With the use of MATLAB solve Partial Differential Equation in Graphical form - A new View

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Abstract

  The current paper deals with the Partial Differential Equation and solution of Partial Differential Equation with the help of MATLAB. Initially, there is a Partial differential Equation with two independent variable and one dependent variable on these two. We are plotted the graph of the solution of Partial Differential Equation with the help of MATLAB and describe it And we try to find out a better solution of Partial Differential Equation using MATLAB its give detail about MATLAB and its modules, plotting the graphs of solution of Partial Differential Equations, also we try to solve and simulate the Partial Differential Equations using MATLAB.

Key words - *MATLAB, Partial Differential Equation, Plotting the graphs, traditional method.*

I. INTRODUCTION

A. Differential Equation
  An Equation involving derivatives of one or more dependent variable with respect to one or more independent variables is called Differential Equation.

B. Partial Differential Equation
  A Differential Equation involving Partial derivatives of one or more dependent variable with respect to more than one independent variable is called Partial Differential Equation.

C. MATLAB (MATRIX LABORATORY)
  MATLAB is a multi-paradigm numerical computing environment and property programming language developed by mathwork.

1. Basic Equations
  Partial Differential Equations are one of an important subjects and it is a foundation of many other aspects of applied science and engineering. It concerns with study of Differential Equations.From the early days of calculus, the Partial Differential Equation has been a great area of theoretical research and particular applications and it continues to be so in our days.

2. Partial Differential Equation
  A Differential Equation involving Partial derivatives of one or more dependent variable with respect to more than one independent variable is called Partial Differential Equation. We will consider the case of two independent variable and one dependent variable, usually take x and y as independent variable and z as dependent variable.

3. Order of Partial Differential Equation
  The order of the highest Partial derivative is occurring in the Partial Differential Equation is called the order of a Partial Differential Equation.

4. Degree of Partial Differential Equation
  The degree of Partial Differential Equation is the degree of highest order derivative which occurs in it after the Equation has been rationalized ,i.e., Made free from radicals and fractions as far as derivatives are concerned the Equation;

D. Solution of Partial Differential Equation

In this paper we re-investigate the problem of Partial Differential Equation with one dependent variable say z with respect to two independent variable say x and y. In the 1st method we attempt for Partial Differential Equation solution of the problem. In this method we are solving Partial Differential Equation by differentiating the dependent variable z partially with respect to x, y, xy, yx, xx, yy. In the second method, we are trying to solve Partial Differential Equation using MATLAB. We are use MATLAB to calculate terms of series coefficients, which are compared with the manually calculated terms. Later invoke suitable expansion scheme for obtaining large number of terms in the perturbation solution.

Now we are trying to solve an example of Partial Differential Equation by a traditional method as well as using MATLAB. Which are try to solving below:-

Solution: of Partial Differential Equation $(y - x)(qy - px) = 1$ by traditional method.

Example: find the complete integral of $(y - x)(qy - px) = 1$ where p=$\frac{\partial z}{\partial x}$; $q = \frac{\partial z}{\partial y}$

Solution: Let x and y be two new variable such that X = x+y and Y = xy   (#)

The given equation is $(y - x)(qy - px) = 1$ (##)

Now, p $= \frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial z}{\partial Y}\frac{\partial z}{\partial x} = 1$ (###)

[Because from (*) $\frac{\partial X}{\partial x} = 1$ and $\frac{\partial Y}{\partial y} = y$]

And q $= \frac{\partial z}{\partial y}$, $\frac{\partial z}{\partial X}\frac{\partial X}{\partial y} + \frac{\partial z}{\partial Y}\frac{\partial Y}{\partial y} = 1$ (####)

$[from$ $(*) \frac{\partial X}{\partial y} = 1, \frac{\partial Y}{\partial y} = x]$

Substituting the above values of p and q in equation (##), we have

$.(y - x)\left[y\left(\frac{\partial z}{\partial X} + x\frac{\partial Z}{\partial Y}\right) - \left(\frac{\partial z}{\partial X} + y\frac{\partial z}{\partial Y}\right)\right] = 1$

Or $(y - x)^2\frac{\partial z}{\partial X} = (y - x)^2\frac{\partial z}{\partial Y}$

Or $\frac{\partial z}{\partial X} = \left(\frac{\partial z}{\partial Y}\right)^2$ or $P = Q^2$ (#####)

Where P $= \frac{\partial z}{\partial X}$ and $Q = \frac{\partial z}{\partial Y}$

Equation (#####) is of the form $f(PQ) = 0$

So, solution of equation (####) is given by $z = ax + b\frac{y+1}{a+b}$ (######)

So from equation (######) , required complete integral is $z = ax + b\frac{y+1}{a+b}$

Or $z = ax + b\frac{y+1}{a+b}$

This is the required complete integral.

Explanation

On the page number 3 we are solved a Partial Differential Equation, which is $(y - x)(qy - px) = 1$ by traditional paper pencil method. In this method we are trying to solving the given Partial Differential Equation.
Here the given Partial Differential Equation is solving and finds the complete integration. So, First of all we find the value of 'p' and 'q' and substitute in the given Differential Equation and this become a famous result. So, by using of that result gives a complete integral directly watch in last page.

The given partial differential equation have two independent variables say x, y and one dependent variable say z. After applying all these operations we are getting the value of z. This is the solution of given Partial Differential Equation by traditional paper pencil method.

III. GRAPHICAL FORMAT OF THE SOLUTION OF PARTIAL DIFFERENTIAL EQUATION
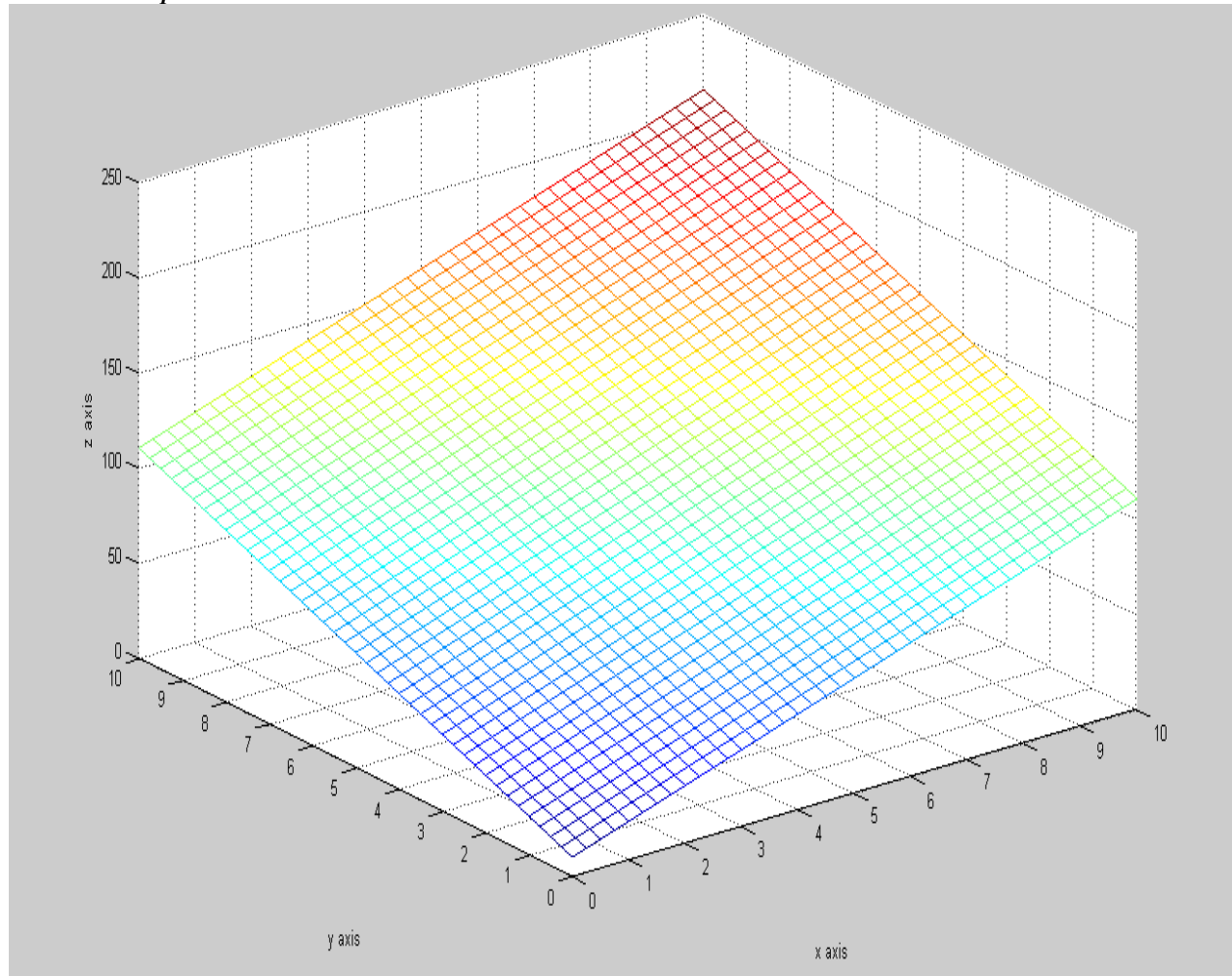### $(y - x)(qy - px) = 1$ USING MATLAB

A. Plotted Graph

```
>> x=0:0.25:10;
>> y=0:0.25:10;
>> [X,Y]=meshgrid(x,y);
>> a=10;
>> b=10;
>> Z=a*X+b*Y+1/a+b;
>> mesh(X,Y,Z)
>> xlabel('x axis');
>> ylabel('y axis');
>> zlabel('z axis')
>>
```

IV. GRAPHICAL FORMAT OF THE SOLUTION OF PARTIAL DIFFERENTIAL EQUATION
$(y - x)(qy - px) = 1$ **USING MATLAB**

A. Plotted Graph

B. Explanation

On page number 4 we are format the graph of solution of Partial Differential Equation, $(y - x)(qy - px) = 1$ for this purpose we must have the range of the independent variables. In this given partial differential equation we have two independent variables say x and y. So, we are taking range of independent variable x; then we are taking range of independent variable y. We must have to convert variable x and y into matrices x and y. By using the command:

>>[X,Y]=meshgrid(x,y);

Now here, we have 2 constant say b and c give this constant value because these constant are arbitrary. After this, convert independent variable z into matrix z by applying operation of matrix. Which are describe below. After this we have plot the graph by the command:

>>mesh(X,Y,Z)

After this give the name of axis by applying the command:

>>xlabel('x axis');

>>ylabel('y axis');

>>zlabel('z axis')

We will plot the graph given partial differential equation using above commands.

All these commands are given below:-

>>x=0:0.25:10;

>>y=0:0.25:10;

>>[X,Y]=meshgrid(x,y);

>>b=2;

>>c=7;

>>Z= a*x+b*y+1/a+b;

>>mesh(X,Y,Z)

>>xlabel('x axis');

>>ylabel('y axis');

>>zlabel('z axis')

       Now we are try to describe the graphical format of the solution of Partial Differential Equation and we are try to difference between geometrical representation of the solution of Partial Differential Equation and the solution of Partial Differential Equation which we are get from traditional paper pencil method.

For good result we are reduces the range of independent variable from "0 to 10" to "0 to 7"

```
>> x=0:0.25:7;
>> y=0:0.25:7;
>> [X,Y]=meshgrid(x,y);
>> a=10;
>> b=10;
>> Z=a*X+b*Y+1/a+b;
>> mesh(X,Y,Z)
>> xlabel('x axis');
>> ylabel('y axis');
>> zlabel('z axis')
>>
```
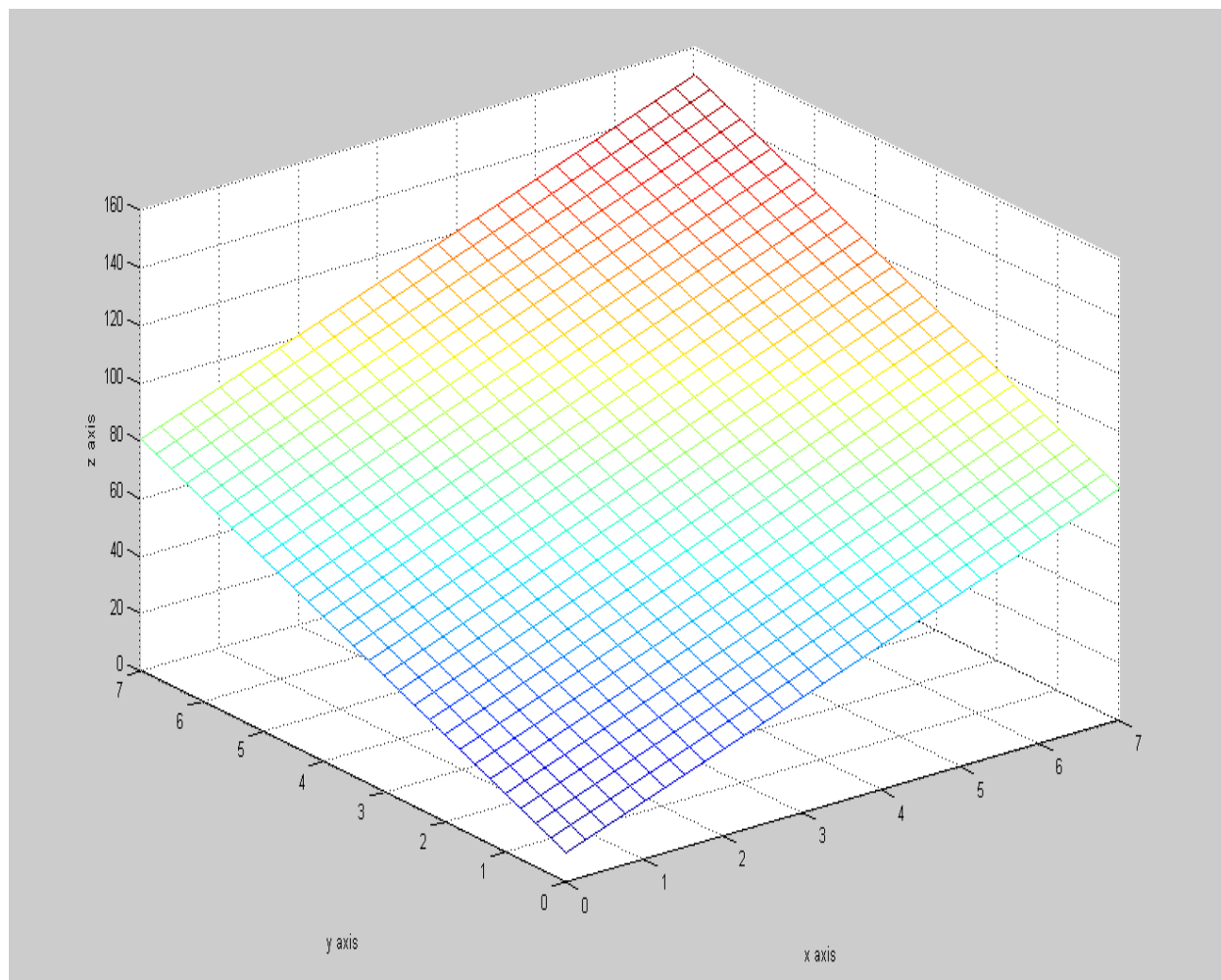
On the page number 8 above graph of the solution of Partial Differential Equation we are taking the range of independent variable x and y is "from 0 to 7" this means that we are plotting the graph between the range "0 to 7" and also the range of dependent variable z is automatically described by MATLAB corresponding to x-axis and y-axis that is Started from 970 and 980, 990, 1000, 1010, 1020, 1030 these range are corresponding to x-axis and y-axis. Here the graph is started from the z-axis of point 998 which is corresponding to x-axis and y-axis in this graph range of independent variable's variable x and y that is from 0 to 7 the space between 0 to 1 is also divided in four parts by the distance 0.25 in both ranges of the axis that is x-axis and y-axis so the graph which describe above is also 4 squares of size 0.25x0.25 in a square of size 1x1. This square of size 1x1 is made the corresponding z-axis on the point 998 which is made corresponding to x-axis and y-axis.

The first square of size 1x1 is made by the points (0,6,994), (0,7,998), (1,6,998) and (1,6,1002) like this the next point of size 1x1 is made by the points (1,6,998), (1,7,1002), (2,6,1002) and (2,7,1006) corresponding to the z-axis with respect to x-axis and y-axis of the complete range of independent variable x and y that is "from 0 to 7".

So, There is next square of size 1x1 is made by the points (2,6,1002), (2,7,1006), (3,6,1006) and (3,7,1010)

And like this we getting a sequence of squares and here last square of size 1x1 corresponding to z-axis is (6,6,1018), (6,7,1022), (7,6,1022) and (7,7,1026).This sequence of squares is getting with respect to z-axis corresponding to x-axis and y-axis.

Also in the above graph we are observe that the next sequence of squares of size 1x1 are made by the points (0,5,978), (0,6,982), (1,5,982) and (1,6,986) like this the next point of size 1x1 is made by the points (1,5,982),

(1,6,986), (2,5,990) and (2,6,994) corresponding to z-axis with respect to x-axis and y-axis of the complete range of independent variable x and y that is "from 0 to 7".

So, there is next square of size 1x1 is made by the points (2,5,986), (2,6,990), (3,5,990) and (3,6,994) And like this we getting a sequence of squares and here last square of size 1x1 corresponding to z-axis is (6,5,1000), (6,6,1004), (7,5,1004) and (7,6,1008). This sequence of squares is getting with respect to z-axis corresponding to x-axis and y-axis.

The next sequence of square of size 1x1 are made by the points (0,4,968), (0,5,972), (1,4,972) and (1,5,976) this sequence is end with the square of size 1x1 which are made by the points (6,4,990), (6,5,994), (7,4,994) and (7,5,998)

When we are observed the graph we notice there this graph have a variation from starting. So, this graph is starting from middle of z-axis with respect to x-axis and y-axis when we are grow with x-axis and also with y-axis the graph will be extended with the values of x-axis is increasing and also with y-axis values on increasing. When we try to observe this variation we are getting a variation from starting the graph. (0,0,12), (0.25,0.25,11.8), (0.5,0.5,11.6), (0.75,0.75,11.4), (1,1,11.2), (1.25,1.25,11), (1.50,1.50,11) these points are gives the rotation of graph downward from left hand side.

Also the points (1.75,1.75,11), (2,2,11.2), (2.25,2.25,11.4), (2.5,2.5,11.6), (2.75,2.75,11.8),(3,3,12) are gives the rotation of graph variation upward from left hand side. Also the point (3.75,3.75,12), (4,4,11.8), (4.25,4.25,11.6), (4.5,4.5,11.4), (4.75,4.75,11.2), (5,5,11), (5.25,5.25,11) these points are gives the rotation of graph downward from right hand side. Also the points (5.75,5.75,11), (6,6,11.2), (6.25,6.25,11.4), (6.5,6.5,11.6), (6.75,6.75,11.8), (7,7,12) are gives the rotation of graph variation upward from right hand side.

When we observe the graph from left hand side to right hand side we get the graph is starting from middle of the value of z-axis and grow downward with respect to x-axis and y-axis and also it is grow upward with respect to x-axis and y-axis parallel. When it is grow downward and upward parallel with respect to x-axis and y-axis on pick of the height in both upward and downward it is changed from downward to upward and upward to downward. We are getting a rhombus type graph here.

From this graph we are observed that the geometrical format of the solution of given Partial Differential Equation is have 28 vertical lines with respect to x-axis and 28 horizontal lines with respect to y-axis.the 1st line is made on y-axis where the value of x-axis on y-axis is '0' and values of y-axis is changed from 0 to 7 and here the value of z is changed from 970 to 1030. In the next line of the point '1' on the x-axis here also the point of x-axis is constant and points of y-axis is changed from 0 to 7, here also the points of z-axis is starting from 970 to 1030 In between these two lines of the points of x-axis that is 0 and 1 here we have also other lines in the distance of 0.25 and 0.25.

Also 2nd line is made on y-axis where the value of x-axis on y-axis is '1' and values of y-axis are changed from 0 to 7 and here the value of z is changed from 970 to 1030. In the process of 970, 980, 990, 1000, 1010, 1020 and 1030. In between these two lines of the points of x-axis that is 0 and 1 here we have also other lines in the distance of 0.25 and 0.25.

Like this in this graph we getting 8 like as we are also getting different lines and 7 sequence of squares and in between these 8 lines we have 28 lines of sixe 0.25 with respect to y-axis like as we are also getting 8 different lines and 7 sequence of squares and in between these 8 lines we have 28 lines of sixe 0.25 with respect to x-axis.These 28 lines of x-axis 28 lines of y-axis are cross each on that points these points are the points of Equations which are made by the points of x-axis, y-axis and z-axis.
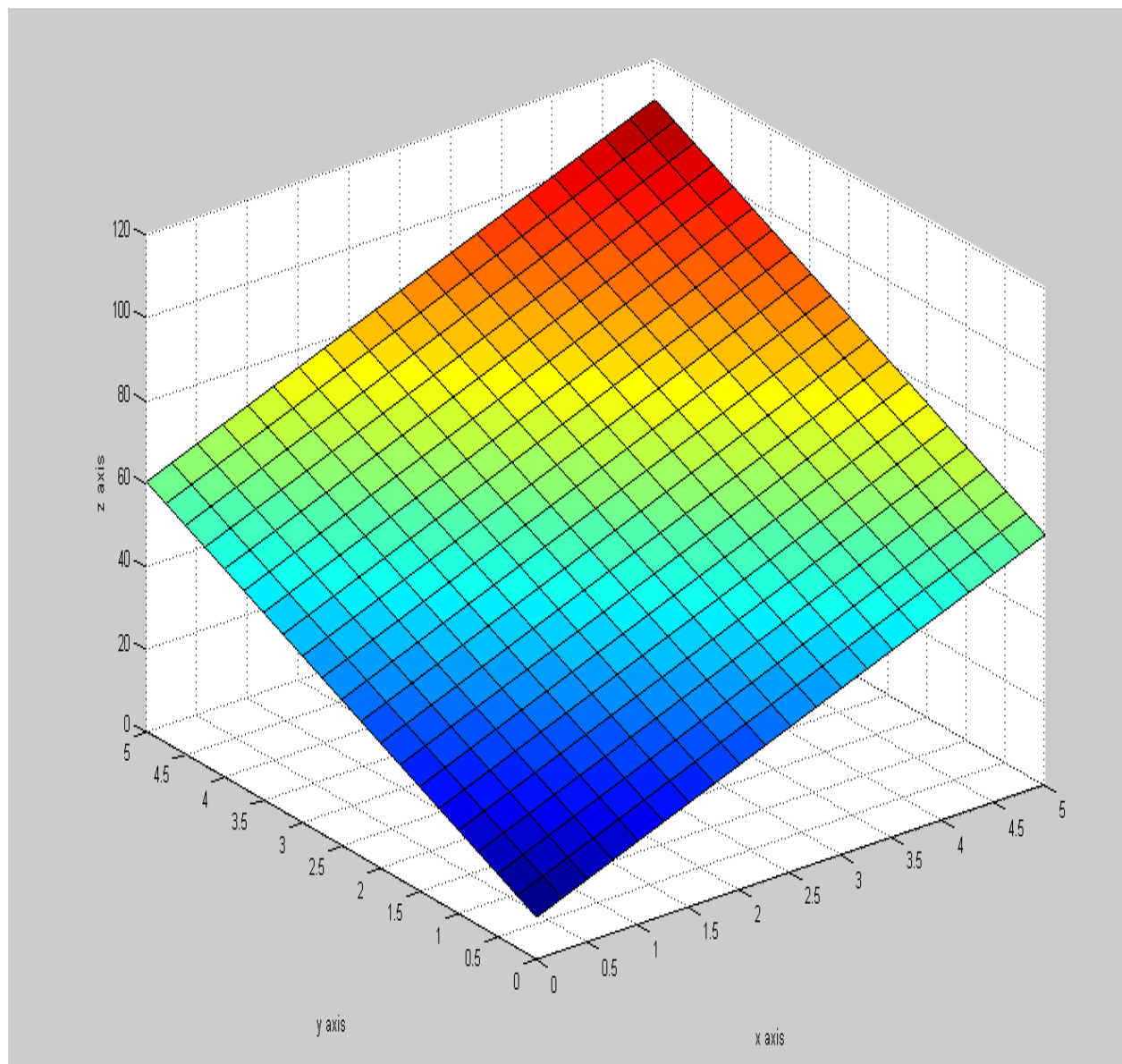
Here we try to more clarify the above graph by using some other commands which are given below:-

>>x=0:0.25:5;

>>y=0:0.25:5;

>>[X,Y]=meshgrid(x,y);

>>b=2;

>>c=7;

>>Z= a*x+b*y+1/a+b;

>>surf(X,Y,Z)

>>xlabel('x axis');

>>ylabel('y axis');

>>zlabel('z axis')

The graph after using these commands are given below:-

```
>> x=0:0.25:5;
>> y=0:0.25:5;
>> [X,Y]=meshgrid(x,y);
>> a=10;
>> b=10;
>> Z=a*X+b*Y+1/a+b;
>> surf(X,Y,Z)
>> xlabel('x axis');
>> ylabel('y axis');
>> zlabel('z axis')
>> |
```

V. CONCLUSION

The graph of the solution of the partial differential equation is plotted by the use of MATLAB. MATLAB give us a very clear and good graph of the solution of the given partial differential equation that is $.z = -e^{-y} sinx + c3(y) + c2(y)$, we are solving Partial Differential Equation by differentiating the dependent variable z partially with respect to x, y, xy, yx, xx, yy.

In the second method, we are trying to solve Partial Differential Equation using MATLAB. We are use MATLAB to calculate terms of series coefficients, which are compared with the manually calculated terms. Later invoke suitable expansion scheme for obtaining large number of terms in the perturbation solution and getting the good result.

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