.2} =0.3

$d_4(u_3)$ = sup{ 0.6 $\wedge$ 0.4 $\wedge$ 0.3 $\wedge$ 0.2, 0.7 $\wedge$ 0.5 $\wedge$ 0.2 $\wedge$ 0.3}

= sup {0.2, 0.2} =0.2

$d_4(u_4)$ = sup{ 0.4 $\wedge$ 0.3 $\wedge$ 0.2 $\wedge$ 0.5, 0.6 $\wedge$ 0.7 $\wedge$ 0.5 $\wedge$ 0.2}

= sup {0.2, 0.2} =0.2

$d_4(u_5)$ = sup{ 0.3 $\wedge$ 0.2 $\wedge$ 0.5 $\wedge$ 0.7, 0.4 $\wedge$ 0.6 $\wedge$ 0.7 $\wedge$ 0.5}

= sup {0.2, 0.4} =0.4

$d_4(u_6)$ = sup{ 0.2 $\wedge$ 0.5 $\wedge$ 0.7 $\wedge$ 0.6, 0.3 $\wedge$ 0.4 $\wedge$ 0.6 $\wedge$ 0.7}

= sup {0.2, 0.3} =0.3

Definition 3.4: Let G:(σ, μ) be a fuzzy graph on G^*:(V, E). The total d_4-degree of a vertex u Є V is defined as $td_4(u) = \sum \mu^4(uv) + \sigma(u) = d_4(u) + \sigma(u)$

The minimum td_4-degree of G is $t\delta_4(G) = \Lambda\{d_4(v): v \in V\}$.

The maximum td_4-degree of G is $t\Delta_4(G) = \vee\{d_4(v): v \in V\}$.

Example 3.5: Consider G^* : (V, E), where V={ u_1, u_2, u_3, u_4, u_5} and E={ u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_1}. Define G :(σ, μ) by $\sigma(u_1)$= 0.5, $\sigma(u_2)$= 0.6, $\sigma(u_3)$= 0.5, $\sigma(u_4)$= 0.6, $\sigma(u_5)$= 0.5, and $\mu(u_1u_2)$= 0.3, $\mu(u_2u_3)$= 0.4, $\mu(u_3u_4)$= 0.3, $\mu(u_4u_5)$= 0.2, $\mu(u_5u_1)$= 0.1.

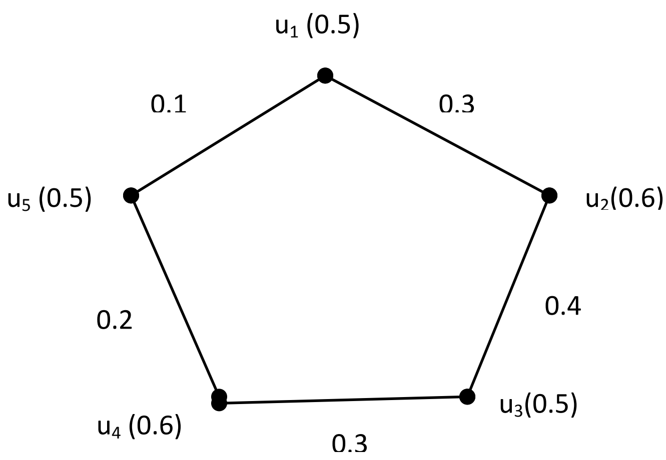


Figure 4

Here, $td_4(u_1)$ = { 0.3 $\wedge$ 0.4 $\wedge$ 0.3 $\wedge$ 0.2 } + { 0.1 $\wedge$ 0.2 $\wedge$ 0.3 $\wedge$ 0.4}+$\sigma(u_1)$

=0.3+0.5=0.8

$td_4(u_2)$ = { 0.4 $\wedge$ 0.3 $\wedge$ 0.2 $\wedge$ 0.1} + { 0.3 $\wedge$ 0.1 $\wedge$ 0.2 $\wedge$ 0.3}+$\sigma(u_2)$

=0.2+0.6=0.8

$td_4(u_3)$ = { 0.3 Λ 0.2 Λ 0.1 Λ 0.3} + { 0.4 Λ 0.3 Λ 0.1 Λ 0.2}+$\sigma(u_3)$

=0.2+0.5=0.7

$td_4(u_4)$ = { 0.2 Λ 0.1 Λ 0.3 Λ 0.4} + { 0.3 Λ 0.4 Λ 0.3 Λ 0.1}+$\sigma(u_4)$

=0.2+ 0.6=0.8

$td_4(u_5)$ = { 0.1 Λ 0.4 Λ 0.4 Λ 0.3} + { 0.2 Λ 0.3 Λ 0.4 Λ 0.3}+$\sigma(u_5)$

=0.3+0.5=0.8.

IV. (4, K)-REGULAR AND TOTALLY (4, K)-REGULAR FUZZY GRAPHS

Definition 4.1: Let G:(σ, μ) be a fuzzy graph on G^*:(V, E). If $d_4(v)$= k for all v€V, then G is said to be (4, k)-regular fuzzy graph .

Example 4.2: Consider G^* : (V, E), where V={ u_1, u_2, u_3, u_4, u_5,} and E={ $u_1u_2, u_2 u_3, u_3u_4, u_4 u_5, u_5 u_1$`}. Define G :$(\sigma, \mu)$ by $\sigma(u_1)$= 0.3, $\sigma(u_2)$= 0.4, $\sigma(u_3)$= 0.5, $\sigma(u_4)$= 0.6, $\sigma(u_5)$= 0.7, and $\mu(u_1u_2)$= 0.3, $\mu(u_2u_3)$= 0.4, $\mu(u_3u_4)$= 0.3, $\mu(u_4u_5)$= 0.6, $\mu(u_5u_1)$= 0.3.

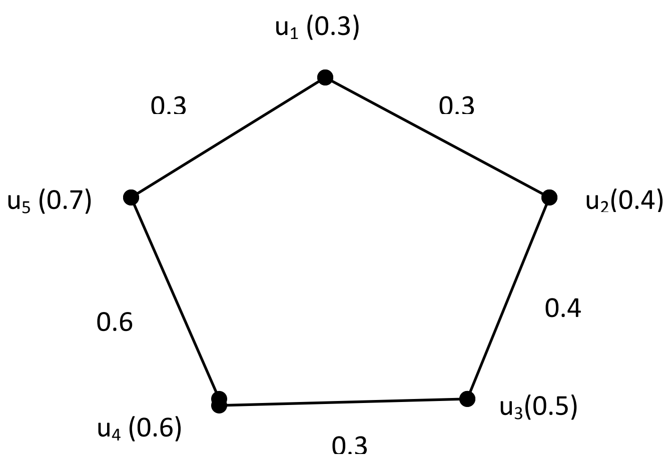


Figure 5

Here, $d_4(u_1)$=0.6, $d_4(u_2)$=0.6, $d_4(u_3)$=0.6, $d_4(u_4)$=0.6, $d_4(u_5)$=0.6.

G is (4, 0.6) - regular fuzzy graphs.

Definition 4.3: If each vertex of G has the same total d_4-degree k, then G is said to be totally (4, k)-regular fuzzy graph.

Example 4.4: Consider G^* : (V, E), where V={ u_1, u_2, u_3, u_4, u_5,} and E={ $u_1u_2, u_2 u_3, u_3u_4, u_4 u_5, u_5 u_1$`}. Define G :$(\sigma, \mu)$ by $\sigma(u_1)$= 0.3, $\sigma(u_2)$= 0.3, $\sigma(u_3)$= 0.3, $\sigma(u_4)$= 0.3, $\sigma(u_5)$= 0.3, and $\mu(u_1u_2)$= 0.1, $\mu(u_2u_3)$= 0.1, $\mu(u_3u_4)$= 0.1, $\mu(u_4u_5)$= 0.1, $\mu(u_5u_1)$= 0.1.

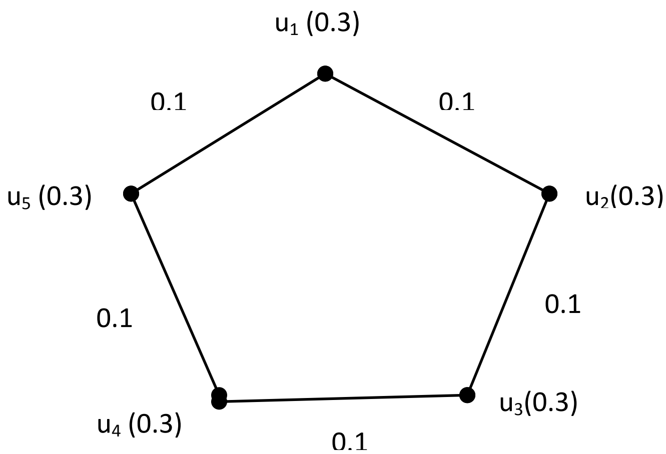


Figure 6

Here, $td_4(u_1)=0.5$, $td_4(u_2)=0.5$, $td_4(u_3)=0.5$, $td_4(u_4)=0.5$, $td_4(u_5)=0.5$.

G is totally regular fuzzy graph.

Example 4.5: Consider G^*: (V, E), where V={ $u_1, u_2, u_3, u_4, u_5, u_6, u_7, u_8, u_9, u_{10}$} and E={ $u_1u_2, u_2u_3, u_3u_4, u_4u_5, u_5u_6, u_6u_7, u_7u_8, u_8u_9, u_9u_{10}, u_{10}u_6, u_1u_7, u_2u_8, u_3u_9, u_4u_{10}$}. Define G :$(\sigma, \mu)$ by $\sigma(u_1)$= 0.3, $\sigma(u_2)$= 0.3, $\sigma(u_3)$= 0.3, $\sigma(u_4)$= 0.3, $\sigma(u_5)$= 0.3, $\sigma(u_6)$= 0.3, $\sigma(u_7)$= 0.3, $\sigma(u_8)$= 0.3, $\sigma(u_9)$= 0.3, $\sigma(u_{10})$= 0.3, and $\mu(u_1u_2)$= 0.2, $\mu(u_2u_3)$= 0.2, $\mu(u_3u_4)$= 0.2, $\mu(u_4u_5)$= 0.2, $\mu(u_5u_6)$= 0.2, $\mu(u_6u_7)$= 0.2, $\mu(u_7u_8)$= 0.2, $\mu(u_8u_9)$= 0.2, $\mu(u_9u_{10})$= 0.2, $\mu(u_{10}u_6)$= 0.2, $\mu(u_1u_7)$= 0.2, $\mu(u_2u_8)$= 0.2, $\mu(u_3u_9)$= 0.2, $\mu(u_4u_{10})$= 0.2, $\mu(u_5u_1)$= 0.2,

Here $td_4(u_1)$ = sup{0.2⋀0.2⋀0.2⋀0.2, 0.2⋀0.2⋀0.2⋀0.2, 0.2⋀0.2⋀0.2⋀0.2, 0.2⋀0.2⋀0.2⋀0.2}+$\sigma(u_1)$

= sup{0.2, 0.2, 0.2, 0,2} +0.3

= 0.5

$td_4(u_2)$ =0.5, $td_4(u_3)$ =0.5, $td_4(u_4)$ =0.5, $td_4(u_5)$ =0.5, $td_4(u_6)$ =0.5, $td_4(u_7)$ =0.5,

$td_4(u_8)$ =0.5, $td_4(u_9)$ =0.5, $td_4(u_{10})$ =0.5.

G is totally regular graph.

Example 4.6: Non regular fuzzy graphs which is (4, k) - regular.

In example 4.2, we have G is (4, 0.6) regular fuzzy graph but not regular graph.

Theorem 4.7: Let G: (σ, μ) be a fuzzy graph on G^*: (V,E). Then σ is constant function iff the following conditions are equivalent:

(1) G: (σ, μ) is (4, k)- regular fuzzy graph.

(2) G: (σ, μ) is totally (4, k+c)- regular fuzzy graph.

Proof: Suppose that σ is constant function.

Let $\sigma(u)$=c, constant for all u Є V.

Assume that G: (σ, μ) is a (4, k)- regular fuzzy graph.

Then d_4(u)=k for all u ЄV.

So t_4d(u)= d_4(u)+ σ(u) for all uЄV.

$\Longrightarrow$ td_4 (u)= K+c for all uЄV.

Hence G: (σ, μ) is totally (4, k+c)- regular fuzzy graph.

Thus, (1) $\Longrightarrow$ (2) is proved.

Suppose G: (σ, μ) is totally (4, k+c)-regular fuzzy graph.

Then td_4(u)= k +c for all uЄV.

$\Longrightarrow$$d_4$(u)+ σ(u)=k+c for all u ЄV.

$\Longrightarrow$$d_4$(u)+ c =k+c for all u ЄV.

$\Longrightarrow$$d_4$(u)=k for all u ЄV.

Hence G: (σ, μ) is a (4, k)- regular fuzzy graph.

Thus, (2) $\Longrightarrow$ (1) is proved.

Hence (1) and (2) are equivalent.

Conversely it is assume that (1) and (2) are equivalent.

Suppose that σ is not constant function.

Then σ(u)$\neq$ σ(w), for at least one pair u, w Є V. Let G: (σ, μ) be a (4,k)- regular fuzzy graph. Then , d_4(u)= d_4(w)=k .

So, td_4(u)= d_4(u)+ σ(u)=k+ σ(u) and td_4(w)= d_4(w)+ σ(w)=k+ σ(w).

Since σ(u)$\neq$ σ(w) implies K+ σ(u)$\neq$ $K + \sigma$(w)

$\Longrightarrow$ td_4(u) $\neq$ td_4(w) .

So, G: (σ, μ) is not totally (4, k)- regular fuzzy graph, Which is contradiction to our assumption.

Let G: (σ, μ) be a totally (4, k)-regular fuzzy graph.

Then, td_4(u)= td_4(w) .

$\Longrightarrow$$d_4$(u)+ σ(u)= d_4(w)+ σ(w).

$\Longrightarrow$$d_4$(u)- d_4(w)= σ(w)- σ(u) $\neq$0.

$\Longrightarrow$$d_4$(u) $\neq$ d_4(w).

So, G: (σ, μ) is not (4, k)- regular fuzzy graph, Which is contradiction to our assumption.

Hence σ is constant function.

Theorem 4.8: If a fuzzy graph G: (σ, μ) is both (4 ,k)- regular and totally (4, k)-regular then σ is constant function.

Proof: Let G be (4, k_1)-regular and totally (4, k_2)-regular fuzzy graph.

Then $d_4(u)= k_1$ and $td_4(u)=k_2$ for all u∈V.

Now, $td_4(u)=k_2$, for all u∈V.

$\Rightarrow d_4(u)+ \sigma(u)= k_2$, for all u∈V.

$\Rightarrow k_1+ \sigma(u)= k_2$, for all u∈V.

$\Rightarrow \sigma(u)= k_2 - k_1$, for all u∈V.

Hence σ is constant function.

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