Distance Transformation

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Abstract

In this paper, we define the transformation of a graph Gbased on distance between the non-adjacent vertices, motivated by the methodology asses the biological coherence of a gene network.

Keywords- *Distance Transformation, Identity Transformation, Linear Transformation*

I.INTRODUCTION

The first transformation was defined by Alexander Kelmans, hence known as Kelmans transformation [2]. In [1], Francisco Gomez-Vela, coherence is calculated by converting data into distance matrices. If the minimum distance between two genes is greater than γ, then no path between the genes will be assumed. In this paper, we define a transformation related to shortest path and called T_γ transformation. $T_\gamma$transformation is defined as, there exist a mapping $T_\gamma: G \to G^*$ which satisfies following conditions;

i) $|\ V(G)\ | \ = \ |V(G^*)\ |$

ii) u^* and $v^*$are adjacent in G^* if either$u$ and v are adjacent in G or $d(u,v) = \gamma$. We study properties ofT_γ transformation of certain graphs. For basic definitions of Graph Theory we use [3].

II. ON T_γ TRANSFORMATION

A. *Definition 1.1.*

Let G be a (p, q) graph. A graph G^* is said to be $T_\gamma$transformation of G, if there exist a mapping $T_\gamma : G \to G^*$ such that

i) $|\ V(G)\ | \ = \ |V(G^*)\ |$

ii) Edge set of G^* consists of edges of G together with (u^*, v^*), where$u$ and v are not adjacent in G, and $d(u, v) = \gamma$, for all $u, v \in V(G)$.

Example.

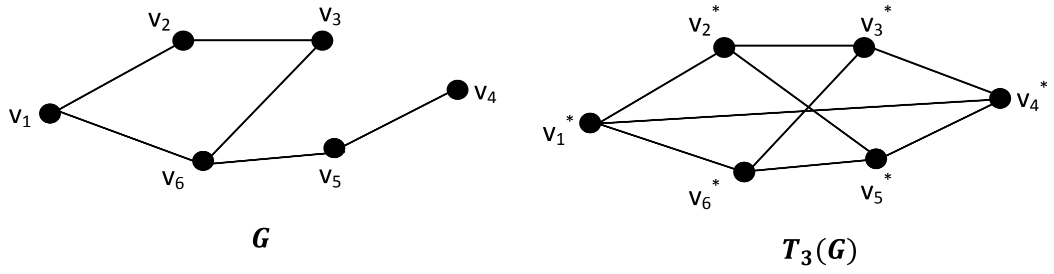

Figure.1

Consider the graph G given in the figure 1. In G, v_1 and v_4 are not adjacent, but $d\,(v_1, v_4) = 3$. In T_3 transformation of G, $v_1{}^*$and $v_4{}^*$ are adjacent. Like that v_2 and v_5 are not adjacent in G, but $d(\,v_2, v_5) = 3$, in T_3 transformation $v_2{}^*$ and $v_5{}^*$ are adjacent and so on.

B. *Definition 1.2.*

If $T_\gamma(G) = G$, then the transformation is said to be an identity transformation.

C. *Definition 1.3.*

If $T_\gamma(G) = K_p$, then the transformation is said to be complete.

D. *Definition1.4.*

If $T_\gamma(G) \cong G + e$, where$e \notin E(G)$ then the transformation is said to be linear.

E. *Theorem 1.5.*

$T_\gamma(C_p) \cong C_p + p$ chords, if $\gamma = 2, p \geq 5$.

Proof. Let C_p be a cycle with p vertices and $\gamma = 2$

Let $v_1, v_2, \ldots . v_p$ be the vertices of a cycle C_p

1.Case 1. p is even:

Let (v_i, v_j) be any arbitrary vertices of C_p. If $d(v_i, v_j) = 2$, then $(v_i{}^*, v_j{}^*)$ is an edge in $T_2(C_p)$.That is $(v_i{}^*, v_j{}^*)$ is adjacent in $T_2(C_p)$, if $|i-j| \equiv 0 \, mod \, 2$. For each vertex$v_i{}^*$ there exist exactly two vertices$v_j{}^*$and$v_k{}^*$ where $d(v_i{}^*, v_j{}^*) = d(v_i{}^*, v_k{}^*) = 2$. Since v_i is a vertex in cycle, $d(v_i) = 2$. Therefore $d(v_i{}^*) = d(v_i) + 2 = 4$, which implies T_2 transformation of cycle is 4-regular, if p is even. Total number of edges added in $T_2(C_p)$ is p. Therefore in $T_2(C_p)$, p is even, we can add p edges which are not in C_p.

2. Case 2. p is odd:

$(v_i{}^*, v_j{}^*)$ is adjacent in T_2 transformation, if $|i-j| \equiv 0 \, mod \, 2$ and $|i-j| \equiv 1 \, mod \, 2$. For each vertex $v_i{}^*$ there exist exactly two vertices at distance 2. Therefore $d(v_i{}^*) = d(v_i) + 2 = 4$, which implies T_2 transformation of cycle is 4 - regular, if p is odd. Total number of edges added is$\frac{p+1}{2} + \frac{p-1}{2} = p$.Therefore in $T_2(C_p)$, we can add p edges which are not in C_p. Thus T_2 transformation of C_p is a 4-regular graph and $T_2(C_p) = C_p + p$ chords, if $p \geq 5$.■

F. *Observation 1.6.*

1. $T_2(C_4) \cong K_4$.
2. $T_2(C_4) \cong C_4 + 2$ chords.
3. $T_2(G) \cong G^{(2)}$

G. *Theorem 1.7.*

Let $C_p$be a cycle with p vertices. Then

i) $T_{\frac{p}{2}}(C_p) \cong C_p + \frac{p}{2}$ chords, p is even.

ii)$T_{\frac{p-1}{2}}(C_p) \cong C_p + p$ chords, p is odd.

Proof. Let $v_1, v_2, \ldots . . v_p$ be the vertices of a C_p.

i) $\gamma = \frac{p}{2}$, p is even.

For each vertex v_i in C_p, there is exactly one vertex is at a distance $\frac{p}{2}$. That is $d(v_1, v_{\frac{p}{2}+1}) = \frac{p}{2}$, $d(v_2, v_{\frac{p}{2}+2}) = \frac{p}{2}$ etc. In general, $(v_i{}^*, v_j{}^*)$ is adjacent in $T_{\frac{p}{2}}(C_p)$, if $|i-j| \equiv 0 \, mod \, \frac{p}{2}$. Therefore in $T_{\frac{p}{2}}(C_p)$ each vertex is adjacent with exactly one vertex and $d(v_i{}^*) = 2 + 1 = 3$. Total number of edgesadded is $\frac{p}{2}$. Therefore $T_{\frac{p}{2}}(C_p)$ is 3-regular and $T_{\frac{p}{2}}(C_p) \cong C_p + \frac{p}{2}$ chords, when p is even.

ii) $\gamma = \frac{p-1}{2}$, p is odd.

In $T_{\frac{p-1}{2}}$ transformation of $C_p (v_i{}^*, v_j{}^*)$ is adjacent if $|i-j| \equiv 0 \, mod \, \frac{p+1}{2}$ and $|\, i-j| \equiv 0 \, mod \, \frac{p-1}{2}$. That is each vertex $v_i{}^*$isadjacent with 2 vertices in $T_{\frac{p-1}{2}}(C_p)$.Therefore degree of each vertex $v_i{}^* = 2 + 2 = 4$. Total number of edges added is p. Therefore $T_{\frac{p-1}{2}}(C_p)$ is 4-regular and $T_{\frac{p-1}{2}}(C_p) \cong C_p + p$chords, where p is odd. ■

Example. (p is even) Consider the graph C_8 as shown in the figure 40. $d(v_1,v_5) = d(v_2,v_6) = d(v_3,v_7) = d(v_4,v_8)4$. In $T_4(C_8)$, (v_1^*,v_5^*),(v_2^*,v_6^*),(v_3^*,v_7^*), (v_4^*,v_8^*) are edges. Each vertex in $T_4(C_8)$ has degree 3. Therefore $T_4(C_8)$ is 3-regular and $T_4(G) = G + 4$ chords.

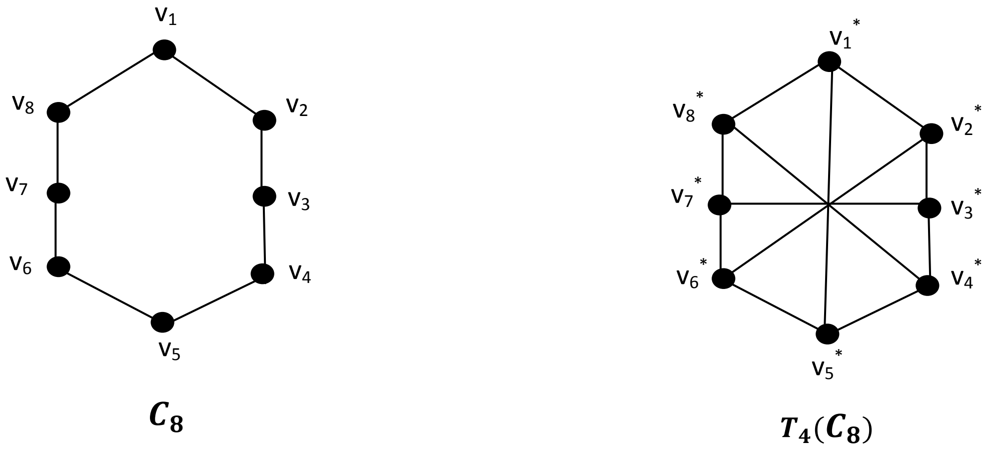

Figure.2

H. Theorem 1.8.

If $\Delta(G) = p - 1$, then $T_2(G) \cong K_p$.

Proof. Let $v_1, v_2, \ldots v_p$ be the vertices of a graph G. If $\Delta(G) = p - 1$, there exists at least one vertex has degree $p - 1$ and hence diameter of $G = 2$. It follows that the distance between any two non-adjacent vertices is 2. In T_2 transformation, (u^*, v^*) is an edge where $(u, v) \notin E(G)$. It is true for all u and v in G.Therefore $T_2(G) \cong K_p$. ■

I.Theorem 1.9.

Let G be a (p, q) graph with diameter d.

Then $T_2(G) \cup T_3(G) \ldots \cup T_d(G) \cong K_p$

Proof. Let G be a (p, q) graph with diameter d, which implies max $d(u, v) = d$. Let v_i and v_j be any two arbitrary vertices in G, where $d(v_i, v_j) = 2$. In $T_2(G)$, $v_i^*$and $v_j^*$are adjacent. Similarly, for $T_3(G)$, all v_i^*, andv_j^* are adjacent if $d(v_i, v_j) = 3$ etc., in $T_d(G)$, all (v_i^*, v_j^*) are adjacent if $d(v_i, v_j) = d$. Therefore, in $T_2(G) \cup T_3(G) \ldots \cup T_d(G)$, every pair of vertices are adjacent, and so $T_2(G) \cup T_3(G) \ldots \cup T_d(G) \cong K_p$.■

I. Theorem 1.10.

Let P_n be a path with n vertices. The following holds.

i) γ transformation of a path is linear, if $\gamma = n - 1$.

ii) $T_\gamma(P_n) = P_n + (n - 2)$ chords, if $\gamma = 2$.

Proof. *i)* Let P_n be a path with vertices $v_1, v_2, \ldots v_n$. Consider the vertex v_1, clearly $d(v_1, v_n) = n - 1$. There are no other vertices v_i, v_j in P_n having $d(v_i, v_j) = n - 1$. That is $T_{n-1}(P_n) = P_n + (v_1^*, v_n^*)$.Therefore $T_{n-1}(P_n) = P_n + 1$ chord, i.e., $T_\gamma(P_n)$ is linear, when $\gamma = n - 1$.

ii) Consider $T_\gamma(P_n)$ for $\gamma = 2$.InP_n, $d(v_1, v_3) = d(v_2, v_4) = d(v_3, v_5) = d(v_4, v_6) \ldots d(v_{n-3}, v_{n-1}) = d(v_{n-2}, v_n) = 2$. That is $d(v_i, v_j) = 2$, *if* $|i - j| \equiv 0 \bmod 2$. Therefore $n - 2$ pair of non-adjacent vertices have distance 2 in P_n. Therefore $T_\gamma(P_n) = P_n + n - 2$ chords, for $\gamma = 2$. ■

III. CONCLUSION

In this paper we studied the transformation of certain graphs. In$T_\gamma(G)$ the distance between two non-adjacent vertices is less thanγ. Using the properties of $T_\gamma(G)$, we can study the gene network coherences.

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