On $(a,d)-1$- Vertex- Antimagic Labeled Graphs

Beena S
Associate Professor
Department of Mathematics
NSS College, Nilamel, Kollam, Kerala, India

Abstract

Let G be a graph on $p-$ vertices. Then a vertex labeling $f:V(G)\to\{1,2,...,p\}$ is called $(a,d)-1-$ vertex antimagic - vertex (1-VAV) labeling of G, if the vertex weight induced by f at each vertex of V are $a, a+d, a+2d,..., a+(p-1)d$ where a and d are some fixed positive integers. In this work, we have proved every graph is an induced subgraph of a regular $(a,d)-1-$ VA graph and some of its properties.

Key words : $(a,d)-1$ *VAV labeling,* Σ' *labeling, regular graph.*

I. INTRODUCTION

All graphs considered in this paper are finite, undirected simple graphs.

Definition [1] 1. Let $G=(V,E)$ be a graph on p vertices. Let $N[u]=N(u)\cup\{u\}$ be the closed neighborhood of u in G. The graph G is said to be Σ' labeled graph or Σ' graph if there exists a bijection $f:V(G)\to\{1,2,..., p\}$ such that for all $u\in v(G)$, $\sum_{v\in N[u]} f(v)$ is a constant independent of u. The constant sum is denoted by S' and f is called a Σ' labeling of G.

Theorem [2] 2. Every graph is an induced subgraph of some connected r - regular graph for some $r>0$.

II. MAINRESULTS

Proposition 3. If the graph G has two distinct vertices u and v such that $N(u)=N(v)$, then G is not $(a,d)-1-$ VA graph.

Proof: If possible, suppose that an $(a,d)-1-$ VA graph has two distinct vertices u and v such that $N(u)=N(v)$. Then $wt(u)=wt(v)$, which is not possible. Since vertex weights are distinct for a $(a,d)-1$ –VA graph. Hence G is not $(a,d)-1-$ VA graph.

Corollary 4. $K_{m_1,m_2,...m_r}$ is $(a,d)-1-$ VA graph if and only if $m_1=1\forall i$.

Proof: Suppose that $K_{m_1,m_2,...m_r}$ is $(a,d)-1-$ VA graph. If possible, let $m_i>1$ for some i, then the vertices in the partite set with m_i - vertices have equal neighborhood. Therefore, $K_{m_1,m_2,...m_r}$ is $(a,d)-1$ VA graph then $m_i=1\ \forall i$.

Conversely, suppose that $m_i=1\ \forall i$. Therefore $K_{m_1,m_2,...m_r}\cong K_r$, which is a $(a,d)-1-$ VA graph where $a=\frac{r(r-1)}{2}$ and $d=1$.

Theorem 5. Every Σ' graph is is $(a,\ d)-1-$ VA graph.

Proof: Let $v_1,v_2,...,v_p$ be the vertices of a Σ' graph G with vertex sum S'. Let $f(v_i)=i$ be the labeling. Then $i+wt(v_i)=S'$. That is, $wt(v_i)=S'-i\ \forall i=1,2,..., p$. Hence G is a $(a,d)-1-$ VA graph with $a=S'-p$ and $d=1$.

Remark. The converse of the above theorem need not be true.

For example, consider the wheel W_5.

Let $G=(V,E)$ be any connected r - regular graph, where $r>0$ with vertex set $\{v_1,v_2,...,v_n\}$. Let G_1^* be another graph with $|V(G_1^*)|=n$ having vertices say $\{v_1',v_2',...,v_n'\}$. Join v_i' to v_i and to the vertices which are adjacent to v_i in G, also v_i' is joined to v_j' whenever v_i and v_j are joined in G. Note that G_1^* is $(2r+1)$- regular graph of order $2n$.

Theorem 6. Every connected r - regular graph, where $r>0$ is an induced subgraph of a connected $(a,d)-1-$ VA graph.

Proof: Let $G=(V,E)$ be any connected r - regular graph, where $r>0$ with vertex set $\{v_1,v_2,...,v_n\}$. Then the graph G_1^* constructed as above is $(2r+1)$- regular graph of order $2n$. Define a function $f:V(G_1^*)\rightarrow\{1,2,...,2n\}$ as follows: $f(v_i)=i$ and $f(v_i')=(2n+1)-i$ for $1\le i\le n$. Clearly f is a bijection and $wt(v_i)=(r+1)(2n+1)-i$, $wt(v_i')=r(2n+1)$, $1\le i\le n$. Therefore, f is a $(r(2n+1)+1,1)-1$ -VAV labeling of G_1^*. Thus G_1^* is a connected $(r(2n+1)+1,1)-1$ -VA graph.

Theorem 7. Every connected graph H is an induced subgraph of a regular $(a,d)-1-$ VA graph.

Proof: By theorem 2 , the graph H is an induced subgraph of some connected r -regular graph G for some $r>0$. Then by theorem 6, the graph G_1^* is a $(2r+1)$ - regular $(a,d)-1-$ VA graph. As G is an induced subgraph of G_1^*, it follows that H is also an induced subgraph of G_1^*. Hence every graph H is an induced subgraph of a regular $(a,d)-1$ VA graph.

Theorem 8. The graph K_1+P_n has $(a,d)-1$ VAV labeling if and only if $n=1,2,4$.

Proof: Suppose that K_1+P_n has $(a,d)-1$ VAV labeling. Let $v_1,v_2,...,v_n$ be the vertices of P_n in this order and vertex of K_1 be v. Note that $wt(v)\ge\frac{n(n+1)}{2}$ and $wt(v_i)\le 3n$ .Therefore $d\ge\frac{n(n+1)}{2}-3n$. That is, $n(n+1)-6n\le 2$, which implies $n(n-5)\le 2$, which is true only when $n=1,2,3,4,5$.

(i) When $n=1$, K_1+P_1 has (1, 1) – 1 - VAV labeling.

(ii) When $n=2$, $K_1+P_2\cong K_3$ has (3, 1) – 1 –VAV labeling.

(iii) When $n=3$ $n=3$

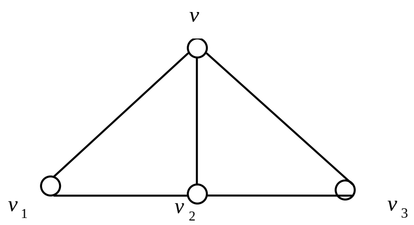

Figure 1

From the figure 1, $N(v_1)=\{v,v_2\}=N(v_3)$. Therefore by proposition 3, K_1+P_3 has no $(a,1)-1$ VAV labeling.

(iv) When $n=4$, K_1+P_4 has $(7,\ 1)-1-$ VAV labeling.

(v) When $n=5$. Let $f(v)=x$, $f(v_1)=a$, $f(v_2)=b$, $f(v_3)=c$, $f(v_4)=d$, $f(v_5)=e$. Then the possible values of x are 6 or 5. If possible, assume that $x=6$. Then $wt(x)=15$. Therefore, $10\le wt(a)<15$. $wt(a)\ge 10$ implies that $b\ge 4$. That is $b=4$ or 5.

Let $b=4$, then $wt(a)=10$. Therefore, $10<wt(e)<15$. That is $10<d+6<15$, Which implies $4<d<9$. Therefore $d=5$. Hence $wt(c)=15=wt(x)$, which is impossible. Similarly if $b=5$, we get $d=4$ and $wt(c)=wt(x)$, which is not possible. Therefore, $x\ne 6$. If possible, let $x=5$. Then $wt(x)=16$. Therefore,

$11 \leq wt(a) < 16$. That is, $11 \leq 5 + b < 16$. This implies $6 \leq b < 11$. Therefore $b = 6$ and $wt(a) = 11$. Now, $11 < wt(e) < 16$ implies $11 < d + 5 < 16$. That is, $6 < d < 9$, Which is not possible. Therefore, $K_1 + P_5$ has no $(a, 1) - 1 -$ VAV labeling. Hence $K_1 + P_n$ has $(a, d) - 1 -$ VAV labeling if and only if $n = 1,2,4$.

III CONCLUSIONS

If the graph G has two distinct vertices u and v such that $N(u) = N(v)$, then G is not $(a, d) - 1 -$ VA graph. Every Σ' graph is $(a, d) - 1 -$ VA graph and the converse is need not be true. Every connected graph H is an induced subgraph of a regular $(a, d) - 1 -$ VA graph. The graph $K_1 + P_n$ has $(a, d) - 1 -$ VAV labeling if and only if $n = 1,2,4$

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