On g_γ-τ-Connectedness and g_γ-τ Disconnectedness in Topological Spaces

K.Priyanka#1, M.Rajeshwari*2
[1]*Assistant Professor, Department of Mathematics ,*
NadarSaraswathi College Of Arts and Science, Theni.
M.Sc., Mathematics, Department of Mathematics ,
NadarSaraswathi College Of Arts and Science, Theni.

Abstract

The aims of this paper is to introduce new approach of separate sets, connected sets and disconnected sets called g_γ–τ-Separate sets, g_γ–τ-Connected sets and g_γ–τ-Disconnected sets of topological spaces with the help of g_γ–open sets and g_γ–closedsets. On the basis of new introduce approach, some relationship of g_γ–τ-Connected sets,g_γ–τ-Disconnected sets with g_γ–τ -separate sets have been investigated thoroughly.

Keywords - g_γ–τ –Separate sets, g_γ–τ- Disconnected sets, g_γ–τ- Connected sets,g_γ–Open sets,g_γ–Closed sets ,g_γ–Interior, g_γ–Closure.

I. INTRODUCTION

In 1991, II-Ogata [17] introduced the concept of an operation γon a topological spaces based on the idea of the γ-operation as initiated by S.Kasahara[15] and consequently be introduced γ-open sets. Several research papers published in recent years using γ-operator due to II-Ogata [17]. The properties of γ-open sets and closed sets generalized many classical notions was introduce by Jayanthi V, Janaki C[14].There are several natural approaches that can take to rigorously the concept of connectedness for the topologicalspaces. These concepts play a significant role in application of geographic information system studied by Egenhofer and Franzosa (1991), topological modeling studied by Clementini et al (1994) and notion planning in robotics studied by Farber et al (2003). The generalization of open sets and closed sets as like g_γ-open sets and g_γ-closed sets was introduced by II-Ogata [17] which is nearly to open sets and closed sets respectively. These notions are play significant role in general topology. In this paper, the new approaches of separate sets, disconnected sets and connected sets called g_γ-separate sets,g_γ-disconnected sets and g_γ-connected sets with the help of g_γ-open sets and g_γ-closed sets are firstly introduced. Further some relationship concerning g_γ-τ-connected sets and g_γ-τ-disconnected sets with g_γ-τ-separate sets are also investigated. Throughout this paper (X,τ) and (X,τ_{g_γ}) will always be topological spaces. For a subset A of a topological space X,cl_{g_γ}(A),int_{g_γ}(A) denotes the g_γ- closure and g_γ $-$interior respectively and g_γ-open set for topologyτ_{g_γ} on X.

MAIN RESULTS

II-SOME DEFINITIONS ON g_γ-τ-CONNECTEDNESS AND g_γ-τ-DISCONNECTEDNESS IN TOPOLOGICAL SPACES

Definition: 2.1

A subset A of a topological spaces (X,τ) is called a generalized closed sets if, *cl*(A)⊆H , whenever A⊆H and H is open in X. The complement of generalized closed sets are called a generalized open sets.

Definition: 2.2

Let (X,τ_{g_γ}) be a topological space and A be a subsetof (X,τ_{g_γ}) is said to beg_γ $-$open if ,*A⊆Int(cl(A))* ∪ *cl(Int(A))* .The family of all τ_{g_γ} of g_γ-open sets of (X,τ_{g_γ}) is a topology on X, then τ_{g_γ} is finer than τ . τ_{g_γ} be a collection of g_γ -open sets.

Definition: 2.3

Let (X, τ_{g_γ}) be a topological space and A be a subsetof (X, τ_{g_γ}) is said to be g_γ–closedif*cl(int(A))*∩*int(cl(A))*⊆*A*.The complement of g_γ–open set is called g_γ–closed set.

Definition: 2.4

The generalized γ –interior of a set Adenoted by $g_\gamma int(A)$ is the union of allg_γ-open sets contained in A.

Definition: 2.5

The generalized γ-closure of a set A denoted by, g_γ cl (A) is the intersection of all g_γ-closed sets contains A.

Definition: 2.6

Let(X,τ) and (X,τ_{g_γ}) be a topological spaces. Then the subset A of X is said to be g_γ–τ-connected, if X cannot be expressed as a disjoint union of two non-empty open sets inX.

Definition: 2.7

Let(X,τ) and (X,τ_{g_γ}) be a topological spaces. Then the subset A of X is said to be g_γ–τ-disconnected, if X can be expressed as a disjoint union of two non-empty open sets in X. If there exist a non-empty disjoint open sets G_{g_γ} and H_{g_γ} in τ_{g_γ} such that,

$G_{g_\gamma} \cup H_{g_\gamma}$ =X and$G_{g_\gamma} \cap H_{g_\gamma}=\emptyset$.

Definition: 2.8

Let(X,τ) and (X,τ_{g_γ}) be a topological spaces. Then the subset A and B of (X,τ_{g_γ}) are said to be g_γ–τ-separate sets if and only if, A and B are non-empty set.

A$\cap$ cl_{g_γ} (B) and B$\cap$ cl_{g_γ} (A) are non-empty.

Definition: 2.9

Let (X,τ) and (X,τ_{g_γ}) be a topological spaces. Then the subset A of X is said to be g_γ–τ-disconnected,if there exists G_{g_γ} and H_{g_γ} in τ_{g_γ} such that,

i)A$\cap$ $G_{g_\gamma}$and A$\cap$ $H_{g_\gamma} \neq \emptyset$

ii)(A$\cap$ G_{g_γ}) $\cap$ (A$\cap$ H_{g_γ})= $\emptyset$

iii)(A$\cap$ G_{g_γ}) $\cup$ (A$\cap$ H_{g_γ})=A.

Definition: 2.10

Let (X,τ_{g_γ}) be a topological space and A be a non-empty subset of X. Let G_{g_γ} be arbitrary in τ_{g_γ}.Then the collection,

$$\tau^A_{g_\gamma}=\{G_{g_\gamma} \cap A; G_{g_\gamma} \in \tau_{g_\gamma}\}$$

is a topology on A, called the subspace or relative topology of τ_{g_γ}.

Definition: 2.11

Let τ_{g_γ} and τ'_{g_γ}be a topological spaces on X. If $\tau'_{g_\gamma} \supseteq \tau_{g_\gamma}$. Then it is known as $\tau'_{g_\gamma}$is finer than (or) stronger than τ_{g_γ}. If $\tau'_{g_\gamma} \subseteq \tau_{g_\gamma}$. Then it is known as τ_{g_γ} is coarser than (or) weaker than τ'_{g_γ}. Then τ_{g_γ} is comparable with τ'_{g_γ}. If either $\tau'_{g_\gamma} \supset \tau_{g_\gamma}$ than $\tau'_{g_\gamma}$is called strictly finer than (or) stronger than τ_{g_γ} .If $\tau'_{g_\gamma} \subset \tau_{g_\gamma}$, than $\tau_{g_\gamma}$is called strictly coarser than τ'_{g_γ}.

Definition: 2.12

Two subsets A and B form a separation or partition of a set E in a topological space (X, τ_{g_γ}) if and only if,

i) E = A $\cup$ B

ii) A and B are non-empty

iii) A $\cap$ B = ϕ

iv) Neither A contains a limit point of B nor B contains a limit of A.If A and B form a separation of E, then we write E = A/B.

III-SOME RESULTS ON g_γ-τ-CONNECTEDNESS AND g_γ-τ-DISCONNECTEDNESS IN TOPOLOGICAL SPACES

Theorem: 3.1

A topological space (X,τ_{g_γ}) is g_γ–τ- connected if and only if it cannot be expressed as the union of two non-empty sets that are separated in X.

Proof:
Assumption:
X is g_γ–τ-connected.
Suppose X is not connected. Then there are non-empty, disjoint open sets Y and Z such that X = Y$\cup$ Z. Then Y and Z are closed so that Cl(Y) $\cap$ Z = Y$\cap$ Z = ϕ and
Y$\cap$Cl(Z) = Y$\cap$ Z = ϕ.
Therefore U and V are separated in X.
Suppose, now that there are non-empty subsets A and B such that X = A $\cup$ B and
Cl(A)$\cap$ B = A$\cap$Cl(B) = ϕ.
Since X = A $\cup$ B and Cl(A) $\cap$ B = ϕ.
$\Rightarrow$ Cl(A)$\subseteq$ A so that A is closed.
Similarly, Cl(B) $\subseteq$ B is closed.
Therefore A and B are also open and hence X is not connected.
Which is contradiction to ourassumption,
Therefore X is g_γ–τ-connected.

Remark: 3.1
If A and B form a separation of the topological space (X, τ_{g_γ}) then A and B are both open and closed.

Theorem: 3.2

Let (X, τ) and (X,τ_{g_γ}) be a topological spaces, then (X, τ) is g_γ–τ- connected if and only if there is no non-empty proper subset of X which is both g_γ -open and g_γ –closed.
Proof:

Necessity:

Let (X,τ_{g_γ}) be g_γ–τ-connected. Then by definition of g_γ–τ-connected, there exist a two non-empty sets M_{g_γ} and N_{g_γ}in τ_{g_γ}. Then N_{g_γ} is open in τ_{g_γ}. Show that M_{g_γ}=X - N_{g_γ}, but it is g_γ-open. Hence $M_{g_\gamma}$is no non-empty proper subset of X which is both g_γ -open and g_γ –closed.

Sufficiency:

Suppose A is no non-empty proper subset of X such that it is g_γ -open and g_γ –closed. Now A is non – empty g_γ-open, show that X – A is non-empty g_γ-open, suppose B=X – A. Thus A and B are non-empty disjoint g_γ-open subset of X.
Consequently, X is g_γ–τ- connected.

Theorem: 3.3
If Y is a connected subset of a topological space (X, τ_{g_γ}), which has a disconnection X=A∪B, then either A $\subseteq$ Y or B $\subseteq$ Y.
Proof:
LetY= Y $\cap$ X = Y $\cap$ (A $\cup$ B)
Y = (Y$\cap$ A) $\cup$ (Y $\cap$ B)
Since X=A∪B,
By definition of g_γ- τ separated sets,
[(Y $\cap$ A) $\cup$ Cl(Y $\cap$ B)] $\cup$ [Cl(Y $\cap$ A) $\cap$ (Y $\cap$ B)]$\subseteq$ [A $\cap$ Cl(B)] $\cup$ [Cl(A) $\cap$ B] $\neq\phi$
Since A and B are separations.
Thus if we assume that both Y $\cap$ A and Y $\cap$ B are non-empty,
we have , Y = (Y $\cap$ A) $\cup$ (Y $\cap$ B). Hence either Y $\cap$ A is non- empty so that either B $\subseteq$ Y, or Y$\cap$ B is non-empty so that either A$\subseteq$ Y.

Theorem : 3.4
If (X, τ_{g_γ}) isindiscrete topological space, then any subset of X is connected.
Proof:

Let C$\subseteq$ X.

Then the subset C is connected.

In thiscase to prove by contradiction.

Let C=A∪B

Where A and B are disjoint non-empty subset (open $\in \tau_{indiscrete}$) of X.

By definition of indiscrete topology,

$\boldsymbol{\tau}$**={X,∅}**

Since X≠ ∅, τ_{g_γ}={X,∅} is in τ

Put A=X and B=∅

C=A∪B

C =X∪ ∅

C =X

Which is a contradiction,

∴ C is connected. Every subset of X is connected.

Every indiscrete space is connected.

Let every X be an indiscrete space, then X is only non-empty open set.

Theorem: 3.5

If (X, τ) is a disconnected space and (X,τ_{g_γ}) is a topological space, then (X,τ_{g_γ}) is $g_\gamma - \tau -$ disconnected.

Proof:

Let (X, τ) is a disconnected and τ_{g_γ} is strictly finer than τ,then by definition of g_γ-open and g_γ-closed set. $\tau_{g_\gamma} \supset \tau$,

τ is a subspace of τ_{g_γ}. Since τ is disconnected, $\tau_{g_\gamma}$is also disconnected.

∴ $g_\gamma - \tau -$disconnected.

Theorem : 3.6

A subset Y of a topological space $X$is$g_\gamma - \tau$ -disconnected if and only if it is union of two $g_\gamma - \tau -$separate sets.

Proof:

Necessity:

Suppose Y= A∪B, where $A$and B are $g_\gamma - \tau$ -separate sets of X.

By the definition of $g_\gamma - \tau$ – disconnected, A∪B is $g_\gamma - \tau$ –disconnected. Hence Y is $g_\gamma - \tau$ –disconnected.

Sufficiency:

Let Y is $g_\gamma - \tau$ –disconnected. To prove that there exists two $g_\gamma - \tau$ –separate subsetsof A,B in X such that Y =A∪B. By assumption, Y is $g_\gamma - \tau$ –disconnected show that there exists a $g_\gamma - \tau$ –disconnection $G_{g_\gamma} \cup H_{g_\gamma}$ of Y. Thereforeby the definition of $g_\gamma - \tau$ -disconnected, we can say that there exists $G_{g_\gamma}, H_{g_\gamma}$in τ_{g_γ} such that,

Y∩ G_{g_γ} and Y∩ H_{g_γ} are non-empty;

(Y∩ G_{g_γ}) ∩(Y∩ H_{g_γ})=∅;

(Y∩ G_{g_γ}) ∪(Y∩ H_{g_γ})=Y.

Since (Y∩ G_{g_γ})and (Y∩ H_{g_γ}) are separated sets,

If we write, A=(Y∩ G_{g_γ})and B= (Y∩ H_{g_γ}), then by definition of $g_\gamma - \tau$ –separate set ,
$Y = A \cup B$.
Finally, we can say that there exist two$g_\gamma - \tau$ –separate sets A and B in X such that $Y = A \cup B$.

REFERENCES

[1] Andrijević D (1986) Semi-preopen sets,MatematičkiVesnik 38: 24-32.
[2] Arya SP (1990) Characterizations of s-normal spaces, Indian J Pure Appl Math 21: 717-719.
[3] Aslim G, CaksuGuler A, Noiri T (2006) On π gs-closed sets in topological spaces. ActaMathematicaHungarica 112: 275-283.
[4] A.V.Arhangelśkii and R.Wiegandt, Connectedness and disconnectedness in topology, Top. App.5 (1975).
[5] A.V.Arhangelśkii, On P-spaces extremal, disconnecctdness, and some results of Isbell J.R., and V.I.Malychin, Quest. Answ. Gen. Topol., 17 (2) (1999) 257-267.
[6] Clementini, E.; Sharma, J.; Egenhofer, M. J.Modelling topological spatial relations Strategies for queryprocessing. Computers and Graphics, v. 18,n. 6,p. 815-822, 1994.
[7] Dontchev J (1995) On generalizing semi-preopen sets. MemFacSci Kochi UnivSer A (Math) 16: 35-48.
[8] Dontchev J, Noiri T (2000) Quasi-normal spaces and π g-closed sets. Acta Math Hungar 89: 211-219.
[9] Egenhofer, M. J.; Franzosa, R. D. Point-SetTopological Spatial Relations.International Journal ofGeographical Information System, v. 5, n. 2, p. 161-174, 1991.
[10] Farber,M.;Tabachnikov,S.;Yuzvinsky,S.Topological robotics: Motion planning in projectivespaces. International Mathematical Research Notices, v. 34, p. 1853-1870, 2003.
[11] J.A.Guthrie, H.E.Stone and M.L.Wage, Maximal connected expansions of the reals, Proc. Amer. Math. Soc., 69 (1) (1978) 159-165.
[12] J.A.Guthrie, D.F.Reynolds and H.E.Stone, Connected expansions of topologies, Bull, Austral. Math. Soc., 9 (1973) 259-265.
[13] J.A.Guthrie and H.E.Stone, Spaces whose connected expansions preserve connectedsubsets, Fund. Math., 80 (1) (1973) 91-100.
[14] Jayanthi V, Janaki C (1963) On gr-closed set in topological spaces. Amer Math Monthly 70: 36-41.
[15] S.Kasahara, operation compact space .Math. Japon.,(1979),97-105.
[16] Kumar MV (2000) Between closed sets and g-closed sets. MemFacSci Kochi Univ (Math) 21: 1-19.
[17] II.ogata operations on topological spaces associated topology, Math.Japan.,36(1)(1991),175-184.
[18] Priyanka.K; Ananthi.V ;On $\gamma - \tau -$connectedness and disconnectedness in topological spaces,Bulletin of Mathematics and Statistical Research, Vol.6.Issue.4.2018(Oct-Dec).
[19] Sanjay Mishra; On$\alpha - \tau -$disconnectedness and$\alpha - \tau -$connectedness in topological spaces,Article inActaScientiarum Technology · July 2015
[20] G.T.Whyburn, Concerning the cut points of continua, Trans. Amer. Math. Soc., 30 (1928) 597-609.
[21] Zaitsev VI (1968) Some classes of topological spaces and their bicompact extensions. InDokladyAkademiiNauk 178: 778-779.