Boolean Algebra and Logic Gates

Subhrajit Sarma[#1], Rama Kanta Bhuyan*[2]
[#]*Student, Department of Mathematics, Gauhati University, Guwahati, Assam, India*
[*]*Associate Professor, Department of Mathematics, Tihu College, Tihu,Assam, India*

Abstract
Boolean algebra or switching algebra is used to simplify Boolean equations and functions before they are used to design a logic gate. Boolean algebra is nothing butalogic to perform mathematical operations on binary numbers i.e., '0' and '1'. Boolean algebra contains three basic operators viz. AND, OR and NOT. In this paper, we are trying to show these operations and some basic laws(i.e., commutative, associative and distributive) and the corresponding logic gate representations.

Keywords - *Boolean algebra, Boolean functions, logic gate, basic operations, basic laws*

I. INTRODUCTION

Boolean algebra is the mathematical foundation of logic design. It was invented by great mathematician George Boole in the year 1847. In 1937, the logic design of the Boolean algebra was given by Claude Shannon in his award winning paper "A symbolic Analysis of Relay and Switching Circuits" (Balabanian and Carlson, 2001; Camara, 2010)

Boolean functions and their different operations are shown by certain diagrams (Muller, 1954). These diagrams are called logic gates. From the beginning, Boolean algebra and logic gates has drawn special attention to logicians, mathematicians, philosophers, computer scientists and programmers, electronic engineers etc. Boolean algebra is the backbone of computer technology.

II. BOOLEAN ALGEBRA AND FUNCTION

Boolean algebra is an abstract mathematical structure where the variables take only two values- true or false, usually denoted by 1 and 0 respectively (Gregg, 1998).Instead of elementary algebra, where the primary operations are addition and multiplication, the main operations of Boolean algebra are conjunction (denoted as ∧), disjunction (denoted by∨) and negation (denoted by¬). Boolean algebra describes logical relations in the same way elementary algebra describes numerical relations.

Boolean function of $f(x_1, x_2, x_3, \dots, x_n)$,is a mapping $\{0,1\}^n \rightarrow \{0,1\}$ where $x_i's$ are Boolean variables .There are many different ways to represent a Boolean function (eg. Truth table, Boolean expression, logic circuits etc.) (Millan et al., 1998

III. LOGIC GATE

One of the most important applications of Boolean algebra is to design and simplify electronic circuits mainly used in computers. In a digital computer, the smallest memory unit is called a bit. All the programs and data in the computer are stored in the combinations of a bit. A bit can take only one value from 0 and 1.

The gates or devices that implement a Boolean function are called a logic gate. It does a logical operation on one or more bits of input and gives a simple bit as an output. These gates are the fundamental building blocks of any kind of digital system. Some commonly used logic gates are- AND, OR, NOT, NAND, NOR, XORetc (Snider et al., 19999)

IV. BASIC OPERATIONS OF BOOLEAN ALGEBRA AND LOGIC GATE

i) NOT operation
The notation used for NOT is "''or "¬".

A	A'

0	1
1	0

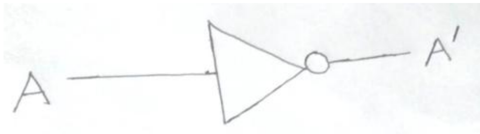


Fig-1- schematic logic gate diagram of NOT operation

ii) AND operation

The notation used for AND is "." Or "∧"

A	B	C=A.B
1	1	1
1	0	0
0	1	0
0	0	0

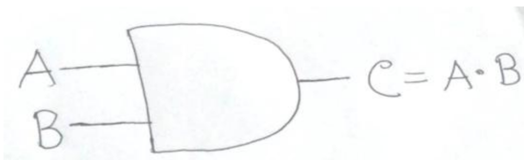


Fig-2- schematic logic gate diagram of AND operation

iii) OR operation

The notation used for OR is "+" or "∨"

A	B	C=A+B
1	1	1
1	0	1
0	1	1
0	0	0

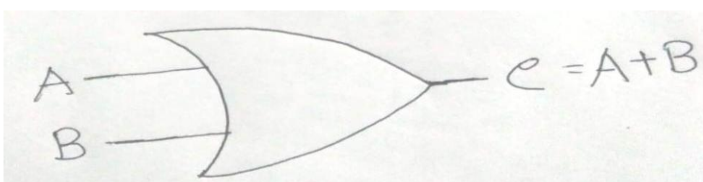


Fig-3- schematic logic gate diagram of OR operation

V. Basic Laws of Boolean algebra and corresponding logic gate

i) Commutative Law

a) A.B=B.A

A	B	A.B	B.A
1	1	1	1
1	0	0	0
0	1	0	0
0	0	0	0

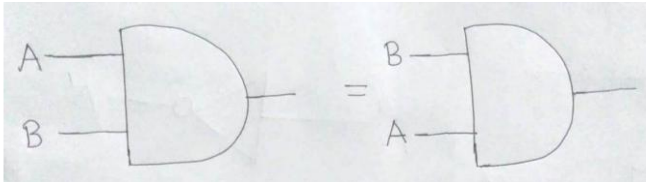
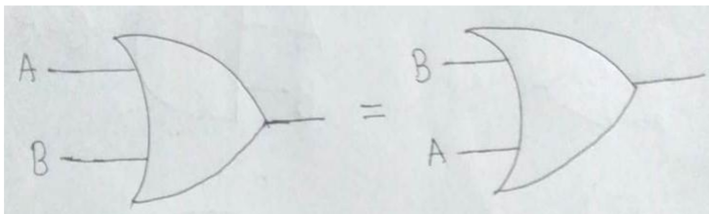


Fig-4a- schematic logic gate diagram of commutative law (A.B=B.A)

b) A+B=B+A

A	B	A+B	B+A
1	1	1	1
1	0	1	1
0	1	1	1
0	0	0	0



c)

Fig-4a- schematic logic gate diagram of commutative law (A+B=B+A)

ii) Associative Law

a) (A.B).C=A.(B.C)=A.B.C

A	B	C	A.B	(A.B).C	B.C	A.(B.C)	A.B.C
1	1	1	1	1	1	1	1
1	1	0	1	0	0	0	0
1	0	1	0	0	0	0	0
1	0	0	0	0	0	0	0
0	1	1	0	0	1	0	0
0	1	0	0	0	0	0	0
0	0	1	0	0	0	0	0
0	0	0	0	0	0	0	0

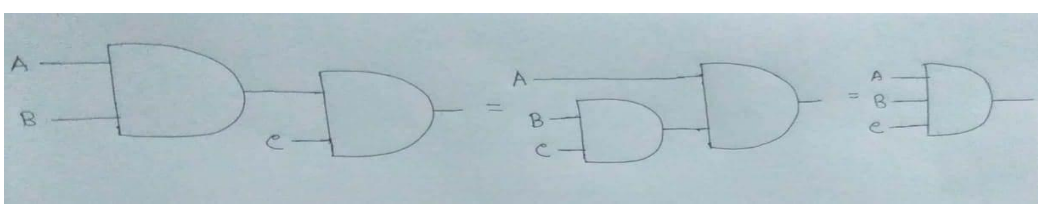


Fig-5a- schematic logic gate diagram of associative law (A.B).C=A.(B.C)=A.B.C

b) (A+B)+C=A+(B+C)=A+B+C

A	B	C	A+B	(A+B)+C	B+C	A+(B+C)	A.+B+C
1	1	1	1	1	1	1	1
1	1	0	1	1	1	1	1
1	0	1	1	1	1	1	1
1	0	0	1	1	0	1	1
0	1	1	1	1	1	1	1

0	1	0	1	1	1	1	1
0	0	1	0	1	1	1	1
0	0	0	0	0	0	0	0

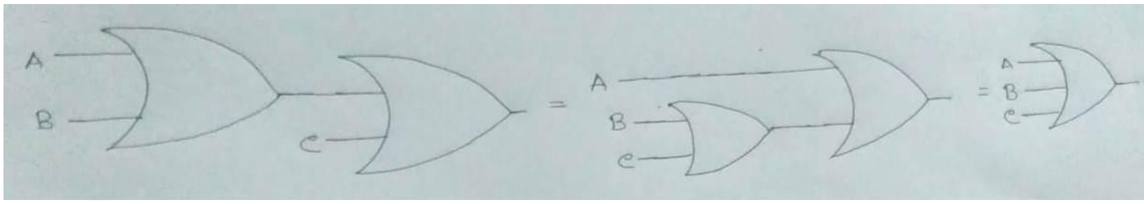


Fig-5b- schematic logic gate diagram of associative law (A+B)+C=A+(B+C)=A+B+C

iii) Distributive Laws

a) A.(B+C)=A.B+A.C

A	B	C	B+C	A.(B+C)	A.B	A.C	A.B+A.C
1	1	1	1	1	1	1	1
1	1	0	1	1	1	0	1
1	0	1	1	1	0	1	1
1	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0
0	1	0	1	0	0	0	0
0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0

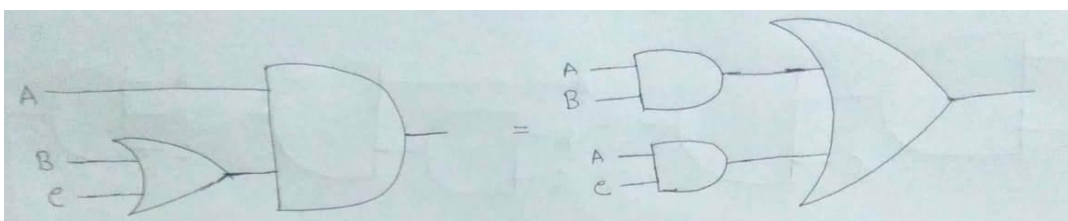


Fig-6a- schematic logic gate diagram of distributive law A.(B+C)=A.B+A.C

b) A+B.C=(A+B).(A+C)

A	B	C	B.C	A+B.C	A+B	A+C	(A+B).(A+C)
1	1	1	1	1	1	1	1
1	1	0	0	1	1	1	1
1	0	1	0	1	1	1	1
1	0	0	0	1	1	1	1
0	1	1	1	1	1	1	1
0	1	0	0	0	1	0	0
0	0	1	0	0	0	1	0
0	0	0	0	0	0	0	0

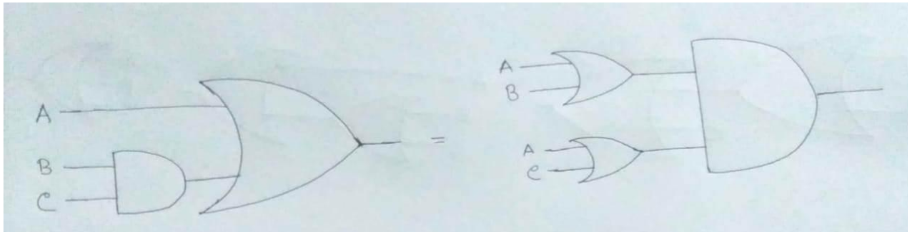


Fig-6b- schematic logic gate diagram of distributive law A+B.C=(A+B).(A+C)

V. CONCLUSION

Boolean algebra is an interesting area under the mathematical science having a wide range of applications in engineering. It is the core of circuits which are used in electronics. Thus Boolean algebra opens up a new directions of research connecting different branches of technology with mathematics.

ACKNOWLEDGEMENT

The authors are thankful to Miss Riju Sarma and Department of Mathematics, Gauhati University and Department of Mathematics, Tihu College, Tihu for their valuable help and support in the present investigation.

REFERENCES

[1] N. Balabanian and B. Carlson, Digital logic design principles, John Wiley, pp- 39-40, 2001.
[2] J. A. Camara, Electrical and Electronics reference manual for the electrical and computer PE exam, 2010.
[3] J. Gregg, Ones and zeros: Understanding Boolean algebra, digital circuits, and the logic of sets, Wiley- IEEE press, 1998.
[4] D. E. Muller, Application of Boolean algebra to switching circuit design and to error detection, Transactions of the IRE professional group on electronic computers,pp- 6-12, 1954.
[5] W. Millan, A. Clark, Ed dawson, Heuristic design of cryptographically strong balanced Boolean functions, International Conference on the theory and applications of cryptographic techniques, pp- 489-499, 1998.
[6] G. L. Snider, A. O. Orlov, I. Amlani, G. H. Bernstenin, C. S. Lent, J.L. Merz, W. Porod, Quantum- dot cellular automata: Line and majority logic gate, Japanese journal of Applied physics, Vol. 38, No12, 1999.