Intuitionistic Approach to Heptagonal Fuzzy Number

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Abstract

In this paper we have introduced Heptagonal Intuitionistic Fuzzy Number (H_pIFN) and Symmetric Heptagonal Intuitionistic Fuzzy Number (SH_pIFN) with their graphical representations. Some properties of SH_pIFN are discussed. These numbers can be applied in solving real life problems which involve seven parameters. The arithmetic operations of H_pIFN and SH_pIFN and ranking of H_pIFN are also proposed with numerical examples.

Key words

Heptagonal Intuitionistic Fuzzy Number (H_pIFN), Symmetric Heptagonal Intuitionistic Fuzzy Number (SH_pIFN).

1. Introduction

Lofti A. Zadeh introduced Fuzzy Sets in 1965. It is generalized by Krassimir T. Attanassov in 1986 to introduce the notion of Intuitionistic Fuzzy set(IFS). On the basis of definition of IFS, Intuitionistic Fuzzy Numbers (IFN) was introduced and it has got many applications in decision making problems, medical diagnosis and pattern recognition. Triangular Intuitionistic Fuzzy Numbers, Trapezoidal Intuitionistic Fuzzy Numbers, Pentagonal Intuitionistic Fuzzy Numbers and Hexagonal Intuitionistic Fuzzy Numbers are the most commonly used IFN in various applications. In 2017, A. Mohammed Shapique introduced Heptagonal Fuzzy Number (HFN) which gives additional possibility to represent imperfect knowledge so that many real life situations can be modelled in a detailed way. HFN find its application in many optimization problems which need seven parameters. For example in tumor growth, the growth rate has seven points and it is difficult to represent it by triangular or hexagonal fuzzy number. Hence the HFN is relevant in solving such problems. In section 2, some preliminary definitions are discussed. In section 3, we propose the definition of Heptagonal Intuitionistic Fuzzy Number (H_pIFN) and Symmetric Heptagonal Intuitionistic Fuzzy Number (SH_pIFN) with their graphical representations. The shape of membership and nonmembership functions of symmetric intuitionistic fuzzy numbers are much simpler and regular than that of fuzzy numbers. It makes calculations easier and helps us for more natural interpretations. We also state some properties of SH_pIFN . Some of the arithmetic operations of H_pIFN and SH_pIFN and ranking of H_pIFN are also discussed. In section 4, the arithmetic operations and ranking are illustrated with numerical examples. Section 5 concludes the paper.

2. Preliminaries

2.1 Definition [1] Let X be a given set. A fuzzy set A of X is defined by,

$$\mathbf{A} = \{ (x, \ \mu_A(\mathbf{x})) / \mathbf{x} \in \mathbf{X} \}$$

where $\mu_A(\mathbf{x}) : \mathbf{X} \to [0, 1]$ is the degree of membership of \mathbf{x} in A.

2.2 Definition [2] A fuzzy number is a fuzzy set A on the real line \Re whose membership function satisfies the following 3 conditions :

- 1. Normality i.e, \exists an x_0 such that $\mu_A(x_0) = 1$.
- 2. Piecewise continuity i.e, $\forall \epsilon > 0, \exists \delta > 0$ such that $\mu_A(\mathbf{x}) \mu_A(\mathbf{x}_0) < \epsilon$ whenever $||\mathbf{x} \mathbf{x}_0|| < \delta$.
- 3. Convexity i.e, $\mu_A(\lambda x_1 + (1 \lambda)x_2) \geq \min(\mu_A(x_1), \mu_A(x_2)) \forall x_1, x_2 \in \Re, \lambda \in [0, 1].$

2.3 Definition [3] Let X be a given set. An Intuitionistic fuzzy set \tilde{A} in X is given by, $\tilde{A} = \{(x, \mu_A(x), \nu_A(x)) \mid x \in X\}$

where μ_A , $\nu_A : X \to [0, 1]$ and $0 \le \mu_A(x) + \nu_A(x) \le 1$. $\mu_A(x)$ is the degree of membership of the element x in A and $\nu_A(x)$ is the degree of nonmembership of x in A. For each $x \in X$, $\pi_A(x) = 1 - \mu_A(x) - \nu_A(x)$ is called the degree of hesitation.

2.4 Definition [4] An Intutionistic Fuzzy Number is an intuitionistic fuzzy set \tilde{A} on \Re which satisfies all the 3 conditions of a fuzzy number along with the following condition, Concavity for the nonmembership function i.e,

 $\nu_A (\lambda \mathbf{x}_1 + (1 - \lambda)\mathbf{x}_2) \leq \max (\nu_A(\mathbf{x}_1), \nu_A(\mathbf{x}_2)) \forall \mathbf{x}_1, \mathbf{x}_2 \in \Re, \lambda \in [0, 1].$

2.5 Definition [5] A Heptagonal Fuzzy Number denoted by A_{Hp} is a fuzzy number with membership function μ_{Hp} : $\Re \to [0, 1]$ is given by

$$\mu_{H_p}(x) = \begin{cases} 0, & x < a_1; \\ \frac{1}{2}(\frac{x-a_1}{a_2-a_1}), & a_1 \le x \le a_2; \\ \frac{1}{2}, & a_2 \le x \le a_3; \\ \frac{1}{2} + \frac{1}{2}(\frac{x-a_3}{a_4-a_3}), & a_3 \le x \le a_4; \\ \frac{1}{2} + \frac{1}{2}(\frac{a_5-x}{a_5-a_4}), & a_4 \le x \le a_5; \\ \frac{1}{2}, & a_5 \le x \le a_6; \\ \frac{1}{2}(\frac{a_7-x}{a_7-a_6}), & a_6 \le x \le a_7. \\ 0, & x \ge a_7; \end{cases}$$

It is denoted by $A_{Hp} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7)$ where $a_i, 1 \leq i \leq 7$ are real numbers which satisfy the condition $a_1 \leq a_2 \leq a_3 \leq a_4 \leq a_5 \leq a_6 \leq a_7$.

3. Heptagonal Intuitionistic Fuzzy Number

3.1 Definition A Heptagonal Intuitionistic Fuzzy Number (H_pIFN) denoted by $\tilde{A}_{HPI} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; a'_1, a'_2, a'_3, a_4, a'_5, a'_6, a'_7)$ where $a_i, a'_i, 1 \leq i \leq 7$ are real numbers such that $a'_1 \leq a_1 \leq a'_2 \leq a_2 \leq a'_3 \leq a_3 \leq a_4 \leq a_5 \leq a'_5 \leq a_6 \leq a'_6 \leq a_7 \leq a'_7$ whose membership and nonmembership functions are defined as

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	0,	$x < a_1;$
$\mu_{H_p}(x) = \langle$	$\frac{1}{2}(\frac{x-a_1}{a_2-a_1}),$	$a_1 \leq x \leq a_2;$
	$\frac{1}{2}$,	$a_2 \leq x \leq a_3;$
	$\frac{1}{2} + \frac{1}{2}(\frac{x-a_3}{a_4-a_3}),$	$a_3 \leq x \leq a_4;$
	$\frac{1}{2} + \frac{1}{2}(\frac{a_5 - x}{a_5 - a_4}),$	$a_4 \leq x \leq a_5;$
	$\frac{1}{2}$,	$a_5 \leq x \leq a_6;$
	$\frac{1}{2}(\frac{a_7-x}{a_7-a_6}),$	$a_6 \leq x \leq a_7.$
	0,	$x\geq a_7\ ;$
ĺ	1,	$x < a^{'}_{1};$
$ u_{H_p}(x) = \langle$	$1-\frac{1}{2}(\frac{x-a_{1}^{'}}{a_{2}^{'}-a_{1}^{'}}),$	$a'_{1} \le x \le a'_{2};$
	$\frac{1}{2}$,	$a'_{2} \le x \le a'_{3};$
	$\frac{1}{2}(\frac{a_4-x}{a_4-a_3'}),$	$a_{\ 3}^{'}\leq x\leq a_{4};$
	$\frac{1}{2}(\frac{x-a_4}{a_5'-a_4}),$	$a_4 \leq x \leq a_{5}^{'};$
	$\frac{1}{2}$,	$a'_{5} \le x \le a'_{6};$
	$\frac{1}{2} + \frac{1}{2}(\frac{x-a_{6}^{'}}{a_{7}^{'}-a_{6}^{'}}),$	$a'_{6} \le x \le a'_{7}.$
	1	$\mathbf{v} > \mathbf{a'}_{\tau} \cdot$

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3.2 Remark The graphical representation of H_p IFN is given below

3.3 Definition A Symmetric Heptagonal Fuzzy Number (SH_pFN) is denoted by $A_{SHP} = (a, l, l, m, m, n, n)$. Its membership function is defined as

$$\mu_{SH_{pI}}(x) = \begin{cases} 0, & x < a - l; \\ \frac{1}{2}(\frac{x - (a - l)}{l - m}), & a - l \le x \le a - m; \\ \frac{1}{2}, & a - m \le x \le a - n; \\ \frac{1}{2} + \frac{1}{2}(\frac{x - (a - n)}{n}), & a - n \le x \le a; \\ \frac{1}{2} + \frac{1}{2}(\frac{(a + n) - x}{n}), & a \le x \le a + n; \\ \frac{1}{2}, & a + n \le x \le a + m; \\ \frac{1}{2}, & a + n \le x \le a + m; \\ \frac{1}{2}(\frac{(a + l) - x}{l - m}), & a + m \le x \le a + l; \\ 0, & x \ge a + l; \end{cases}$$

For convenience let us denote A_{SHP} as $A_{SHP} = (a, \underline{l}, \underline{m}, \underline{n})$

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3.4 Definition A Symmetric Heptagonal Intuitionistic Fuzzy Number (SH_pIFN) is denoted as $\tilde{A}_{SHPI} = (a, \underline{l}, \underline{m}, \underline{n}; \underline{l}', \underline{m}', \underline{n}')$. Its membership and nonmembership function is given by

$$\mu_{SH_{pI}}(x) = \begin{cases} 0, & x < a - 1; \\ \frac{1}{2}(\frac{x - (a - l)}{l - m}), & a - 1 \le x \le a - m; \\ \frac{1}{2}, & a - m \le x \le a - n; \\ \frac{1}{2} + \frac{1}{2}(\frac{x - (a - n)}{n}), & a - n \le x \le a; \\ \frac{1}{2} + \frac{1}{2}(\frac{(a + n) - x}{n}), & a \le x \le a + n; \\ \frac{1}{2}, & a + n \le x \le a + n; \\ \frac{1}{2}, & a + n \le x \le a + n; \\ \frac{1}{2}, & a + n \le x \le a + n; \\ \frac{1}{2}(\frac{(a + l) - x}{l - m}), & a + m \le x \le a + 1; \\ 0, & x \ge a + 1; \end{cases}$$

$$\nu_{SH_{pI}}(x) = \begin{cases} 1, & x < a - l'; \\ 1 - \frac{1}{2}(\frac{x - (a - l')}{l' - m'}), & a - l' \le x \le a - m'; \\ \frac{1}{2}, & a - m' \le x \le a - n'; \\ \frac{1}{2}, & a - m' \le x \le a - n'; \\ \frac{1}{2}, & a - n' \le x \le a - n'; \\ \frac{1}{2}, & a - n' \le x \le a + n'; \\ \frac{1}{2}, & a + n' \le x \le a + n'; \\ \frac{1}{2}, & a + n' \le x \le a + n'; \\ \frac{1}{2}, & a + n' \le x \le a + n'; \\ \frac{1}{2}, & a + n' \le x \le a + n'; \\ \frac{1}{2}, & a + n' \le x \le a + n'; \\ \frac{1}{2}, & a + n' \le x \le a + n'; \\ \frac{1}{2}, & a + n' \le x \le a + n'; \\ \frac{1}{2}, & a + n' \le x \le a + n'; \end{cases}$$

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3.6 Properties of SH_pIFN

For a SH_pIFN of the form $\tilde{A}_{SHPI} = (a, \underline{l}, \underline{m}, \underline{n}; \underline{l}', \underline{m}', \underline{n}')$ such that l - m = l' - m' we have $n \le n', m \le m', l \le l'$

Proof

For $x \in [a - n, a]$, we have

$$\frac{1}{2} + \frac{1}{2}\left(\frac{x-a+n}{n}\right) + \frac{1}{2}\left(\frac{a-x}{n'}\right) \le 1$$
$$\Rightarrow 1 + \frac{x-a}{n} + 1 + \frac{a-x}{n'} \le 2$$
$$\Rightarrow (x - a)\left(\frac{1}{n} - \frac{1}{n'}\right) \le 0$$
$$\Rightarrow (x - a)(n' - n) \le 0$$
$$\Rightarrow n' - n \ge 0$$
$$\Rightarrow n' \ge n.$$

For $x \in [a - l, a - m]$, we have

$$\frac{1}{2}\left(\frac{x-(a-l)}{l-m}\right) + 1 - \frac{1}{2}\left(\frac{x-(a-l')}{l'-m'}\right) \le 1$$
$$\Rightarrow \left(\frac{x-a+l}{l-m} - \frac{x-a+l'}{l'-m'}\right) \le 0$$

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$$\Rightarrow \frac{l-l'}{l-m} \le 0$$

$$\Rightarrow l \leq l'$$

For $x \in [a + m, a + l]$, we have

$$\begin{split} &\frac{1}{2}(\frac{a+l-x}{l-m}) + \frac{1}{2} + \frac{1}{2}\left(\frac{x-a-m'}{l'-m'}\right) \leq 1\\ &\Rightarrow \frac{a+l-x}{l-m} + 1 + \frac{x-a-m'}{l'-m'} \leq 2\\ &\Rightarrow \frac{l-m'}{l-m} \leq 1\\ &\Rightarrow m \leq m' \end{split}$$

3.7 Arithmetic Operations

Consider two H_p IFN

 $\tilde{A}_{HPI} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; a'_1, a'_2, a'_3, a_4, a'_5, a'_6, a'_7)$ $\tilde{B}_{HPI} = (b_1, b_2, b_3, b_4, b_5, b_6, b_7; b'_1, b'_2, b'_3, b_4, b'_5, b'_6, b'_7)$

1. Addition

$$\begin{split} \tilde{A}_{HPI} + \tilde{B}_{HPI} &= (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4, a_5 + b_5, a_6 + b_6, a_7 + b_7; \\ a_1' + b_1', a_2' + b_2', a_3' + b_3', a_4 + b_4, a_5' + b_5', a_6' + b_6', a_7' + b_7') \\ 2. Subtraction \\ \tilde{A}_{HPI} - \tilde{B}_{HPI} &= (a_1 - b_7, a_2 - b_6, a_3 - b_5, a_4 - b_4, a_5 - b_6, a_3 - b_2, a_7 - b_1; \\ a_1' - b_7', a_2' - b_6', a_3' - b_5', a_4 - b_4, a_5' - b_3', a_6' - b_2', a_7' - b_1') \\ 3. Scalar Multiplication \\ K.\tilde{A}_{HPI} &= (K.a_1, K.a_2, K.a_3, K.a_4, K.a_5, K.a_6, K.a_7; K.a_1', K.a_2', K.a_3', K.a_4, K.a_5', K.a_6', K.a_7'), when K > 0 \\ K.\tilde{A}_{HPI} &= (K.a_7, K.a_6, K.a_5, K.a_4, K.a_3, K.a_2, K.a_1; K.a_7', K.a_6', K.a_5', K.a_4, K.a_3', K.a_2', K.a_1'), when K < 0. \\ 4. Equality \\ \tilde{A}_{HPI} &= \tilde{B}_{HPI} \text{ iff } a_i = b_i, a_i' = b_i', 1 \le i \le 7. \\ 5. \text{ Non - negativity } \tilde{A}_{HPI} \text{ is non - negative iff } a_1' \ge 0. \end{split}$$

Consider two SH_pIFN $\tilde{A}_{SHPI} = (a, \underline{l}_1, \underline{m}_1, \underline{n}_1; \underline{l}_1', \underline{m}_1', \underline{n}_1')$ $\tilde{B}_{SHPI} = (b, \underline{l}_2, \underline{m}_2, \underline{n}_2; \underline{l'}_2, \underline{m'}_2, \underline{n'}_2)$

1. Addition $\tilde{A}_{SHPI} + \tilde{B}_{SHPI} = (a + b, \underline{l_1 + l_2}, \underline{m_1 + m_2}, \underline{n_1 + n_2}; \underline{l'_1 + l'_2}, \underline{m'_1 + m'_2}, \underline{n'_1 + n'_2},)$ 2. Subtraction $\tilde{A}_{SHPI} - \tilde{B}_{SHPI} = (a - b, \underline{l_1 + l_2}, \underline{m_1 + m_2}, \underline{n_1 + n_2}; \underline{l'_1 + l'_2}, \underline{m'_1 + m'_2}, \underline{n'_1 + n'_2},)$ 3. Scalar Multiplication $K\tilde{A}_{SHPI} = (Ka, \underline{Kl_1}, \underline{Km_1}, \underline{Kn_1}; \underline{Kl_1'}, \underline{Km_1'}, \underline{Kn_1'})$

3.8 Ranking of H_p**IFN** Consider a Heptagonal Intuitionistic Fuzzy Number $\tilde{A}_{HPI} = (a_1, a_2, a_3, a_4, a_5, a_6, a_7; a'_1, a'_2, a'_3, a_4, a'_5, a'_6, a'_7)$ The magnitude of \tilde{A}_{HPI} is a real number given by

$$\operatorname{Mag}(\hat{A}) = \frac{a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_1' + a_2' + a_3' + a_4 + a_5' + a_6' + a_7'}{7}$$

Note :

1. $\tilde{A}_{HPI} \preceq \tilde{B}_{HPI}$ when $\operatorname{Mag}(\hat{A}) \leq \operatorname{Mag}(\hat{A})$ 2. $\tilde{A}_{HPI} \succeq \tilde{B}_{HPI}$ when $\operatorname{Mag}(\hat{A}) \geq \operatorname{Mag}(\hat{A})$

4. Numerical Examples

Consider the Heptagonal intuitionstic fuzzy numbers

$$\begin{split} \tilde{A}_{HPI} &= (0, 1, 2, 6, 7, 9, 10; 0, 1, 2, 6, 8, 9, 11) \\ \tilde{B}_{HPI} &= (-1, 2, 3, 4, 7, 8, 9; -2, 0, 2, 4, 8, 9, 10) \\ \text{Then} \\ 1. \quad \tilde{A}_{HPI} + \tilde{B}_{HPI} = (-1, 3, 5, 10, 14, 17, 19; -2, 1, 4, 10, 16, 18, 21) \\ 2. \quad \tilde{A}_{HPI} - \tilde{B}_{HPI} = (-9, -7, -5, 2, 4, 7, 11; -10, -8, -6, 2, 6, 9, 13) \\ 3. \quad \text{For K} = 1.5, \text{K}\tilde{A}_{HPI} = (0, 1.5, 3, 9, 10.5, 13.5, 15; 0, 1.5, 3, 9, 12, 13.5, 16.5) \\ \text{For K} = -1, \text{K}\tilde{A}_{HPI} = (-10, -9, -7, -6, -2, -1, 0; -11, -9, -8, -6, -2, -1, 0) \\ 4. \quad \text{Mag}(\hat{A}) = 10.28, \text{Mag}(\hat{B}) = 9, \text{ since Mag}(\hat{A}) \geq \text{Mag}(\hat{B}), \text{ we have } \tilde{A}_{HPI} \succeq \tilde{B}_{HPI}. \end{split}$$

Consider the Symmetric Heptagonal Intuitionistic Fuzzy Numbers

 $\tilde{A}_{SHPI} = (0.4, 0.1, 0.2, 0.3; 0.2, 0.3, 0.35)$ $\tilde{B}_{SHPI} = (0.5, 0.15, 0.25, 0.4; 0.2, 0.35, 0.45)$ $1. \tilde{A}_{SHPI} + \tilde{B}_{SHPI} = (0.9, 0.25, 0.45, 0.7; 0.4, 0.65, 0.8)$

- 2. \tilde{A}_{SHPI} \tilde{B}_{SHPI} = (-0.1, <u>0.25</u>, <u>0.45</u>, <u>0.7</u>; <u>0.4</u>, <u>0.65</u>, <u>0.8</u>)
- 3. For K = 1.5, $K\tilde{A}_{SHPI} = (0.6, 0.15, 0.3, 0.45; 0.3, 0.45, 0.52)$

5. Conclusion

In this paper we have introduced Heptagonal Intuitionistic Fuzzy Number and Symmetric Heptagonal Intuitionistic Fuzzy Number and their arithmetic operations. Ranking of these fuzzy numbers are also discussed which help us in decision making problems. Their graphical representations and numerical examples clarify the concept.

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