Nano Generalized Regular Continuity Innano Topological Space

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Abstract

In this paper a new form of continuous maps called Nano generalized regular (Ngr) Continuous maps has been analyzed and their relations with various other forms of continuous maps are analyzed. Further, Nano generalized regular closure and Nano generalized regular interior in Nano topological spaces are analyzed under the Continuous maps.

Key words - Ngr - open sets, Ngr- closed sets, Ngr - continuity, Ngr-interior, Ngr - closure .

I. INTRODUCTION

In 1970, Levine [9] introduced the concept of generalized closed sets as a generalization of closed sets in Topological space. In 2011, Sharmistha Bhattacharya [11] have introduced the notation of generalized regular closed sets in topological space. The notion of nano topology was introduced by Lellis Thivagar [7] which was defined in terms of approximations and boundary region of a subset of a universe using an equivalence relation on it and also defined nano closed sets, nano interior and nano closure.

II. PRELIMINARIES

Definition 2.1: [8]

Let U be a non empty finite set of objects called the universe and R be an Equivalence relation on U named as indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be an approximation space. Let $X \subseteq U$. Then,

i) The lower approximation of X with respect to R is the set of all objects which Can be for certain classified as X with respect to R and is denoted by $L_R(X)$. $L_R(X) = \cup \{R(x): R(x) \subseteq X,\ x \in U\}$ where R(x) denotes the equivalence Class determined by $x \in U$.

ii) The upper approximation of X with respect to R is the set of all objects which Can be possibly classified as X with respect to R and is denoted by $U_R(X)$.
$$U_R(X) = \cup \{R(x): R(x) \cap X \neq \emptyset, x \in U\}.$$

iii) The Boundary region of X with respect to R is the set of all objects which Can be classified neither as X nor as not-X with respect to R and it is denoted by $B_R(X)$. $B_R(X) = U_R(X) - L_R(X)$.

PROPERTY 2.2: [8]

If (U, R) is an approximation space and X, $Y \subseteq U$, then

1. $L_R(X) \subseteq X \subseteq U_R(X)$
2. $L_R(\emptyset) = U_R(\emptyset) = \emptyset$
3. $L_R(U) = U_R(U) = U$
4. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
5. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
6. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
7. $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
8. $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
9. $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
10. $U_R[U_R(X)] = L_R[U_R(X)] = U_R(X)$
11. $L_R[L_R(X)] = U_R[L_R(X)] = L_R(X)$

Definition 2.3: [8]

Let U be the Universe, R be an equivalence relation on U and the Nano Topology $\tau_R(X) = \{U, \emptyset, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. Then $\tau_R(X)$ satisfies the following axioms:

i. U and $\emptyset \in \tau_R(X)$
ii. The Union of the elements of any sub-collection of $\tau_R(X)$is in $\tau_R(X)$.
iii. The intersection of the elements of any finite subcollection of $\tau_R(X)$is in $\tau_R(X)$

Then $\tau_R(X)$ is a topology on U called the Nano topology on U with respect to X. $(U, \tau_R(X))$ is called the Nano topological space. Elements of the Nano topology are known as Nano open sets in U. Elements of $[\tau_R(X)]^C$ are called Nano closed sets. If $\tau_R(X)$is the Nano topology on U with respect to X, then the set $\mathcal{B} = \{ U, L_R(X), B_R(X) \}$ is the basis for $\tau_R(X)$.

Definition 2.4: [8]

If $(U, \tau_R(X))$ is a Nano topological space with respect to X where X⊆ U and if A ⊆ U, then the Nano interior of the set A is defined as the union of all nano open Subsets contained in A and is denoted by NInt(A). NInt(A) is the largest Nano open subset of A. The set A is said to be Nano Open if NInt(A) = A.

Definition 2.5: [8]

If $(U, \tau_R(X))$ is a Nano topological space with respect to X where X ⊆ U and if A ⊆ U, then the Nano Closure of the set A is defined as the intersection of all nano closed sets containing A and is denoted by NCl(A). NCl(A) is the smallest nano closed set containing A. The set A is said to be Nano closed if NCl(A) = A

Definition 2.6: [3]

A subset A of a Nano topological space $(U, \tau_R(X))$ is called Nano regular generalized closed set ifNrCl(A)⊆ V whenever A⊆V and V is Nano regular open.

The complement of Nano regular generalized closed set is called Nano regular generalized open set.

Definition 2.7: [3]

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The complement of Nano generalized regular closed set is called Nano generalized regular open set.

Definition 2.8: [1]

If $(U, \tau_R(X))$ is a Nano topological space with respect to X where X ⊆ U and if A ⊆ U, then the Nano generalized regular interior of the set A is defined as the union of all nano generalized regular open Subsets contained in A and is denoted by NgrInt(A). NgrInt(A) is the largest Nano generalized regular open subset of A and NgrInt(A) ⊆ A.

Definition 2.9: [1]

If $(U, \tau_R(X))$ is a Nano topological space with respect to X where X ⊆ U and if A⊆ U, then the Nano generalized regular closure of the set A is defined as the intersection of all nano generalized regular closed sets containing A and is denoted by NgrCl(A). NgrCl(A) is the smallest nano generalized regular closed set containing A.

Remark 2.10: [8]

Throughout this paper, U and V are non-empty, finite universes; X ⊆ U and Y ⊆ V; U/ R and V/ R'denote the families of equivalence classes by equivalence relations R and R'respectively on U and V. $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ are the nano topological spaces with respect to X and Y respectively.

Definition 2.11: [5]

Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two nano topological spaces. Then a mapping f: $(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano continuous on U if the inverse image of every nano open set in V is nano open in U.

Definition 2.12: [5]

The function f: $(U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is nano open if the image of every nano open set in$(U, \tau_R(X))$ is nano open in $(V, \tau_{R'}(Y))$.

Definition 2.13: [5]

The function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is nano closed if the image of every nano closed set in (U, $\tau_R(X)$) is nano closed in (V, $\tau_{R'}(Y)$).

Definition 2.14: [5]

The function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is nano g – closed if the image of every nano g – closed set in (U, $\tau_R(X)$) is nano g – closed in(V, $\tau_{R'}(Y)$).

III. NANO GENERALIZED REGULAR CONTINUITY IN NANO TOPOLOGICAL SPACE

In this section, the concept of nano generalized – regular continuous map is introduced and certain characterizations of these maps are discussed.

Definition 3.1:
Let (U, $\tau_R(X)$) and (V,$\tau_{R'}(Y)$) be two nano topological spaces. Define a map f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) such that the inverse image of every nano open subsets in V is Ngr - open in U, then the map is called Nano generalized regular continuous (briefly Ngr – continuous).

Theorem 3.2:
A function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr –continuous if and only if the inverse image of every nano closed set in (V, $\tau_{R'}(Y)$) is Ngr –closed in (U, $\tau_R(X)$).

Proof:
Let f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be Ngr – continuous function and A be a nano closed set in (V, $\tau_{R'}(Y)$). That is, V – A is nano open set in V. Since f is Ngr – continuous, the inverse image of every nano open set in V is Ngr – open in U. Hence $f^{-1}(V-A)$ is Ngr – open in U. That is, $f^{-1}(V-A) = f^{-1}(V) - f^{-1}(A)$ = U – $f^{-1}(A)$ is Ngr – open in U. Hence $f^{-1}(A)$is Ngr – closed in U. Thus, the inverse image of every nano closed set in (V, $\tau_{R'}(Y)$) is Ngr – closed in (U, $\tau_R(X)$) if f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be Ngr – continuous function.

Conversely, let the inverse image of every nano closed set in (V, $\tau_{R'}(Y)$) is Ngr –closed in (U, $\tau_R(X)$). Let B be a nano open set in V. Then V – B is nano closed in V. By the given hypothesis, $f^{-1}(V-B)$ is Ngr – closed in U. That is, $f^{-1}(V-B) = f^{-1}(V) - f^{-1}(B)$ = U – $f^{-1}(B)$ is Ngr – closed in U. Hence, $f^{-1}(B)$ is Ngr – open in U. Thus, the inverse image of every nano open set in (V, $\tau_{R'}(Y)$) is Ngr – open in (U, $\tau_R(X)$). Hence by definition f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be Ngr – continuous function.

Example 3.3:

Let U = {a, b, c, d} be the Universe with U/R= {{a}, {b}, {c, d}} and let X = {a, b, c} ⊆ U. Thenthe nano Open Sets are$\tau_R(X)$ = {U, ∅, {a, b}, {c, d}} and the nano closed sets are$[\tau_R(X)]^C$= {∅, U, {c, d}, {a, b}}. Nano generalized regular closed sets are {U, ∅, {a}, {b}, {c}, {d},{a, b}, {c, d}}.Nano generalized regular open sets are {U, ∅, {b, c, d}, {a, c, d}, {a, b, d}, {a, b, c},{c, d}, {a, b}}. Let V = {x, y, z, w} be the Universe with V/R′ = {{x, y}, {z}, {w}} and let Y = {x, y} ⊆ V Then,$\tau_{R'}(Y)$ = {V, ∅, {x, y}} and $[\tau_{R'}(Y)]^C$ = {V, ∅, {z, w}}. Now, let us define a functionf: (U, $\tau_R(X)$) →(V, $\tau_{R'}(Y)$) as f(a) = x; f(b) = y; f(c) =z; f(d) = w. The inverse images are $f^{-1}(V)$ = U; $f^{-1}(\emptyset) = \emptyset$; $f^{-1}(\{x,y\}) = \{a,b\}$. i.e., The inverse images of every nano open set in V is Ngr – Open in U. Thus, the function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nano generalized regular continuous.

Theorem 3.4:

If the function f: (U, $\tau_R(X)$) → (V,$\tau_{R'}(Y)$) is Nano continuous, then it is Ngr –continuous but not conversely.

Proof:
Let the function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be nano continuous on U. Also, every nano closed set is Ngr – closed but not conversely. Since f is nano continuous on (U, $\tau_R(X)$), the inverse image of every nano closed set in (V, $\tau_{R'}(Y)$) is nano closed in (U, $\tau_R(X)$). Hence the inverse image of every nano closed set in V is Ngr– closed in U and so f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous.

Conversely, all Ngr–closed sets are not nano closed and hence a Ngr –continuous map need not be nano continuous which can be seen from the following example.

Example 3.5:

Let U = {a, b, c, d} be the Universe with U/R= {{a}, {b}, {c, d}} and let X = {a, b, c} ⊆ U. Then$\tau_R(X)$ = {U, ∅, {a, b}, {c, d}} which are nano Open Sets and$[\tau_R(X)]^C$ ={∅, U, {c, d}, {a, b}} which are Nano Closed sets. Nano regular open sets are { U, ∅, {a}, {b}, {c}, {d}, {a, b}, {b, c}, {c, d}, {a, d}, {a, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}.Nano regular closed sets are {U, ∅, {a}, {b}, {c}, {d}, {a, b}, {b, c}, {c, d}, {a, d}, {a, c}, {b, d}, {a, b, c}, {a, b, d}, {a, c, d}, {b, c, d}}.Nano generalized regular closed sets are {U, ∅, {a}, {b}, {c}, {d}, {a, b}, {c, d}}. Nano generalized regular open sets are {U, ∅, {b, c, d}, {a, c, d}, {a, b, d}, {a, b, c}, {c, d}, {a, b}}.Here, all Ngr–closed sets are not nano closed and hence a Ngr –continuous map need not be nano continuous

Theorem 3.6:

If the function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ng –continuous, then f is Ngr–continuous but not conversely.

Proof:

Since is f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) Ng –continuous, the inverse image $f^{-1}(A)$ of a nano open set A in (V, $\tau_{R'}(Y)$) is Ng –open in (U, $\tau_R(X)$). Hence, $f^{-1}(A)$ is Ngr – open in (U, $\tau_R(X)$). Hence the function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr –continuous.

The converse of the Theorem need not be true in general as can be seen from the following example.

Example 3.7:

Let U = {a, b, c, d} be the Universe with U/R= {{a, c}, {b}, {d}} and let X = {b, c} ⊆ U. Then$\tau_R(X)$ = {U, ∅, {b}, {a, b, c}, {a, c}} which are nano Open Sets and $[\tau_R(X)]^C$ ={∅, U, {a, c, d}, {d}, {b, d}} which are Nano Closed sets. Nano generalized closed sets are{U, ∅, {a, b}, {b, c},{a, b, c}}.Nano generalized open sets are{U, ∅, {c, d}, {a, d}, {d}}.Nano generalized regular closed sets are{U, ∅, {a}, {c}, {a, b, c}}.Nano generalized regular open sets are {U, ∅, {b, c, d}, {a, b, d}, {d}}. Let V = {x, y, z, w} be the Universe with V/R$'$ = {{x}, {y}, {z, w}} and let Y = {x, y} ⊆ V. Then,$\tau_{R'}(Y)$ = {V, ∅, {x, y}} which are nano Open sets and $[\tau_{R'}(Y)]^C$ = {V, ∅, {z, w}}which are nano closed sets.Now,let us define a function f: (U, $\tau_R(X)$)→ (V, $\tau_{R'}(Y)$) as f(a) = z; f(b) = x; f(c) =x;f(d) = y. The inverse images are $f^{-1}(V)$ = U; $f^{-1}(\emptyset) = \emptyset$; $f^{-1}(\{x,y\}) = \{b,c,d\}$. i.e., The inverse images of every nano open set in V is Ngr – Open in U. Thus, the function f: (U, $\tau_R(X)$)→(V, $\tau_{R'}(Y)$) is Nano generalized regular continuous.But $f^{-1}(\{x,y\}) = \{b,c,d\}$ is not Ng – open in (U, $\tau_R(X)$) for the nano open set {x, y} in (V, $\tau_{R'}(Y)$). Thus, the map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is not Ng - continuous even though the map is Ngr – continuous.

Theorem 3.8:

A function f:(U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous if and only if f(NgrCl(A)) ⊆ NCl(f(A)) for every subset A of (U, $\tau_R(X)$) .

Proof:

Let f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be Ngr – continuous. Let A ⊆ U and thus f(A) ⊆ V. Hence NCl(f(A)) is nano closed in V. Since f is Ngr – continuous, $f^{-1}(\mathrm{NCl}(f(A)))$ is also Ngr - closed in (U, $\tau_R(X)$) . Since f(A) ⊆ NCl(f(A)), it follows that A ⊆$f^{-1}(\mathrm{NCl}(f(A)))$. Hence $f^{-1}(\mathrm{NCl}(f(A)))$ is a Ngr – closed set containing A. As NgrCl(A) is the smallest Ngr – Closed set containing A, it follows that NgrCl(A) ⊆$f^{-1}(\mathrm{NCl}(f(A)))$ which implies f(NgrCl(A)) ⊆$\mathrm{NCl}(f(A))$.

Conversely, let f(NgrCl(A)) ⊆ NCl(f(A)) for every subset A of (U, $\tau_R(X)$).Let F be a nano closed set in (V, $\tau_{R'}(Y)$). Now, $f^{-1}(F)$⊆ U and hence, f(NgrCl($f^{-1}(F)$)) ⊆ NCl(f($f^{-1}(F)$)) = NCl(F). It follows that (NgrCl($f^{-1}(F)$)) ⊆f^{-1}(NCl(F)) and thus (NgrCl($f^{-1}(F)$)) ⊆$f^{-1}(F)$ ⊆ (NgrCl($f^{-1}(F)$)). Hence (NgrCl($f^{-1}(F)$)) = $f^{-1}(F)$ which implies that $f^{-1}(F)$ is Ngr – closed in U for every nano closed set F in V. That is the map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous.

Theorem 3.9:

Let (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) be two nano topological spaces whereX ⊆U and Y ⊆ V. Then $\tau_{R'}(Y)$ = $\{V, \emptyset, L_{R'}(Y), U_{R'}(Y), B_{R'}(Y)\}$ and its basis is given by $\mathcal{B}_{R'}$={V, $L_{R'}(Y)$, $B_{R'}(Y)$}. A function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous if and only if the inverse image of every member of $\mathcal{B}_{R'}$ is Ngr – open in U.

Proof:

Let the map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous on U. Let B ∈ $\mathcal{B}_{R'}$. Then B is nano open in V. Since f is Ngr – Continuous, $f^{-1}(B)$ is Ngr – open in U and hence $f^{-1}(B) \in \tau_R(X)$. Hence the inverse image of every member of $\mathcal{B}_{R'}$ is Ngr – open in U.

Conversely, let the inverse image of every member of $\mathcal{B}_{R'}$ be Ngr – open in U. Let G be nano open in V. Now $G = \cup\{B : B \in B_1\}$ where $B_1 \subset \mathcal{B}_{R'}$. Then $f^{-1}(G) = f^{-1}[\cup\{B : B \in B_1\}] = \cup\{f^{-1}(B) : B \in B_1\}$ where each $f^{-1}(B)$ is Ngr – open in U and their union which is $f^{-1}(G)$is also Ngr – open in U. Hence the inverse image of a nano open set in V is Ngr – open in U and thus f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous on U.

Theorem 3.10:

A function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous if and only if $f^{-1}(\mathrm{NInt}(B)) \subseteq \mathrm{NgrInt}(f^{-1}(B))$for every subset B of (V, $\tau_{R'}(Y)$).

Proof:

Let f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous. By the given hypothesis, B⊆V. Then, NInt(B) is nano open in V. As f is Ngr – Continuous, f^{-1}(NInt(B)) is Ngr – open in (U, $\tau_R(X)$). Hence it follows that NgrInt(f^{-1}(NInt(B)) = f^{-1}(NInt(B)). Also, for B ⊆ V, NInt(B) ⊆ B always. Then, f^{-1}(NInt(B))⊆f^{-1}(B). Since f is Ngr continuous, it follows that NgrInt(f^{-1}(NInt(B)))⊆ NgrInt(f^{-1}(B)), and hence f^{-1}(NInt(B))⊆ NgrInt(f^{-1}(B)).

Conversely, let f^{-1}(NInt(B)) ⊆ NgrInt(f^{-1}(B))for every subset B of V. Let B be nano open in V and hence NInt(B) = B. Givenf^{-1}(NInt(B)) ⊆ NgrInt(f^{-1}(B)),i.e.,f^{-1}(B) ⊆ NgrInt(f^{-1}(B)). Also,NgrInt(f^{-1}(B)) ⊆ f^{-1}(B). Hence it follows that f^{-1}(B) = NgrInt(f^{-1}(B)) which implies that f^{-1}(B) is Ngr – open in U for every subset B of V. Therefore,f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – Continuous.

Theorem 3.11:

A function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous if and only if NgrCl(f^{-1}(B)) ⊆ f^{-1}(NCl(B)) for every subset B of (V, $\tau_{R'}(Y)$).

Proof:

Let B ⊆ V and f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous. Then NCl(B) is nano closed in (V, $\tau_{R'}(Y)$) and hence f^{-1}(NCl(B))is Ngr – closed in (U, $\tau_R(X)$). Therefore, NgrCl(f^{-1}(NCl(B))) = f^{-1}(NCl(B)). Since B ⊆ NCl(B) ,then f^{-1}(B) ⊆ f^{-1}(NCl(B)),i.e., NgrCl(f^{-1}(B)) ⊆ NgrCl(f^{-1}(NCl(B))) = f^{-1}(NCl(B)). Hence NgrCl(f^{-1}(B)) ⊆ f^{-1}(NCl(B)).

Conversely, let NgrCl(f^{-1}(B)) ⊆ f^{-1}(NCl(B)) for every subset B ⊆ V. Now, let B be a nano closed set in (V, $\tau_{R'}(Y)$), then NCl(B) = B. By the given hypothesis, NgrCl(f^{-1}(B)) ⊆ f^{-1}(NCl(B)) and hence NgrCl(f^{-1}(B)) ⊆ f^{-1}(B). But, we also have f^{-1}(B)⊆NgrCl(f^{-1}(B)) and hence NgrCl(f^{-1}(B)) = f^{-1}(B). Thus f^{-1}(B) is Ngr -Closed set in (U, $\tau_R(X)$) for every nano closed set Bin (V, $\tau_{R'}(Y)$). Hence f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous.

Theorem 3.12:

Let (U, $\tau_R(X)$) and (V, $\tau_{R'}(Y)$) be two nano topological spaces with respect to X ⊆ U and Y ⊆ V respectively. Then for any function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$), the following are equivalent.

i. f is Ngr – continuous.
ii. The inverse image of every nano closed set in V is Ngr – closed in (U, $\tau_R(X)$).
iii. f(NgrCl(A)) ⊆ NCl(f(A)) for every subset A of (U, $\tau_R(X)$).
iv. The inverse image of every member of $\mathcal{B}_{R'}$ is Ngr – open in (U, $\tau_R(X)$).
v. NgrCl(f^{-1}(B)) ⊆ f^{-1}(NCl(B)) for every subset B of (V, $\tau_{R'}(Y)$).

Proof:

The proof of this theorem is obvious

Remark 3.13:

The composition of two Ngr – Continuous functions need not be Ngr – continuous and this can be shown by the following example:

Example 3.14:

Let (U, $\tau_R(X)$),(V, $\tau_{R'}(Y)$) and (W, $\tau_{R''}(Y)$) be three nano topological spaces and Let U = {a, b, c, d} be the Universe with U/R= {{a, c}, {b}, {d}} and let X = {b, c} ⊆ U.Then$\tau_R(X)$ = {U, ∅, {b}, {a, b, c}, {a, c}} which are nano Open Sets. Nano generalized regular open sets are {U, ∅, {b, c, d}, {a, b, d}, {d}. Now, letV = {x, y, z, w} be the Universe with V/R$'$ = {{x},{y},{z, w}} and let Y = {x, y} ⊆ V.Then,$\tau_{R'}(Y)$ = {V, ∅, {x, y}} which are nano Open sets.Nano generalized regular open sets are {V, ∅, {y, z, w}, {x, z, w}, {z, w}}. Now, let W = {p, q, r, s} be the Universe with W/R$''$ = {{p}, {q}, {r, s}} and letZ = {r, s} ⊆ W. Then,$\tau_{R''}(Z)$ = {W, ∅, {r, s}} which are nano Open sets.

Now, let us define a function f: (U, $\tau_R(X)$)→(V, $\tau_{R'}(Y)$) as f(a) = z; f(b) = x; f(c) =x;f(d) = y.The inverse images are f^{-1}(V) = U; f^{-1}(∅) = ∅; f^{-1}({x,y}) = {b, c, d}. i.e., The inverse images of every nano open set in V is Ngr – Open in U.Thus, the function f: (U, $\tau_R(X)$)→ (V, $\tau_{R'}(Y)$) is Nano generalized regular continuous.Another map g: (V, $\tau_{R'}(Y)$) → (W, $\tau_{R''}(Z)$) as g(x) = p;g(y) = q; g(z) =r; g(w) = s. The inverse images are g^{-1}(W) = V; g^{-1}(∅) = ∅; g^{-1}({r, s}) = {z, w}. i.e., The inverse images of every nano open set in W is Ngr – Open in V.Thus, the function g: (V, $\tau_{R'}(Y)$) → (W, $\tau_{R''}(Z)$) is Nano generalized regular continuous.

Here, f and g are Ngr – continuous function. But their Composite g o f: (U, $\tau_R(X)$) → (W, $\tau_{R''}(Z)$) is not Ngr – continuous because (g o f)$^{-1}$({r, s}) = f^{-1}[g^{-1}({r, s})] = f^{-1}({z, w}) = {a} is not Ngr – open in (U,

$\tau_R(X)$) for a nano open set {r, s} in (W, $\tau_{R''}(Z)$). Hence, the composition of two Ngr – continuous functions need not be Ngr – continuous.

Theorem 3.15:

If the map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nrg – continuous, if and only if it is Ngr – continuous.

Proof:

Let the map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be Nrg – continuous and let A be a nano open set in(V, $\tau_{R'}(Y)$. As the map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nrg – continuous,$f^{-1}(A)$is Nrg – open in(U, $\tau_R(X)$). Then $f^{-1}(A)$ is Ngr – open in (U, $\tau_R(X)$) and hence the map f: (U, $\tau_R(X)$) →(V, $\tau_{R'}(Y)$) which is Nrg – continuous is Ngr – continuous.

Conversely, Let the map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) be Ngr– continuous and let A be a nano open set in (V, $\tau_{R'}(Y)$. As the map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Ngr – continuous, $f^{-1}(A)$is Ngr – open in(U, $\tau_R(X)$). Then $f^{-1}(A)$ is Nrg – open in(U, $\tau_R(X)$) and hence the map f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) which is Ngr – continuous is Nrg – continuous.

Example 3.16:

Let U = {a, b, c, d} be the Universe with U/R= {{a, c}, {b}, {d}} and let X = {b, c} ⊆ U.Then$\tau_R(X)$ = {U, ∅, {b}, {a, b, c}, {a, c}}which are Nano Open Sets and $[\tau_R(X)]^C$ ={∅, U, {a, c, d}, {d}, {b, d}} which are Nano Closed sets. Nano generalized regular open sets are{U, ∅, {b, c, d}, {a, b, d}, {d}}. Nano regular generalized open sets are {U, ∅, {a}, {c}, {d}, {a, b}, {b, c}, {c, d}, {a, d}, {a, c}, {a, b, c}, {a, b, d}, {b, c, d}, {a, c, d}}. Now, let V = {x, y, z, w} be the Universe with V/R$'$ = {{x}, {y}, {z, w}} and let Y = {x, y} ⊆ V. Then, $\tau_{R'}(Y)$ = {V, ∅, {x, y}} which are Nano Open sets.Now, let us define a function f : (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) as f(a) = z; f(b) = x; f(c) =x; f(d) = y .The inverse images are $f^{-1}(V)$ = U; $f^{-1}(\emptyset) = \emptyset$; $f^{-1}(\{x,y\}) = \{b,c,d\}$. i.e., The inverse images of every nano open set in V is Ngr – Open in U.Also, the inverse images of every nano open set in V is Nrg – Open in U.Thus, the function f: (U, $\tau_R(X)$) → (V, $\tau_{R'}(Y)$) is Nano generalized regular continuous and also Nano regular generalized continuous.

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