$\pi g\gamma^*$ closed Sets in Intuitionistic Fuzzy Topological Spaces

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Abstract

This paper is devoted to the study of intuitionistic fuzzy topological spaces. In this paper $\pi g\gamma^*$ closed sets and $\pi g\gamma^*$ open sets in intuitionistic fuzzy topological spaces are introduced. Also we have analyzed some properties of $\pi g\gamma^*$ closed sets and $\pi g\gamma^*$ open sets in intuitionistic fuzzy topological spaces

Key words and phrases - *Intuitionistic fuzzy topology, intuitionistic fuzzy $\pi g\gamma^*$ closed sets, intuitionistic fuzzy $\pi g\gamma^*$open sets.*

I. INTRODUCTION

The concept of intuitionistic fuzzy sets was introduced by Atanassov[1]. Coker [3] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper we have introduced intuitionistic fuzzy $\pi g\gamma^*$closed sets and intuitionistic fuzzy $\pi g\gamma^*$open sets.

II. PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$
where the functions $\mu_A(x): X \to [0,1]$ and $\nu_A(x): X \to [0,1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non-membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$. Denote by IFS(X), the set of all intuitionistic fuzzy sets in X.

Definition 2.2: [1] Let A and B be IFSs of the form $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{ \langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$, for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.

In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.

The complement A^c of an IFOS A in an IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4: [3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and an intuitionistic fuzzy closure are defined by

$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,

$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = (int(A))^c$ and $int(A^c) = (cl(A))^c$.

Definition 2.5: [6] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy semi closed set* (IFSCS in short) if $int(cl(A)) \subseteq A$.

Definition 2.6: [6] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be an *intuitionistic fuzzy semi open set* (IFSOS in short) if $A \subseteq cl(int(A))$.

Every IFOS in (X, τ) is an IFSOS in X.

Definition 2.7: [13] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy pre closed set* (IFPCS in short) if $cl(int(A)) \subseteq A$,

(ii) *intuitionistic fuzzy pre open set* (IFPOS in short) if $A \subseteq int(cl(A))$.

Definition 2.8: [13] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy α-open set* (IFαOS in short) if $A \subseteq int(cl(int(A)))$,

(ii) *intuitionistic fuzzy α-closed set* (IFαCS in short) if $cl(int(cl(A)) \subseteq A$.

Definition 2.9:[5] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy γ-open set* (IFγOS in short) if $A \subseteq int(cl(A)) \cup cl(int(A))$,

(ii) *intuitionistic fuzzy γ-closed set* (IFγCS in short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$.

Definition 2.10: [6] An IFS A of an IFTS (X, τ) is an

(i) *intuitionistic fuzzy regular open set* (IFROS in short) if $A = int(cl(A))$,

(ii) *intuitionistic fuzzy regular closed set* (IFRCS in short) if $A = cl(int(A))$.

Definition 2.11: [7] The union of IFROSs is called *intuitionistic fuzzy π-open set* (IFπOS in short) of an IFTS (X, τ). The complement of IFπOS is called *intuitionistic fuzzy π -closed set* (IFπCS in short).

Definition 2.12: [12] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized closed set* (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.13: [13] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized open set* (IFGOS in short) if A^c is an IFGCS in X.

Definition 2.14: [11] Let an IFS A of an IFTS (X, τ). Then semi closure of A (scl(A) in short) and semi interior of A (sint(A)) in short) are defined as

$scl(A) = \cap\{ K / K$ is an IFSCS in X and $A \subseteq K \}$

$sint(A) = \cup\{ K / K$ is an IFSOS in X and $K \subseteq A\}$.

Definition 2.15: [11] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized semi closed set* (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.16: [11] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized semi open set* (IFGSOS in short) if A^c is an IFGSCS in X.

The family of all IFGSCSs (resp. IFGSOSs) of an IFTS (X, τ) is denoted by IFGSC(X)(resp. IFGSO(X)).

Result 2.17: [11] Let A be an IFS in (X, τ), then
(i) scl(A) = A ∪ int(cl(A))
(ii) sint(A) = A ∩ cl(int(A))
If A is an IFS of X then $scl(A^c) = (sint(A))^c$

Definition 2.18: [9] Let an IFS A of an IFTS (X, τ). Then pre closure of A (pcl(A) in short) and pre interior of A (pint(A)) are defined as
pcl (A) = ∩{ K / K is an IFPCS in X and A ⊆ K }.
pint(A) = ∪{ K / K is an IFPOS in X and K ⊆ A}.

Definition 2.19: [9] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy generalized pre closed set* (IFGPCS in short) if pcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X. The complement of IFGPCS is called an *intuitionistic fuzzy generalized pre open set* (IFGPOS in short).

Definition 2.20: [9] Let an IFS A of an IFTS (X, τ). Then alpha closure of A (αcl(A) in short) and alpha interior of A (αint(A)) in short) are defined as
αcl (A) = ∩{ K / K is an IFαCS in X and A ⊆ K }.
αint(A) = ∪{ K / K is an IFαOS in X and K ⊆ A}.

Result 2.21: [9] Let A be an IFS in (X, τ). Then
(i) αcl(A) = A ∪ cl(int(cl(A)))
(ii) αint(A) = A ∩ int(cl(int(A)))

Definition 2.22: [9] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy alpha generalized closed set* (IFαGCS in short) if αcl(A) ⊆ U whenever A ⊆ U and U is an IFOS in X. The complement of IFαGCS is called an *intuitionistic fuzzy generalized open set* (IFαGOS in short).

Definition 2.23: [8] An IFS A of an IFTS (X, τ) is an *intuitionistic fuzzy $\gamma^\star$ generalized closed set* (IF$\gamma^\star$GCS in short) if cl(int(A)) ∩ int(cl(A)) ⊆ U whenever A ⊆ U and U is an IFOS in (X, τ). The complement of IF$\gamma^\star$GCS is called an *intuitionistic fuzzy generalized open set* (IFαGOS in short).

Definition 2.24: [12] Two IFSs are said to be q-coincident (A q B in short) if and only if there exists an element x ∈ X such that $\mu_A(x) > \nu_B(x)$ or $\nu_A(x) < \mu_B(x)$.

Definition 2.25: [12] For any two IFSs A and B of X, ($A_{\bar{q}}$ B) iff $A \subseteq B^c$.

III. INTUITIONISTIC FUZZY $\pi g\gamma^\star$ CLOSED SETS

In this section we introduce intuitionistic fuzzy $\pi g\gamma^\star$ closed sets and studied some of its properties.

Definition 3.1: An IFS A in (X, τ) is said to be an *intuitionistic fuzzy $\pi g\gamma^\star$ closed set* (IF$\pi g\gamma^\star$CS in short) if cl(int(A)) ∩ int(cl(A)) ⊆ U whenever A ⊆ U and U is an IFπOS in (X, τ).

Example 3.2: Let X = { a, b } and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X, where T = ⟨ x, (0.8, 0.4), (0.2, 0.3) ⟩. Then the IFS A = ⟨ x, (0.2, 0.1), (0.8, 0.8) ⟩ is an IF$\pi g\gamma^\star$CS in (X, τ).

Theorem 3.3: Every IFCS is an IF$\pi g\gamma^\star$CS but not conversely.

Proof: Let A ⊆ U and U is an IFπOS in (X, τ). Since cl(int(A)) ∩ int(cl(A)) ⊆ cl(A) and A is an IFCS, cl(int(A)) ∩ int(cl(A)) ⊆ cl(A) = A ⊆ U. Therefore A is an IF$\pi g\gamma^\star$CS in X.

Example 3.4: Let X = { a, b } and let τ = {$0_\sim$, T, $1_\sim$} where T = ⟨ x, (0.4, 0.3), (0.6, 0.6) ⟩. Then the IFS A = ⟨ x, (0.5, 0.3), (0.5, 0.6) ⟩ is an IF$\pi g\gamma^\star$CS but not an IFCS in X.

Theorem 3.5: Every IFαCS is an IF$\pi g\gamma^\star$CS but not conversely.

Proof: Let A ⊆ U and U is an IFπOS in (X, τ). That is U is IFOS in X. By hypothesis cl(int(A)) ∩ int(cl(A)) ⊆ cl(int(cl(A))) ∩ cl(int(cl(A))) ⊆ A ∩ A ⊆ U. Hence cl(int(A)) ∩ int(cl(A)) ⊆ U. Therefore A is an IF$\pi g\gamma^\star$CS in X.

Example 3.6: Let X = { a, b } and let τ = {$0_\sim$, T_1, T_2, $1_\sim$}, where T_1 = ⟨ x, (0.4, 0.2), (0.6, 0.7) ⟩, T_2 = ⟨ x, (0.8, 0.8), (0.2, 0.2) ⟩. Then the IFS A = ⟨x, (0.4, 0.4), (0.5, 0.5) ⟩ is an IF$\pi g\gamma^\star$CS but not an IFαCS in X.

Theorem 3.7: Every IFSCS is an IF$\pi g\gamma^\star$CS but not conversely.

Proof: Let A ⊆ U and U is an IFπOS in (X, τ). By hypothesis cl(int(A)) ∩ int(cl(A)) ⊆ cl(int(A)) ∩ A ⊆ U. Hence cl(int(A)) ∩ int(cl(A)) ⊆ U. Therefore A is an IF$\pi g\gamma^\star$CS in X.

Example 3.8: Let X = { a, b } and let τ = {$0_\sim$, T, $1_\sim$}, where T = ⟨ x, (0.3, 0.4), (0.6, 0.6) ⟩. Then the IFS A = ⟨x, (0.3, 0.3), (0.7, 0.6) ⟩ is an IF$\pi g\gamma^\star$CS but not an IFSCS in X.

Theorem 3.9: Every IFPCS is an IF$\pi g\gamma^\star$CS but not conversely.

Proof: Let A ⊆ U and U is an IFπOS in (X, τ). By hypothesis cl(int(A)) ∩ int(cl(A)) ⊆ int(cl(A)) ⊆A ⊆ U. Hence cl(int(A)) ∩ int(cl(A)) ⊆ U. Therefore A is an IF$\pi g\gamma^\star$CS in X.

Example 3.10: Let X = { a, b } and let τ = {$0_\sim$, T_1, T_2, $1_\sim$}, where T_1 = ⟨ x, (0.3, 0.3), (0.4, 0.6) ⟩, T_2 = ⟨ x, (0.7, 0.8), (0.3, 0.2) ⟩. Then the IFS A = ⟨x, (0.4, 0.4), (0.5, 0.5) ⟩ is an IF$\pi g\gamma^\star$CS but not an IFPCS in X.

Theorem 3.11: Every IFGCS is an IF$\pi g\gamma^\star$CS but its converse may not be true.

Proof: Let IFS A is an IFGCS in (X, τ). Let A ⊆ U and U is an IFπOS in X. That is U is IFOS in X. By hypothesis cl(A) ⊆ U. clearly cl(int(A)) ∩ int(cl(A)) ⊆ cl(A) ⊆ U. Which implies cl(int(A)) ∩ int(cl(A)) ⊆ U whenever A ⊆ U and U is an IFπOS. Therefore A is an IF$\pi g\gamma^\star$CS in X.

Example 3.12: Let X = { a, b } and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X, where T = ⟨ x, (0.2, 0.8), (0.3, 0.1) ⟩. Then the IFS A = ⟨ x, (0.1, 0), (0.4, 0.9) ⟩ is an IF$\pi g\gamma^\star$CS but not an IFGCS in X.

Theorem 3.13: Every IFGSCS is an IF$\pi g\gamma^\star$CS but its converse may not be true.

Proof: Let A $\subseteq$ U and U is an IFπOS in (X, τ). That is U is IFOS in X. By hypothesis scl(A) $\subseteq$ U. That is A$\cup$(int(cl(A)) $\subseteq$ U. Which implies int(cl(A)) $\subseteq$ U. Therefore cl(int(A)) $\cap$ int(cl(A)) $\subseteq$ U. Hence A is an IF$\pi g\gamma^{\star}$CS in X.

Example 3.14: Let X = { a, b } and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X, where T = ⟨ x, (0.4, 0.5), (0.3, 0.2) ⟩. Then the IFS A = ⟨ x, (0.3, 0.3), (0.6, 0.6) ⟩ is an IF$\pi g\gamma^{\star}$CS in X but not an IFGSCS in X.

Theorem 3.15: Every IFπCS is an IF$\pi g\gamma^{\star}$CS in (X, τ) but not conversely in general.

Proof: Let A be an IFπCS in (X, τ). Since every IFπCS is an IFCS, A is an IF$\pi g\gamma^{\star}$CS in (X, τ).

Example 3.16: Let X = {a, b} and T = ⟨ x, (0.5, 0.5), (0.5, 0.6) ⟩. Then τ = {$0_\sim$, T, $1_\sim$} is an IFT on X. Here the IFS A = ⟨ x, (0.5, 0.5), (0.5, 0.4) ⟩ is an IF$\pi g\gamma^{\star}$CS but not an IFπCS in (X, τ).

Theorem 3.17: Every IF$\pi g\gamma$*CS is an IFGPCS but its converse may not be true.

Proof: Let A $\subseteq$ U and U is an IFπOS in (X, τ). That is U is IFOS in X. By hypothesis pcl(A) $\subseteq$ U. Which implies int(cl(A)) $\subseteq$ U. That is cl(int(A))$\cap$int(cl(A)) $\subseteq$ U. Therefore A is an IF$\pi g\gamma$*CS in X.

Example 3.18: Let X = { a, b } and T = ⟨ x, (0.1, 0.8), (0.5, 0.1) ⟩ and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X. the IFS A = ⟨ x, (0.1, 0.3), (0.8, 0.7) ⟩ is an IF$\pi g\gamma$*CS but not an IFGPCS in X.

Theorem 3.19: Every IFγCS is an IF$\pi g\gamma$*CS in (X, τ) but not conversely in general.

Proof: Let A be an IFγCS in (X, τ). Let A $\subseteq$ U and U be an IFπOS in (X, τ). Now cl(int(A)) $\cap$ int(cl(A)) $\subseteq$ A $\subseteq$ U by hypothesis. Hence A is an IF$\pi g\gamma$*CS in (X, τ).

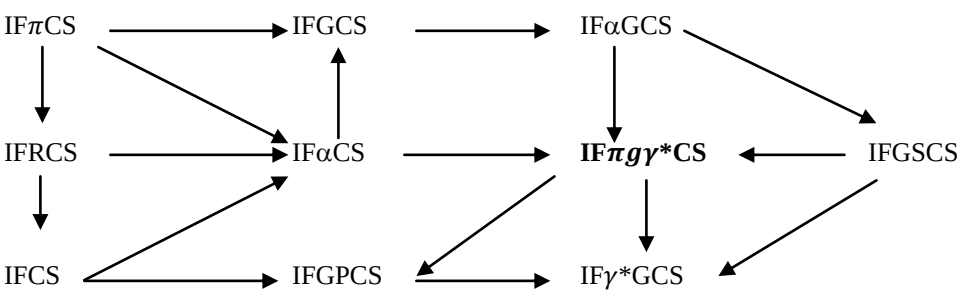
Example 3.20: Let X = {a, b}, T_1 = ⟨x, (0.5, 0.6), (0.5, 0.4)⟩ and T_2 =⟨x, (0.2, 0.2), (0.8, 0.8)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} is an IFT on X. Here the IFS A =⟨x, (0.4, 0.6), (0.6, 0.4)⟩ is an IFγ*GCS but not an IFγCS in (X, τ), as cl(int(A)) $\cap$ int(cl(A)) = = ⟨x, (0.5, 0.4), (0.5, 0.6)⟩ $\nsubseteq$ A.

Theorem 3.21: Every IF$\pi g\gamma$*CS is an IFγ*GCS in X. But not conversely in general.

Proof: Let A $\subseteq$ U and U is an IFπOS in (X, τ). That is U is IFOS in X. By hypothesis γcl(A) $\subseteq$ U. Which implies cl(int(A))$\cap$int(cl(A)) $\subseteq$ U. Therefore A is an IF$\pi g\gamma$*CS in X.

Example 3.22: Let X = { a, b } and T = ⟨ x, (0.6, 0.6), (0.4, 0.3) ⟩ and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X. the IFS A = ⟨ x, (0.4, 0.4), (0.6, 0.5) ⟩ is an IF$\pi g\gamma$*CS. Since cl(int(A))$\cap$int(cl(A)) $\nsubseteq$ U is IFOS in X , A is not an IFγ*GCS in X.

The following diagram implications are true:



Theorem 3.23 : Let (X, τ) be an IFTS. Then for every A ∈ IF$\pi g\gamma$*C(X) and for every B ∈ IFS(X), A ⊆ B ⊆ cl(int(A)) ⇒ B ∈ IF$\pi g\gamma$*C(X).

Proof: Let B ⊆ U and U be an IFπOS in X. Since A ⊆ B, A ⊆ U. Also B ⊆ cl(int(A)), cl(int(B)) ⊆ cl(int(A)). Also int(cl(B)) ⊆ int(cl(A)). Therefore cl(int(B)) ∩ int(cl(B)) ⊆ cl(int(A)) ⊆ U, by hypothesis. Hence B ∈ IF$\pi g\gamma$*C(X).

Theorem 3.24: If A is both an IFOS and an IF$\pi g\gamma$*CS in (X, τ) then A is an IFγCS in (X, τ).

Proof: Since A is an IFπOS and A ⊆ A, by hypothesis int(cl(A)) ∩ cl(int(A)) ⊆ A. Hence A is an IFγCS in (X, τ).

Theorem 3.25: If A is both an IFOS and an IF$\pi g\gamma$*CS in (X, τ) then A is an IFβCS in (X, τ).

Proof: Let A be an IFπOS and an IF$\pi g\gamma$*CS in X. Then int(cl(A)) ∩ cl(int(A)) ⊆ A, as A ⊆ A. Clearly int(cl(int(A))) = int(cl(int(A))) ∩ cl(int(A)) ⊆ int(cl(A)) ∩ cl(int(A)) ⊆ A. Therefore int(cl(int(A))) ⊆ A. Hence A is an IFβCS in (X, τ).

Theorem 3.26: If A is an IFπOS and an IF$\pi g\gamma$*CS in (X, τ), then A is an IFSCS in (X, τ).

Proof: Let A be an IFπOS and an IF$\pi g\gamma$*CS in X. That is A is an IFOS in X. Then int(cl(A)) ∩ cl(int(A)) ⊆ A, as A ⊆ A. Clearly int(cl(A)) = cl(A) ∩ int(cl(A)) = cl(int(A)) ∩ int(cl(A)) ⊆ A. This implies int(cl(A)) ⊆ A. Hence A is an IFSCS in (X, τ).

Theorem 3.27: For an IFS A in (X, τ), A is both an IFOS and an IF$\pi g\gamma$*CS in X, then A is an IFROS in X.

Proof: Let A be an IFπOS and an IF$\pi g\gamma$*CS in X. That A is a IFOS in X. Then int(cl(A)) = int(cl(A)) ∩ cl(A) = int(cl(A)) ∩ cl(int(A)) ⊆ A. Since A is an IFOS, it is an IFPOS and A ⊆ int(cl(A)). Therefore A = int(cl(A)) and A is an IFROS in X.

Theorem 3.28: If an IFS A of an IFTS (X, τ) is an intuitionistic fuzzy nowhere dense, then A is an IF$\pi g\gamma$*CS in X.

Proof: If A is an intuitionistic fuzzy nowhere dense, then by Definition, int(cl(A)) = $0_{\sim}$. Let A⊆ U where U is an IFπOS in X. Then cl(int(A)) ∩ int(cl(A)) = cl(int(A)) ∩ $0_{\sim}$ = $0_{\sim}$ ⊆ U and hence A is an IF$\pi g\gamma$*CS in X.

Theorem 3.29: Let A ⊆Y ⊆ X and suppose that A is an IF$\pi g\gamma$*CS in *X* then A is an IF$\pi g\gamma$*CS relative to Y.

Proof: Given that A ⊆Y ⊆ X and A is an IF$\pi g\gamma$*CS in X. Now let A ⊆Y ∩ U where U is an IFπOS in X. Since A is an IF$\pi g\gamma$*CS in X, A ⊆ U implies cl(int(A)) ∩ int(cl(A)) ⊆ U. It follows that Y ∩ [cl(int(A)) ∩ int(cl(A))] = $cl_y(int_y(A))$ ∩ $int_y(cl_y(A))$ ⊆ Y ∩ U = U. Thus A is an IF$\pi g\gamma$*CS relative to Y.

Theorem 3.30: Let F ⊆ A ⊆ X where A is an IFπOS and an IF$\pi g\gamma$*CS in X. Then F is an IF$\pi g\gamma$*CS in A if and only if F is an IF$\pi g\gamma$*CS in X.

Proof: Necessity: Let U be an IFπOS in X and F ⊆ U. Also let F be an IF$\pi g\gamma$*CS in A. Then clearly F ⊆ A ∩ U and A ∩ U is an IFπOS in A. Hence $int_A(cl_A(F))$ ∩ $cl_A(int_A(F))$ ⊆ A ∩ U and by theorem, A is an IFγCS. Therefore int(cl(A)) ∩ cl(int(A)) ⊆ A. Now int(cl(F)) ∩ cl(int(F)) ⊆ {int(cl(F)) ∩ cl(int(F))} ∩ A = $int_A(cl_A(F))$ ∩ $cl_A(int_A(F))$ ⊆ A ∩ U ⊆ U. That is int(cl(F)) ∩ cl(int(F)) ⊆ U, whenever F ⊆ U. Hence F is an IF$\pi g\gamma$*CS in X.

Sufficiency: Let V be an IFπOS in A such that F$\subseteq$V. Since A is an IFπOS in X, V is an IFπOS in X. Therefore int(cl(F)) $\cap$ cl(int(F)) $\subseteq$ V as F is an IF$\pi g\gamma$*CS in X. Thus, $\text{int}_A(\text{cl}_A(F)) \cap \text{cl}_A(\text{int}_A(F)) = \{\text{int}(\text{cl}(F)) \cap \text{cl}(\text{int}(F))\} \cap A \subseteq V \cap A \subseteq V$. Hence F is an IF$\pi g\gamma$*CS in A.

Remark 3.31: The intersection of any two IF$\pi g\gamma$*CSs need not be an IF$\pi g\gamma$*CS in (X, τ) in general.

Example 3.32: Let X = {a, b}, T_1 = ⟨x, (0.4, 0.2), (0.6, 0.8)⟩ and T_2 = ⟨x, (0.4, 0.4), (0.5, 0.5)⟩. Then $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$ is an IFT on X. Here the IFSs A = ⟨x, (0.4, 0.5), (0.5, 0.4)⟩ and B = ⟨x, (0.5, 0.2), (0.4, 0.6)⟩ are IF$\pi g\gamma$*CSs in (X, τ) but A $\cap$ B = ⟨x, (0.4, 0.2), (0.5, 0.6)⟩ is not an IF$\pi g\gamma$*CS in (X, τ)

Theorem 3.33: An IFS A of an IFTS (X, τ) is an IF$\pi g\gamma$*CS if and only if $A_{\bar{q}}$ F $\Rightarrow$ $(\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)))_{\bar{q}}$ F for every IFπCS F of X.

Proof: Necessity: Let F be an IFπCS in X and $A_{\bar{q}}$ F, then A $\subseteq F^c$, by Definition, F^c is an IFπOS. Then int(cl(A)) $\cap$ cl(int(A)) $\subseteq F^c$, by hypothesis. Hence by Definition, $(\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)))_{\bar{q}}$ F.

Sufficiency: Let U be an IFπOS such that A $\subseteq$ U. Then U^c is an IFπCS and A $\subseteq (U^c)^c$. By hypothesis, $A_{\bar{q}}$ $U^c \Rightarrow (\text{int}(\text{cl}(A)) \cap \text{cl}(\text{int}(A)))_{\bar{q}} U^c$. Hence int(cl(A)) $\cap$ cl(int(A)) $\subseteq (U^c)^c$ = U. Therefore int(cl(A)) $\cap$ cl(int(A)) $\subseteq$ U and A is an IF$\pi g\gamma$*CS in X.

IV. INTUITIONISTIC FUZZY $\pi g\gamma^\star$ OPEN SETS

In this section we have introduced intuitionistic fuzzy $\pi g\gamma^\star$ open sets and studied some of its properties.

Definition 4.1: An IFS A is said to be an *intuitionistic fuzzy* $\pi g\gamma^\star$ *open set* (IF$\pi g\gamma$*OS in short) in (X, τ) if the complement A^c is an IF$\pi g\gamma$*CS in X.
The family of all IF IF$\pi g\gamma$*OSs of an IFTS (X, τ) is denoted by IF$\pi g\gamma$*C(X).

Theorem 4.2: For any IFTS (X, τ), we have the following:

- Every IFOS is an IF$\pi g\gamma$*OS,
- Every IFαOS is an IF$\pi g\gamma$*OS,
- Every IFROS is an IF$\pi g\gamma$*OS,
- Every IFPOS is an IF$\pi g\gamma$*OS,
- Every IFγOS is an IF$\pi g\gamma$*OS
- Every IFπOS is an IF$\pi g\gamma$*OS. But the converses are not true in general.

Proof: Straight forward

Example 4.3: Let X = { a, b } and let $\tau = \{0_\sim, T, 1_\sim\}$ where T = ⟨ x, (0.4, 0.3), (0.6, 0.6) ⟩. Then the IFS A = ⟨ x, (0.5, 0.6), (0.6, 0.3) ⟩ is an IF$\pi g\gamma^\star$OS but not an IFOS in X.

Example 4.4: Let X = { a, b } and let $\tau = \{0_\sim, T_1, T_2, 1_\sim\}$, where T_1 = ⟨ x, (0.4, 0.2), (0.6, 0.7) ⟩, T_2 = ⟨ x, (0.7, 0.8), (0.2, 0.2) ⟩. Then the IFS A = ⟨x, (0.5, 0.5), (0.3, 0.4) ⟩ is an IF$\pi g\gamma^\star$OS but not an IFαOS in X.

Example 4.5: Let X = { a, b } and let τ = {$0_\sim$, T, $1_\sim$}, where T = ⟨ x, (0.3, 0.4), (0.6, 0.6) ⟩. Then the IFS A = ⟨x, (0.7, 0.6), (0.3, 0.4) ⟩ is an IF$\pi g\gamma^\star$OS but not an IFSOS in X.

Example 4.6: Let X = { a, b } and let τ = {$0_\sim$, T_1, T_2, $1_\sim$}, where T_1 = ⟨ x, (0.3, 0.3), (0.4, 0.6) ⟩, T_2 = ⟨ x, (0.7, 0.8), (0.3, 0.2) ⟩. Then the IFS A = ⟨x, (0.5, 0.5), (0.4, 0.4) ⟩ is an IF$\pi g\gamma^\star$OS but not an IFPOS in X.

Example 4.7: Let X = {a, b}, T_1 = ⟨x, (0.5, 0.6), (0.5, 0.4)⟩ and T_2 =⟨x, (0.2, 0.2), (0.8, 0.8)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} is an IFT on X. Here the IFS A =⟨x, (0.6, 0.4), (0.4, 0.6)⟩ is an IFγ*GOS but not an IFγOS in (X, τ).

Example 4.8: Let X = {a, b} and T = ⟨ x, (0.5, 0.5), (0.5, 0.6) ⟩. Then τ = {$0_\sim$, T, $1_\sim$} is an IFT on X. Here the IFS A = ⟨ x, (0.5, 0.4), (0.5, 0.5) ⟩ is an IF$\pi g\gamma^\star$OS but not an IFπOS in (X, τ).

Theorem 4.9: For any IFTS (X, τ), we have the following:

- Every IFGOS is an IF$\pi g\gamma$*OS,
- Every IFGSOS is an IF$\pi g\gamma$*OS,
- Every IFGPOS is an IF$\pi g\gamma$*OS,
- Every IF$\pi g\gamma$*OS is an IFγ*GOS. But the converses are not true in general.

Proof: Straight forward

Example 4.10: Let X = { a, b } and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X, where T = ⟨ x, (0.2, 0.8), (0.3, 0.1) ⟩. Then the IFS A = ⟨ x, (0.4, 0.9), (0.1, 0) ⟩ is an IF$\pi g\gamma^\star$OS but not an IFGOS in X.

Example 4.11: Let X = { a, b } and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X, where T = ⟨ x, (0.4, 0.5), (0.3, 0.2) ⟩. Then the IFS A = ⟨ x, (0.6, 0.6), (0.3, 0.3) ⟩ is an IF$\pi g\gamma^\star$OS in X but not an IFGSOS in X.

Example 4.12: Let X = { a, b } and T = ⟨ x, (0.1, 0.8), (0.5, 0.1) ⟩ and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X. the IFS A = ⟨ x, (0.8, 0.7), (0.1, 0.3) ⟩ is an IF$\pi g\gamma$*OS but not an IFGPOS in X.

Example 4.13: Let X = { a, b } and T = ⟨ x, (0.6, 0.6), (0.4, 0.3) ⟩ and let τ = {$0_\sim$, T, $1_\sim$} is an IFT on X. the IFS A = ⟨ x, (0.4, 0.4), (0.6, 0.5) ⟩ is an IF$\pi g\gamma$*CS. Since cl(int(A))∩int(cl(A)) ⊈ U is IFOS in X , A is not an IFγ*GCS in X.

Theorem 4.14: Let (X, τ) be an IFTS. Then for every IFS A and for every B∈IFRC(X), B ⊆ A ⊆ cl(int(B)) ∩ int(cl(B)) implies A is an IF$\pi g\gamma$*CS in X.

Proof: Let B be an IFRCS in X. Then B = cl(int(B)). By hypothesis, A ⊆ (cl(int(B)) ∩ int(cl(B))) = B ∩ int(cl(B)) ⊆ int(cl(B)) ⊆ int(cl(A)) as B ⊆ A. Therefore A is an IFPOS and by Proposition, A is an IF$\pi g\gamma$*OS in X.

V. APPLICATIONS OF INTUITIONISTIC FUZZY $\pi g\gamma$* CLOSED SETS

In this section we provide some applications of intuitionistic fuzzy $\pi g\gamma$* closed sets.

Definition 5.1: An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi\gamma$*$T_{1/2}$ (in short IF$\pi\gamma$*$T_{1/2}$) space if every IF$\pi g\gamma$*CS in X is an IFγCS in X.

Definition 5.2: An IFTS (X, τ) is said to be an intuitionistic fuzzy $\pi\gamma$*$cT_{1/2}$ (in short IF$\pi\gamma$*$cT_{1/2}$) space if every IF$\pi g\gamma$*CS in X is an IFCS in X.

Definition 5.3: An IFTS (X, τ) is an intuitionistic fuzzy $\pi\gamma$*$pT_{1/2}$ (IFγ*$pT_{1/2}$) space if every IF$\pi g\gamma$*CS is an IFPCS in X.

Example 5.4: Let X = {a, b}, T_1 = ⟨x, (0.4, 0.5), (0.6, 0.5)⟩ and T_2 = ⟨x, (0.6, 0.5), (0.4, 0.5)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} is an IFT on X. Hence is an IFγ*$pT_{1/2}$ space.

Definition 5.5: An IFTS (X, τ) is an intuitionistic fuzzy $\pi\gamma$*$gT_{1/2}$ (IFγ*$gT_{1/2}$) space if every IF$\pi g\gamma$*CS is an IFGCS in X.

Theorem 5.6: Every IF$\pi\gamma$*$pT_{1/2}$ space is an IF$\pi\gamma$*$T_{1/2}$ space.

Proof: Let (X, τ) be an IF$\pi\gamma$*$pT_{1/2}$ space and let A be an IF$\pi g\gamma$*CS in X. By hypothesis A is an IFPCS in X. Since every IFPCS is an IFγCS, A is an IFγCS in X. Hence (X, τ) is an IF$\pi\gamma$*$T_{1/2}$space.

Theorem 5.7: Every IFπγ*$cT_{1/2}$ space is an IFπγ*$gT_{1/2}$ space.

Proof: Let (X, τ) be an IFπγ*$cT_{1/2}$ space and let A be an IF$\pi g\gamma$*CS in X. By hypothesis A is an IFCS in X. Since every IFCS is an IFGCS, A is an IFGCS in X. Hence (X, τ) is an IFπγ*$gT_{1/2}$ space.

Theorem 5.8: Every IFπγ*$cT_{1/2}$ space is an IFπγ*$T_{1/2}$ space.

Proof: Let (X, τ) be an IFπγ*$cT_{1/2}$ space and let A be an IF$\pi g\gamma$*CS in X. By hypothesis A is an IFCS in X. Since every IFCS is an IFγCS, A is an IFγCS in X. Hence (X, τ) is an IFπγ*$T_{1/2}$ space.

Theorem 5.9: Every IFπγ*$cT_{1/2}$ space is an IFπγ*$pT_{1/2}$ space but not conversely in general.

Proof: Let (X, τ) be an IFπγ*$cT_{1/2}$ space and let A be an IFπγ*GCS in X. By hypothesis A is an IFCS in X. Since every IFCS is an IFPCS, A is an IFPCS in X. Hence (X, τ) is an IFπγ*$pT_{1/2}$ space.

Example 5.10: Let X = {a, b}, T_1 = ⟨x, (0.5, 0.5), (0.5, 0.6)⟩ and T_2 = ⟨x,(0.5, 0.6), (0.5, 0.5)⟩. Then τ = {$0_\sim$, T_1, T_2, $1_\sim$} is an IFT on X. Here (X, τ) is an IFπγ*$pT_{1/2}$ space but not an IFπγ*$cT_{1/2}$ space, since the IFS A = ⟨ x, (0.5, 0.8) (0.4, 0.2)⟩ is an IFπγ*GCS but not an IFCS in X.

Theorem 5.11: An IFTS (X, τ) is an IFπγ*$T_{1/2}$ space if and only if IFγO(X) = IF$\pi g\gamma$*O(X).

Proof: Necessity: Let A be an IF$\pi g\gamma$*OS in (X, τ), then A^c is an IF$\pi g\gamma$*CS in (X, τ). By hypothesis, A^c is an IFγCS in (X, τ). Therefore A is an IFγOS in (X, τ). Hence IFγO(X) = IF$\pi g\gamma$*O(X).

Sufficiency: Let A be an IFπgγ*CS in (X, τ). Then A^c is an IF$\pi g\gamma$*OS in (X, τ). By hypothesis A^c is an IFγOS in (X, τ) and therefore A is an IFγCS in (X, τ). Hence (X, τ) is an IFπγ*$T_{1/2}$ space.

Theorem 5.12: An IFTS (X, τ) is an IFπγ*$cT_{1/2}$ space if and only if IF$\pi g\gamma$*O(X) = IFO(X).

Proof: Necessity: Let A be an IF$\pi g\gamma$*OS in (X, τ), then A^c is an IF$\pi g\gamma$*CS in (X, τ). By hypothesis A^c is an IFCS in (X, τ). Hence A is an IFOS in (X, τ). Thus IFπγ*O(X) = IFO(X).

Sufficiency: Let A be an IF$\pi g\gamma$*CS in (X, τ). Then A^c is an IF$\pi g\gamma$*OS in (X, τ). By hypothesis A^c is an IFOS in (X, τ). Therefore A is an IFCS in (X, τ). Hence (X, τ) is an IF$\pi\gamma$*$cT_{1/2}$ space.

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