3-Modulo Difference Mean Cordial Labeling of Some Graphs

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Abstract

Let G = (V,E) be a simple graph with p vertices and q edges. G is said to have 3 – modulo difference mean cordial labeling if there is a injective map $f: V(G) \to \{0,1,2,3, \ldots \ldots \ldots, 5p\}$ such that for every edge uv , the induced labeling f^* is defined as f*(uv) = $\begin{cases} 1 \; if \; \frac{|f(u)-f(v)|}{2} \equiv 0 (mod\ 3) if \; |f(u)-f(v)| \; is \; even \\ 1 \; if \; \frac{|f(u)-f(v)|+1}{2} \equiv 0 (mod\ 3) if \; |f(u)-f(v)| \; is \; odd \\ 0 \; elsewhere \end{cases}$ with the condition that $|e_f(0) - e_f(1)| \leq 1$,where $e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1.If G admits 3-modulo difference mean cordial labeling then G is a 3-modulo difference mean cordial graph. In this paper, we proved that the graphs Path(P_n), ,Star($K_{1,n}$),$C_n + u_1u_3$ are 3-modulo difference mean cordial graphs.

Keywords-3-modulo difference mean cordial labeling, 3-modulo difference mean cordial graphs.

I. INTRODUCTION

A graph G is a finite non-empty set of objects called vertices together with a set of unordered pairs of distinct vertices of G called edges.Each pair e={u,v} of vertices in E is called edges or a line of G.In this paper,we proved thatthe graphs Path(P_n),Star($K_{1,n}$) , $C_n + u_1u_3$are 3-modulo difference mean cordial graphs.For graph theoretic terminology we follow [2].

II. PRELIMINARIES

Let G = (V,E) be a simple graph with p vertices and q edges. G is said to have

3 – modulo difference mean cordial labeling if there is a injective map

$f: V(G) \to \{0,1,2,3, \ldots \ldots \ldots, 5p\}$ such that for every edge uv , the induced labeling f^* is defined as

f*(uv) = $\begin{cases} 1 \; if \; \frac{|f(u)-f(v)|}{2} \equiv 0 (mod\ 3) if \; |f(u)-f(v)| \; is \; even \\ 1 \; if \; \frac{|f(u)-f(v)|+1}{2} \equiv 0 (mod\ 3) if \; |f(u)-f(v)| \; is \; odd \\ 0 \; elsewhere \end{cases}$ with the condition that

$|e_f(0) - e_f(1)| \leq 1$,where$e_f(0)$ is the number of edges with label 0 and $e_f(1)$ is the number of edges with label 1.If G admits 3-modulo difference mean cordial labeling then G is a 3-modulo difference mean cordial graph. In this paper, we proved that the graphs Path(P_n),Star$(K_{1,n})$, $C_n + u_1u_3$are 3-modulo difference mean cordial graphs.

DEFINITION 2.1:

Path is a graph whose vertices can be listed in the order $(u_1, u_2, u_3, \ldots \ldots u_n)$ such that the edges are $\{u_i, u_{i+1}\}$where $i = 1,2,3, \ldots n-1$.

DEFINITION 2.2:

A Star $K_{1,n}$ is a tree with one internal vertex and n edges.

DEFINITION 2.3:

In any cycle there is a chord between any two non-adjacent vertices.It is denoted by $C_n + u_1u_3$.

III. MAIN RESULT

Theorem 1:

Path (P_n) is a 3-modulo difference mean cordial graph.

Proof:

Let G be a graph

When n is odd , n = 2k + 1 and when n is even , n = 2k.

Let V(G) = { $u_1, u_2, u_3, \dots \dots \dots u_n$}

E(G) = { $u_i u_{i+1} / 1 \leq i \leq n-1$}

Then $|V(G)| = n$ and $|E(G)| = n - 1$

Define $f: V(G) \rightarrow \{0,1,2,3, \dots \dots \dots, 3n\}$

Case (i): n is even

Subcase (i):k is not a multiple of 3.

The vertex labels are,

$$f(u_i) = \begin{cases} 3i & 1 \leq i \leq k \\ 5i & k+1 \leq i \leq n \end{cases}$$

The induced edge labels are,

For 1$\leq i \leq k-1$

$f^*(u_i u_{i+1}) = 2$

For $k+1 \leq i \leq n-1$

$f^*(u_i u_{i+1}) = 3 \equiv 0(mod\ 3)$

$$f^*(u_k u_{k+1}) = k + 3 \not\equiv 0(mod\ 3)$$

It is observed as

$e_f(0) = k$

$e_f(1) = k - 1$

Subcase (ii):k is a multiple of 3.

The vertex labels are,

$$f(u_i) = \begin{cases} 3i & 1 \le i \le k \\ 5i & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1$

$f^*(u_i u_{i+1}) = 2$

For $k+1 \le i \le n-1$

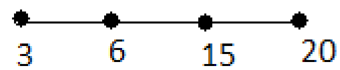
$f^*(u_i u_{i+1}) = 3 \equiv 0 (mod\ 3)$

$$f^*(u_k u_{k+1}) = k+3 \equiv 0 (mod\ 3)$$

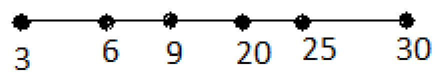
It is observed as

$e_f(0) = k-1$

$e_f(1) = k$



P_4



P_6

Case (ii): n is odd

Subcase (i): k is not a multiple of 3

The vertex labels are,

$$f(u_i) = \begin{cases} 3i & 1 \le i \le k \\ 5i & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1$

$f^*(u_i u_{i+1}) = 2$

For $k+1 \le i \le n-1$

$f^*(u_i u_{i+1}) = 3 \equiv 0 (mod\ 3)$

$$f^*(u_k u_{k+1}) = k+3 \not\equiv 0 (mod\ 3)$$

It is observed as

$e_f(0) = k$

$e_f(1) = k$

Subcase (ii): k is a multiple of 3

The vertex labels are,

$$f(u_i) = \begin{cases} 3i & 1 \le i \le k \\ 5i & k+1 \le i \le n-1 \end{cases}$$

$f(u_n) = 5n - 1$

The induced edge labels are,

For $1 \le i \le k-1$

$f^*(u_i u_{i+1}) = 2$

For $k+1 \le i \le n-2$

$f^*(u_i u_{i+1}) = 3 \equiv 0(mod\ 3)$

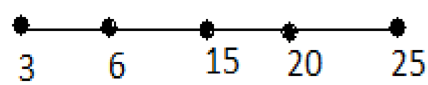
$$f^*(u_k u_{k+1}) = k + 3 \equiv 0(mod\ 3)$$

$f^*(u_{n-1} u_n) = 2$

It is observed as

$e_f(0) = k$

$e_f(1) = k$



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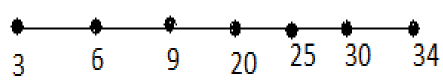


P_7

Clearly $|e_f(0) - e_f(1)| \le 1$

Then f is a 3 – modulo difference mean cordial labeling.

Hence P_n is a 3 – modulo difference mean cordial graph.

Theorem 2:

Star$(K_{1,n})$ is a 3-modulo difference mean cordial graph.

Proof:

Let G be a graph

When n is odd , n = 2k + 1 and when n is even , n = 2k.

Let V(G) = { $u_1, u_2, u_3, \dots \dots \dots u_n, u$}

E(G) = { $uu_i / 1 \le i \le n$}

Then $|V(G)| = n + 1$ and $|E(G)| = n$

Define $f: V(G) \to \{0,1,2,3, \dots \dots \dots ., 3n + 3\}$

The vertex labels are,

$f(u_i) = 3i \quad , \quad 1 \le i \le n$

$f(u) = 0$

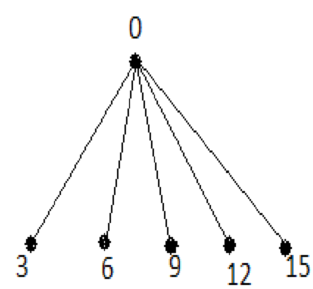
The induced edge labels are,

$$f^*(uu_i) = \begin{cases} \frac{3i+1}{2} \equiv 2(mod\ 3)\ when\ i\ is\ odd \\ \frac{3i}{2} \equiv 0(mod\ 3)\ \ when\ i\ is\ even \end{cases}$$

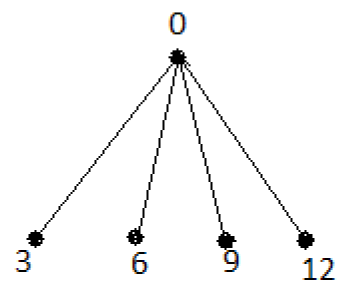
It is observed as,

$$e_f(0) = \begin{cases} k+1 & when\ n\ is\ odd \\ k & when\ n\ is\ even \end{cases}$$

$$e_f(1) = k$$



$K_{1,5}$



$K_{1,4}$

Clearly $\left|e_f(0) - e_f(1)\right| \leq 1$

Then f is a 3 – modulo difference mean cordial labeling

Hence Star is a 3-modulo difference mean cordial graph.

Theorem 3:

$C_n + u_1u_3$ is a 3-modulo difference mean cordial graph only when n is even.

Proof:

Let G be a graph

When n is odd , n = 2k + 1 and when n is even , n = 2k.

Let V(G) = { $u_1, u_2, u_3, \ldots \ldots \ldots u_n$}

E(G) = { $u_iu_{i+1} / 1 \leq i \leq n-1\} \cup \{u_nu_1\} \cup \{u_3u_1\}$

Then $|V(G)| = n$ and $|E(G)| = n$+1

Define $f: V(G) \rightarrow \{0,1,2,3, \ldots \ldots \ldots, 3n\}$

Case(i):$n = 4$

The vertex labels are,

$f(u_i) = 3i \quad , 1 \leq i \leq 3$

$f(u_n) = 5n$

The induced edge labels are,

$f^*(u_1u_2) = 2$

$f^*(u_2u_3) = 2$

$f^*(u_3u_4) = 6 \equiv 0(mod\ 3)$

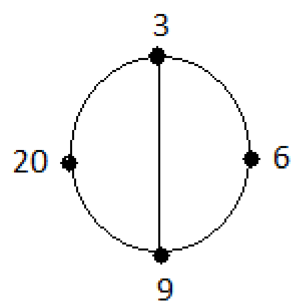
$f^*(u_4u_1) = 9 \equiv 0(mod\ 3)$

$f^*(u_1u_3) = 3 \equiv 0(mod\ 3)$

It is observed as

$e_f(0) = 2$

$e_f(1) = 3$



$C_4 + u_1u_3$

Case(ii):$n > 4$

Subcase (ii):$k \equiv 1(mod\ 3)$.

The vertex labels are,

$$f(u_i) = \begin{cases} 3i & 1 \le i \le k \\ 5i & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For 1$\le i \le k - 1$

$f^*(u_iu_{i+1}) = 2$

For $k + 1 \le i \le n - 1$

$f^*(u_iu_{i+1}) = 3 \equiv 0(mod\ 3)$

$$f^*(u_ku_{k+1}) = k + 3 \equiv 1(mod\ 3)$$

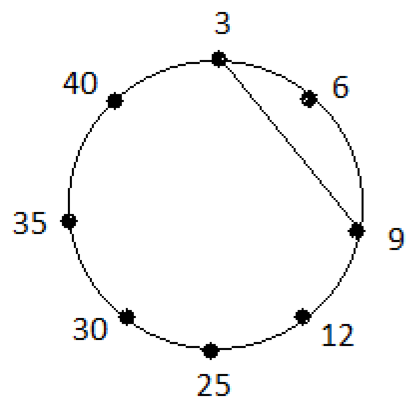
$$f^*(u_nu_1) = \frac{5n - 3}{2} \equiv 1(mod\ 3)$$

$f^*(u_1u_3) = 3 \equiv 0(mod\ 3)$

It is observed as

$e_f(0) = k + 1$

$e_f(1) = k$



$C_8 + u_1u_3$

Subcase (iii): $k \equiv 2(mod\ 3)$.

The vertex labels are,

$$f(u_i) = \begin{cases} 3i & 1 \leq i \leq k \\ 5i & k+1 \leq i \leq n \end{cases}$$

The induced edge labels are,

For $1 \leq i \leq k-1$

$f^*(u_iu_{i+1}) = 2$

For $k+1 \leq i \leq n-1$

$f^*(u_iu_{i+1}) = 3 \equiv 0(mod\ 3)$

$$f^*(u_ku_{k+1}) = k+3 \equiv 2(mod\ 3)$$

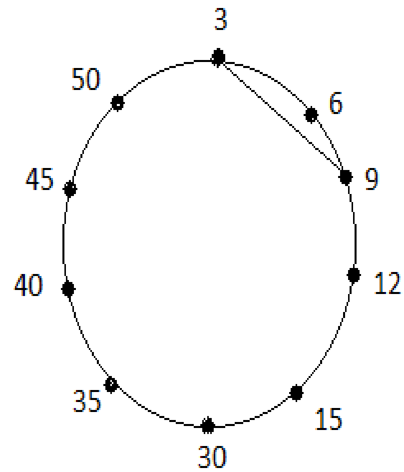
$f^*(u_nu_1) = \dfrac{5n-2}{2} \equiv 0(mod\ 3)$

$f^*(u_1u_3) = 3 \equiv 0(mod\ 3)$

It is observed as

$e_f(0) = k$

$e_f(1) = k+1$



$C_{10} + u_1 u_3$

Subcase (iv): $k \equiv 0(mod\ 3)$.

The vertex labels are,

$$f(u_i) = \begin{cases} 3i & 1 \le i \le k \\ 5i & k+1 \le i \le n \end{cases}$$

The induced edge labels are,

For $1 \le i \le k-1$

$f^*(u_i u_{i+1}) = 2$

For $k+1 \le i \le n-1$

$f^*(u_i u_{i+1}) = 3 \equiv 0(mod\ 3)$

$$f^*(u_k u_{k+1}) = k + 3 \equiv 0(mod\ 3)$$

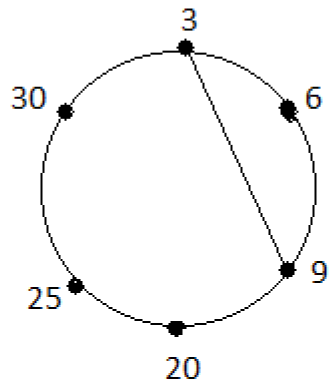
$f^*(u_n u_1) = \frac{5n-2}{2} \equiv 2(mod\ 3)$

$f^*(u_1 u_3) = 3 \equiv 0(mod\ 3)$

It is observed as

$e_f(0) = k$

$e_f(1) = k+1$



$C_6 + u_1u_3$

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