On Generalization of Generalized Regular b-Open Sets in Topological Spaces

B. Sorna Praba[#1], M. J. Jeyanthi*[2]

[#]M.Phil., Scholar, Department of Mathematics, Aditanar College of Arts and Science, Tiruchendur, Tamil Nadu, India

[*]Assistant Professor of Mathematics, Aditanar College of Arts and Science, Tiruchendur, Tamil Nadu, India.

Abstract - *Using the concept of $ggrb$-closed sets, we introduce $ggrb$-open sets. Also we introduce and study the topological properties of $ggrb$- neighborhood, $ggrb$-limit point, $ggrb$-interior, $ggrb$-closure of a set in topological spaces.*

Keywords - *$ggrb$-open set, $ggrb$- neighborhood, $ggrb$-limit point, $ggrb$-interior, $ggrb$-closure.*

I. INTRODUCTION

In 1970, N. Levine [4] introduced the class of generalized closed sets in topological spaces. Different types of generalization of closed sets were introduced and studied by various mathematicians. The notion of generalization of generalized regular b-closed set and its properties are given in [7]. In this paper, we introduce generalization of generalized regular b-open sets and introduce the notions of $ggrb$-neighborhood, $ggrb$-limit points, $ggrb$-interior, $ggrb$-closure of a set and study their properties.

II. PRELIMINARIES

Throughout this paper (X, τ) or X represents a topological space on which no separation axiom is assumed unless otherwise mentioned. For a subset A of a topological space X, $cl(A)$ and $int(A)$ denote the closure of A and interior of A respectively. We recall some definitions and theorems.

Definition 2.1: Let X be a topological space. A subset A of X is said to be

1. Regular open [8] if $A = int(cl(A))$.
2. b-open [1] if $A \subseteq int(cl(A)) \cup cl(int(A))$.

Definition 2.2: A subset A of a topological space X is said to be
1. Generalized closed (g-closed) [4] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
2. Regular b-closed (rb-closed) [5] if $rcl(A) \subseteq U$ whenever $A \subseteq U$ and U is b-open in X.
3. Regular ^generalized closed ($r\hat{}g$-closed) [6] if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.

The complements of the sets mentioned above are their open sets respectively.

Definition 2.3: Let (X,τ) be a topological space and $A \subseteq X$. Then g-closure [2] and r-closure of A is denoted by $gcl(A)$ and $rcl(A)$ and defined as the intersection of all g-closed sets and regular closed sets containing A respectively.

Definition 2.4: Let X be a topological space and $A \subseteq X$. Then g-interior [2] and $r\hat{}g$-interior [6] of A is denoted by $gint(A)$ and $r\hat{}gint(A)$ and defined as the union of all g-open sets and $r\hat{}g$-open sets contained in A respectively.

Definition 2.5: [7] A subset A of a topological space X is called the generalization of generalized regular b-closed ($ggrb$-closed) set if $gcl(A) \subseteq U$ whenever $A \subseteq U$ and U is rb-open in X. We use the notation $GGRBC(X)$ to denote the set of all $ggrb$-closed sets in X.

Remark 2.6: Let A be any subset of X. Then $gcl(X - A) = X - gint(A)$.

Theorem 2.7: [7]

(i) Every closed set is $ggrb$-closed.

(ii) Every g-closed set is $ggrb$-closed.

(iii) Every $ggrb$-closed set is $r\hat{}g$-closed.

Theorem 2.8: [7] If a subset A of a topological space (X,τ) is $ggrb$-closed, then $gcl(A) - A$ does not contain any non-empty rb-closed set in (X,τ).

Theorem 2.9: [7] Union of two $ggrb$-closed set is a $ggrb$-closed set.

III. THE GENERALIZATION OF GENERALIZED REGULAR *b*-OPEN SET

In this section, we introduce $ggrb$-open set and derive its properties. Also we define $ggrb$-neighborhood and discuss some of its properties.

Definition 3.1: A subset A of a topological space (X,τ) is called a $ggrb$-open set in X if $X - A$ is a $ggrb$-closed set in (X,τ). We use the notation $GGRBO(X)$ to denote the set of all $ggrb$-open sets in X.

Theorem 3.2: If A and B are $ggrb$-open sets in a topological space (X,τ), then $A \cap B$ is also a $ggrb$-open set in (X,τ).

Proof: Let A and B be $ggrb$-open sets in (X,τ). Then $X - A$ and $X - B$ are $ggrb$-closed sets in (X,τ). Therefore by Theorem 2.8, $(X - A) \cup (X - B)$ is $ggrb$-closed in (X,τ). But $(X - A) \cup (X - B) = X - (A \cap B)$. Therefore $X - (A \cap B)$ is $ggrb$-closed in (X,τ). Hence $A \cap B$ is a $ggrb$-open set in (X,τ).

Remark 3.3: The union of two $ggrb$-open sets need not be $ggrb$-open. For example, let $X = \{a,b,c,d\}$ and $\tau = \{\emptyset,\{a\},\{b\},\{a,b\},\{a,b,c\},X\}$ be a topology on X. Then $ggrb$-open sets are $\emptyset,\{a\},\{b\},\{c\},\{d\},\{a,b\}$, $\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\}$, $\{a,b,d\}$ and X. Take $A = \{c\}$ and $B = \{d\}$. Then A and B are $ggrb$-open sets but $A \cup B = \{c,d\}$ is not a $ggrb$-open set in X.

Theorem 3.4: A subset A of a topological space X is $ggrb$-open if and only if $U \subseteq gint(A)$ whenever $U \subseteq A$ and U is rb-closed in X.

Proof: Necessity: Suppose A is $ggrb$-open in X. Let U be any rb-closed in X such that $U \subseteq A$. Then $X - A \subseteq X - U$ and $X - U$ is rb-open in X. Since $X - A$ is $ggrb$-closed, we have $gcl(X - A) \subseteq X - U$. But $gcl(X - A) = X - gint(A)$. Therefore $X - gint(A) \subseteq X - U$. Hence $U \subseteq gint(A)$.

Sufficiency: Suppose $U \subseteq gint(A)$ whenever $U \subseteq A$ and U is rb-closed in X. Let U be any rb-open set in X such that $X - A \subseteq U$. Then $X - U \subseteq A$ and $X - U$ is rb-closed in X. By hypothesis $X - U \subseteq gint(A)$ and so $X - gint(A) \subseteq U$. Therefore $gcl(X - A) \subseteq U$. Therefore $X - A$ is a $ggrb$-closed set in X. Hence A is $ggrb$-open in X.

Theorem 3.5: Let A and B be any two subsets of a topological space X. If $gint(A) \subseteq B \subseteq A$ and A is $ggrb$-open in X, then B is $ggrb$-open in X.

Proof: Suppose that $gint(A) \subseteq B \subseteq A$ and A be $ggrb$-open in X. Let F be any rb-closed set in X such that $F \subseteq$ B. Since $F \subseteq B$ and $B \subseteq A$, $F \subseteq A$. Since A is $ggrb$-open, we have $F \subseteq gint(A)$. But $gint(A) \subseteq B$, so $gint(A) \subseteq gint(B)$. Therefore $F \subseteq gint(B)$. Hence B is $ggrb$-open in X.

Theorem 3.6: If $A \subseteq X$ is a $ggrb$-closed set, then $gcl(A) - A$ is a $ggrb$-open set in X.

Proof: Let A be a $ggrb$-closed set in X. Let F be any rb-closed set in X such that $F \subseteq gcl(A) - A$. Then by Theorem 2.7, $F = \emptyset$. Therefore $F \subseteq gint(gcl(A) - A)$. Hence $gcl(A) - A$ is a $ggrb$-open set in X.

Remark 3.7: The converse of Theorem 3.6 need not be true. For example, let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ be a topology on X. Then the g-closed sets are $\emptyset, \{d\}, \{a, d\}, \{b, d\}, \{c, d\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}, X$; $ggrb$-closed sets are $\emptyset, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}$ and X. Take $A = \{a, b\}$. Then $gcl(A) - A = \{a, b, d\} - \{a, b\} = \{d\}$ is $ggrb$-open in X. But A is not a $ggrb$-closed set in X.

Theorem 3.8: Let A and B be subsets of X. If B is $ggrb$-open and $gint(B) \subseteq A$, then $A \cap B$ is a $ggrb$-open set in X.

Proof: Let B be $ggrb$-open in X and $gint(B) \subseteq A$. Clearly, $gint(B) \subseteq B$. Therefore $gint(B) \subseteq A \cap B \subseteq B$. Then by Theorem 3.5, $A \cap B$ is $ggrb$-open in X.

Theorem 3.9: If a subset A is $ggrb$-open in X and G is rb-open in X with $gint(A) \cup (X - A) \subseteq G$, then $X = G$.

Proof: Suppose G is rb-open in X with $gint(A) \cup (X - A) \subseteq G$. This implies $X - G \subseteq \big(X - gint(A)\big) \cap A = gcl(X - A) \cap A = (gcl(X - A)) - (X - A)$. Since $X - A$ is $ggrb$-closed and $X - G$ is rb-closed, by Theorem 2.7, $X - G = \emptyset$. Hence $X = G$.

Definition 3.10: Let X be a topological space and $x \in X$. A subset N of X is said to be a $ggrb$-neighborhood of x if there exists a $ggrb$-open set G such that $x \in G \subseteq N$.

Definition 3.11 : Let (X,τ) be a topological space and A be a subset of X. A subset N of X is said to be a $ggrb$-neighborhood of A if there exists a $ggrb$-open set G such that $A \subseteq G \subseteq N$.

Theorem 3.12: Every neighborhood N of x belongs to X is a $ggrb$-neighborhood of x.

Proof: Let $x \in X$ and N be a neighborhood of x. Then there exists an open set G such that $x \in G \subseteq N$. Since every open set is $ggrb$-open, we have a $ggrb$-open set G such that $x \in G \subseteq N$. Hence N is a $ggrb$-neighborhood of x.

Remark 3.13: In general, a $ggrb$-neighborhood of x need not be a neighborhood of x. For example, let $X = \{a,b,c,d\}$ and $\tau = \{\emptyset,\{a\},\{b\},\{a,b\},\{a,b,c\},X\}$ be a topology on X. Since $d \in \{d\} \subseteq \{a,b,d\}$, we have $\{a,b,d\}$ is a $ggrb$-neighborhood of d. But $\{a,b,d\}$ is not a neighborhood of d, since there is no open set G such that $d \in G \subseteq \{a,b,d\}$.

Theorem 3.14: Every $ggrb$-open set is a $ggrb$-neighborhood of each of its points.

Proof: Suppose N is a $ggrb$-open set in X. Let $x \in N$. Then by Definition 3.10, we have N is a $ggrb$-neighborhood of x. Hence N is a $ggrb$-neighborhood of each of its points.

Remark 3.15: The converse of Theorem 3.14 need not be true. For example let $X = \{a,b,c,d\}$ and $\tau = \{\emptyset,\{a\},\{b\},\{a,b\},\{a,b,c\},X\}$ be a topology on X. Then $GGRBO(X) = \{\emptyset,\{a\},\{b\},\{c\},\{d\},\{a,b\},\{a,c\},\{a,d\},\{b,c\},\{b,d\},\{a,b,c\},\{a,b,d\}, X\}$. Take $N = \{c,d\}$. Then there exist $ggrb$-open sets $\{c\},\{d\}$ such that $c \in \{c\} \subseteq N$ and $d \in \{d\} \subseteq N$. Therefore N is a $ggrb$-neighborhood of c and d but it is not a $ggrb$-open set in X.

Theorem 3.16: Let X be a topological space. If F is a $ggrb$-closed subset of X and $x \in X - F$, then there exists a $ggrb$-neighborhood N of x such that $N \cap F = \emptyset$.

Proof: Let F be a $ggrb$-closed subset of X and $x \in X - F$. Since $X - F$ is a $ggrb$-open set in X, we have $X - F$ is a $ggrb$-neighborhood of each of its points. Hence there exists a $ggrb$-neighborhood N of x such that $N \subseteq X - F$. Hence $N \cap F = \emptyset$.

Definition 3.17: The collection of all $ggrb$-neighborhood of $x \in X$ is called a $ggrb$-neighborhood system at x and is denoted by $ggrb - N(x)$.

Theorem 3.18: Let (X,τ) be a topological space and $x \in X$. Then

(i) $ggrb - N(x) \neq \emptyset$ for every $x \in X$.

(ii) If $N \in ggrb - N(x)$, then $x \in N$.

(iii) $N \in ggrb - N(x)$ and M contains N implies $M \in ggrb - N(x)$.

(iv) $N \in ggrb - N(x)$ and $M \in ggrb - N(x)$ implies $N \cap M \in ggrb - N(x)$.

(v) If $N \in ggrb - N(x)$, then there exists $M \in ggrb - N(x)$ such that $M \subseteq N$ and $M \in ggrb - N(y)$ for every $y \in M$.

Proof: (i) Since X is a $ggrb$-open set, it is a $ggrb$-neighborhood of every $x \in X$. Therefore there exists atleast one $ggrb$-neighborhood, namely X for each $x \in X$. Hence $ggrb - N(x) \neq \emptyset$.

(ii) If $N \in ggrb - N(x)$, then N is a $ggrb$-neighborhood of x. Therefore there exists a $ggrb$-open set G such that $x \in G \subseteq N$. Hence $x \in N$.

(iii) Let $N \in ggrb - N(x)$. Suppose M contains N. Since N is a $ggrb$-neighborhood of x, there exists a $ggrb$-open set G such that $x \in G \subseteq N$. Since $N \subseteq M$, we have $x \in G \subseteq M$. Therefore M is a $ggrb$-neighborhood of x. Hence $M \in ggrb - N(x)$.

(iv) Let $N \in ggrb - N(x)$ and $M \in ggrb - N(x)$. Then there exist $ggrb$-open sets G_1 and G_2 such that $x \in G_1 \subseteq N$ and $x \in G_2 \subseteq M$. This implies $x \in G_1 \cap G_2 \subseteq N \cap M$. Since intersection of two $ggrb$-open set is $ggrb$-open, we have $G_1 \cap G_2$ is $ggrb$-open. Therefore $N \cap M$ is a $ggrb$-neighborhood of x. Hence $N \cap M \in ggrb - N(x)$.

(v) Let $N \in ggrb - N(x)$. Then there exists a $ggrb$-open set M such that $x \in M \subseteq N$. Since M is a $ggrb$-open set, it is a $ggrb$-neighborhood of each of its points. Hence $M \in ggrb - N(y)$ for every $y \in M$.

IV. $ggrb$-INTERIOR AND $ggrb$-CLOSURE OF A SET

In this section, we introduce the concepts of $ggrb$-interior and $ggrb$-closure and derive their properties.

Definition 4.1: Let A be a subset of a topological space (X, τ). Then $ggrb$-interior of A is denoted by $ggrbint(A)$ and defined as the union of all $ggrb$-open sets contained in A.

That is, $ggrbint(A) = \cup \{G : G \subseteq A$ and G is $ggrb$-open in $X\}$.

Definition 4.2: Let A be a subset of a topological space X. A point $x \in X$ is said to be a $ggrb$-interior point of A if there exists a $ggrb$-open set G such that $x \in G \subseteq A$.
Equivalently, by Definition 3.10, x is a $ggrb$-interior point of A if A is the $ggrb$-neighborhood of x.

Theorem 4.3: Let A be a subset of a topological space X. Then $ggrbint(A)$ is the set of all $ggrb$-interior points of A.

Proof: $x \in ggrbint(A) \Leftrightarrow x \in \cup \{G : G \subseteq A$ and G is $ggrb$-open set in $X\}$.

$\Leftrightarrow$ There exists a $ggrb$-open set G such that $x \in G \subseteq A$.

$\Leftrightarrow$ x is a $ggrb$-interior point of A.

This shows that $ggrbint(A)$ consists of all $ggrb$-interior points of A.

Theorem 4.4: Let A and B be subsets of a topological space (X, τ). Then

(i) $ggrbint(X) = X$ and $ggrbint(\emptyset) = \emptyset$.

(ii) $ggrbint(A) \subseteq A$.

(iii) If B is a $ggrb$-open set contained in A, then $B \subseteq ggrbint(A)$.

(iv) If $A \subseteq B$, then $ggrbint(A) \subseteq ggrbint(B)$.

(v) $ggrbint(A) = ggrbint(ggrbint(A))$.

Proof: Let A and B be subsets of (X, τ).

(i) Since X is $ggrb$-open and $ggrbint(X)$ is the union of all $ggrb$-open set contained in X, we have $ggrbint(X) = X$. Also since $\emptyset$ is the only $ggrb$-open set contained in $\emptyset$, $ggrbint(\emptyset) = \emptyset$.

(ii) Directly follows from the definition $ggrb$-interior.

(iii) Let B be any $ggrb$-open set contained in A. Since $ggrbint(A)$ is the union of all $ggrb$-open set contained in A, $ggrbint(A)$ contains every $ggrb$-open sets which is contained in A. Therefore $B \subseteq ggrbint(A)$.

(iv) Suppose $A \subseteq B$. Let $x \in ggrbint(A)$. Then x is a $ggrb$-interior point of A. Therefore there exists a $ggrb$-open set G such that $x \in G \subseteq A$. Since $A \subseteq B$, $x \in G \subseteq B$. Therefore x is $ggrb$-interior point of B. This implies $x \in ggrbint(B)$. Hence $ggrbint(A) \subseteq ggrbint(B)$.

(v) If $G \subseteq A$ and G is $ggrb$-open in X, then by (iii), $G \subseteq ggrbint(A)$. Again by using (iii), $G \subseteq ggrbint(ggrbint(A))$. Therefore $\cup\{G: G \subseteq A, G \text{ is } ggrb\text{-open in } X\} \subseteq ggrbint(ggrbint(A))$.That is, $ggrbint(A) \subseteq ggrbint(ggrbint((A))$. By (ii), $ggrbint(ggrbint(A)) \subseteq ggrbint(A)$. Hence, $ggrbint(A) = ggrbint\big(ggrbint(A)\big)$.

Theorem 4.5 : Let A and B be subsets of a topological space (X, τ). Then

(i) $ggrbint(A) \cup ggrbint(B) \subseteq ggrbint(A \cup B)$.

(ii) $ggrbint(A \cap B) = ggrbint(A) \cap ggrbint(B)$.

Proof: Let A and B be subsets of X.

(i) Since $A \subseteq A \cup B$, $ggrbint(A) \subseteq ggrbint(A \cup B)$. Also $ggrbint(B) \subseteq ggrbint(A \cup B)$, since $B \subseteq A \cup B$. Therefore $ggrbint(A) \cup ggrbint(B) \subseteq ggrbint(A \cup B)$.

(ii) Again $A \cap B \subseteq A$ implies $ggrbint(A \cap B) \subseteq ggrbint(A)$. Also $A \cap B \subseteq B$ implies $ggrbint(A \cap B) \subseteq ggrbint(B)$. Therefore $ggrbint(A \cap B) \subseteq ggrbint(A) \cap ggrbint(B)$. On the other hand, let $x \in ggrbint(A) \cap ggrbint(B)$. Then $x \in ggrbint(A)$ and $x \in ggrbint(B)$. Then there exist $ggrb$-open sets $A_1 \subseteq A$ and $B_1 \subseteq B$ such that $x \in A_1$ and $x \in B_1$. By Theorem 3.2, $A_1 \cap B_1$ is a $ggrb$-open set contained in $A \cap B$ such that $x \in A_1 \cap B_1$. Then by Theorem 4.4 (iii), $A_1 \cap B_1 \subseteq ggrbint(A \cap B)$. This implies $x \in ggrbint(A \cap B)$. Therefore $ggrbint(A) \cap ggrbint(B) \subseteq ggrbint(A \cap B)$. Hence $ggrbint(A \cap B) = ggrbint(A) \cap ggrbint(B)$.

Remark 4.6: In general, $ggrbint(A \cup B)$ is not a subset of $ggrbint(A) \cup ggrbint(B)$. For example, let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ be a topology on X. Take $A = \{a\}$ and $B = \{c\}$. Then $ggrbint(A \cup B) = \{a, c\}$. But $ggrbint(A) \cup ggrbint(B) = \{a\}$.

Theorem 4.7: If a subset A of a topological space (X,τ) is $ggrb$-open, then $ggrbint(A) = A$.

Proof: Let A be a $ggrb$-open set of X. Then by Theorem 4.4 (iii), $A \subseteq ggrbint(A)$. Also, by Theorem 4.4 (ii), $ggrbint(A) \subseteq A$. Hence $ggrbint(A) = A$.

Remark 4.8: The converse of Theorem 4.7 need not be true. For example, let $X = \{a,b,c,d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}, X\}$ be a topology on X. Take $A = \{b,c,d\}$. Then $ggrbint(A) = A$. But A is not a $ggrb$-open set in X.

Theorem 4.9: For any subset A of X, $int(A) \subseteq gint(A) \subseteq ggrbint(A) \subseteq r\char`^gint(A)$.

Proof: Since every open set is g-open, we have $int(A) \subseteq gint(A)$. Since every g-open set is $ggrb$-open, $gint(A) \subseteq ggrbint(A)$. Also every $ggrb$-open set is $r\char`^g$-open so $ggrbint(A) \subseteq r\char`^gint(A)$. Hence $int(A) \subseteq gint(A) \subseteq ggrbint(A) \subseteq r\char`^gint(A)$.

Definition 4.10: Let A be any subset of (X,τ). Then $ggrb$-closure of A is denoted by $ggrbcl(A)$ and is defined as the intersection of all $ggrb$-closed sets containing A.

That is, $ggrbcl(A) = \cap \{F: A \subseteq F$ and $F \in GGRBC(X)\}$.

Theorem 4.11: Let A and B be two subsets of a topological space (X,τ). Then

(i) $ggrbcl(X) = X$ and $ggrbcl(\emptyset) = \emptyset$.

(ii) $A \subseteq ggrbcl(A)$.

(iii) If B is any $ggrb$-closed set containing A, then $ggrbcl(A) \subseteq B$.

(iv) If $A \subseteq B$, then $ggrbcl(A) \subseteq ggrbcl(B)$.

(v) $ggrbcl(A) = ggrbcl(ggrbcl(A))$.

Proof: Let A and B be subsets of X.

(i) Since $\emptyset$ and X are $ggrb$-closed in X, we have $ggrbcl(X) = X$ and $ggrbcl(\emptyset) = \emptyset$.

(ii) By the definition of $ggrb$-closure of A, $A \subseteq ggrbcl(A)$.

(iii) Let B be any $ggrb$-closed set containing A. Since $ggrbcl(A)$ is the intersection of all $ggrb$-closed sets containing A, $ggrbcl(A)$ is contained in every $ggrb$-closed sets containing A. Therefore $ggrbcl(A) \subseteq B$.

(iv) Let A and B be any two subsets of X such that $A \subseteq B$. Let F be any $ggrb$-closed set such that $B \subseteq F$. Since $A \subseteq F$ and F is $ggrb$-closed, we have $ggrbcl(A) \subseteq F$. Therefore $ggrbcl(A) \subseteq \cap \{F: A \subseteq F$ and $F \in GGRBC(X)\} = ggrbcl(B)$. Hence $ggrbcl(A) \subseteq ggrbcl(B)$.

(v) If $A \subseteq F$ and $F \in GGRBC(X)$, then by (iii) $ggrbcl(A) \subseteq F$. Also by (iii), $ggrbcl(ggrbcl(A)) \subseteq F$. Therefore $ggrbcl\big(ggrbcl(A)\big) \subseteq \cap \{F: A \subseteq F, F \in GGRBC(X)$. That is, $ggrbcl(ggrbcl(A)) \subseteq ggrbcl(A)$. By (ii), $ggrbcl(A) \subseteq ggrbcl(ggrbcl(A))$. Hence $ggrbcl(A) = ggrbcl(ggrbcl(A))$.

Theorem 4.12: Let A and B be two subsets of a topological space (X,τ). Then

(i) $ggrbcl(A \cup B) = ggrbcl(A) \cup ggrbcl(B)$.

(ii) $ggrbcl(A \cap B) \subseteq ggrbcl(A) \cap ggrbcl(B)$.

Proof: (i) $A \subseteq A \cup B$ implies $ggrbcl(A) \subseteq ggrbcl(A \cup B)$. Also $B \subseteq A \cup B$ implies $ggrbcl(B) \subseteq ggrbcl(A \cup B)$. Therefore $ggrbcl(A) \cup ggrbcl(B) \subseteq ggrbcl(A \cup B)$. We show that $ggrbcl(A \cup B) \subseteq ggrbcl(A) \cup ggrbcl(B)$. Let $x \in ggrbcl(A \cup B)$. Suppose $x \notin ggrbcl(A) \cup ggrbcl(B)$. Then there exist $ggrb$-closed sets A_1 containing A and B_1 containing B such that $x \notin A_1 \cup B_1$. Therefore $A_1 \cup B_1$ is a $ggrb$-closed set containing $A \cup B$. Then by Theorem 4.11 (iii), $ggrbcl(A \cup B) \subseteq A_1 \cup B_1$. Therefore $x \notin ggrbcl(A \cup B)$. This contradiction shows that $ggrbcl(A \cup B) \subseteq ggrbcl(A) \cup ggrbcl(B)$. Hence $ggrbcl(A \cup B) = ggrbcl(A) \cup ggrbcl(B)$.

(ii) $A \cap B \subseteq A$ implies $ggrbcl(A \cap B) \subseteq ggrbcl(A)$. Also $A \cap B \subseteq B$ implies $ggrbcl(A \cap B) \subseteq ggrbcl(B)$.Therefore $ggrbcl(A \cap B) \subseteq ggrbcl(A) \cap ggrbcl(B)$.

Remark 4.13: In general, $ggrbcl(A) \cap ggrbcl(B)$ is not a subset of $ggrbcl(A \cap B)$. For example, let $X = \{a, b, c\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, X\}$ be a topology on X. Then $ggrb$-closed sets are $\emptyset, \{b\}, \{c\}, \{b, c\}, \{a, c\}$ and X. Take $A = \{a\}$ and $B = \{c\}$. Then $ggrbcl(A) \cap ggrbcl(B) = \{c\}$. But $A \cap B = \emptyset$. Therefore $ggrbcl(A \cap B) = \emptyset$.

Theorem 4.14: If $A \subseteq X$ is a $ggrb$-closed set, then $ggrbcl(A) = A$.

Proof: Let A be any $ggrb$-closed set in X. Then by Theorem 4.11 (iii), $ggrbcl(A) \subseteq A$. Also, $A \subseteq ggrbcl(A)$. Hence $ggrbcl(A) = A$.

Remark 4.15: The converse of Theorem 4.14 need not be true. For example, let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ be a topology on X. Take $A = \{a\}$. Then $ggrbcl(A) = A$. But A is not a $ggrb$-closed set in X.

Theorem 4.16: Let A be any subset of a topological space X. Then $x \in ggrbcl(A)$ if and only if $U \cap A \neq \emptyset$ for every $ggrb$-open set U containing x.

Proof: Necessity: Suppose $x \in ggrbcl(A)$. We show that $U \cap A \neq \emptyset$ for every $ggrb$-open set U containing x. If there is a $ggrb$-open set U in X such that $x \in U$ and $U \cap A = \emptyset$, then $X - U$ is $ggrb$-closed in X such that $A \subseteq X - U$. Therefore $ggrbcl(A) \subseteq X - U$. Since $x \in ggrbcl(A)$, we have $x \in X - U$. This is a contradiction, since $x \in U$. Hence $U \cap A \neq \emptyset$ for every $ggrb$-open set U containing x.

Sufficiency: Suppose $U \cap A \neq \emptyset$ for every $ggrb$-open set U containing x. If $x \notin ggrbcl(A)$, then there exists a $ggrb$-closed set F containing A such that $x \notin F$. Therefore $X - F$ is $ggrb$-open containing x such that $(X - F) \cap A = \emptyset$. This is a contradiction to our assumption. Hence $x \in ggrbcl(A)$.

V. THE GENERALIZATION OF GENERALIZED REGULAR *b*-DERIVED SET

In this section, we define $ggrb$-limit point of a set and give some of its properties.

Definition 5.1: Let A be a subset of a topological space X. A point $x \in X$ is said to be $ggrb$-limit point A if $G \cap (A - \{x\}) \neq \emptyset$ for every $ggrb$-open set G containing x.

Definition 5.2: The set of all $ggrb$-limit points of A is called a $ggrb$-derived set of A and is denoted by $D_{ggrb}(A)$.

Theorem 5.3: Let A and B be subsets of (X, τ). Then the following are hold.

(i) $D_{ggrb}(\emptyset) = \emptyset$.

(ii) $x \in D_{ggrb}(A)$ implies $x \in D_{ggrb}(A - \{x\})$.

(iii) If $A \subseteq B$, then $D_{ggrb}(A) \subseteq D_{ggrb}(B)$.

(iv) $D_{ggrb}(A) \cup D_{ggrb}(B) \subseteq D_{ggrb}(A \cup B)$ and $D_{ggrb}(A \cap B) \subseteq D_{ggrb}(A) \cap D_{ggrb}(B)$.

(v) $D_{ggrb}(D_{ggrb}(A)) - A \subseteq D_{ggrb}(A)$.

(vi) $D_{ggrb}(A \cup D_{ggrb}(A)) \subseteq A \cup D_{ggrb}(A)$.

Proof: (i) Obviously, $D_{ggrb}(\emptyset) = \emptyset$.

(ii) If $x \in D_{ggrb}(A)$, then x is a $ggrb$-limit point of A. Hence every $ggrb$-open set G containing x contains atleast one point of A other than x. Therefore x is a $ggrb$-limit point of $A - \{x\}$. Hence $x \in D_{ggrb}(A - \{x\})$.

(iii) Suppose $A \subseteq B$. Let $x \in D_{ggrb}(A)$. Then $G \cap (A - \{x\}) \neq \emptyset$ for every $ggrb$-open set G containing x. Since $A - \{x\} \subseteq B - \{x\}$, $G \cap (B - \{x\}) \neq \emptyset$. Therefore $x \in D_{ggrb}(B)$. Hence $D_{ggrb}(A) \subseteq D_{ggrb}(B)$.

(iv) Since $A \subseteq A \cup B$ and $B \subseteq A \cup B$, by (iii) $D_{ggrb}(A) \subseteq D_{ggrb}(A \cup B)$ and $D_{ggrb}(B) \subseteq D_{ggrb}(A \cup B)$. Therefore $D_{ggrb}(A) \cup D_{ggrb}(B) \subseteq D_{ggrb}(A \cup B)$. Also since $A \cap B \subseteq A$ and $A \cap B \subseteq B$, by (iii) $D_{ggrb}(A \cap B) \subseteq D_{ggrb}(A)$ and $D_{ggrb}(A \cap B) \subseteq D_{ggrb}(B)$. Therefore $D_{ggrb}(A \cap B) \subseteq D_{ggrb}(A) \cap D_{ggrb}(B)$.

(v) Let $x \in D_{ggrb}(D_{ggrb}(A)) - A$. Then $x \in D_{ggrb}(D_{ggrb}(A))$ and $x \notin A$. Therefore $G \cap (D_{ggrb}(A) - \{x\}) \neq \emptyset$ for every $ggrb$-open set G containing x. Let $y \in G \cap (D_{ggrb}(A) - \{x\})$. Then $y \in G$ and $y \in D_{ggrb}(A)$ with $y \neq x$. Therefore $G \cap (A - \{y\}) \neq \emptyset$. Take $z \in G \cap (A - \{y\})$. Then $z \neq x$, since $x \notin A$. This implies $G \cap (A - \{x\}) \neq \emptyset$. Therefore $x \in D_{ggrb}(A)$. Hence $D_{ggrb}(D_{ggrb}(A)) - A \subseteq D_{ggrb}(A)$.

(vi) Let $x \in D_{ggrb}(A \cup D_{ggrb}(A))$. If $x \in A$, then there is nothing to prove. Suppose $x \notin A$. Since $x \in D_{ggrb}(A \cup D_{ggrb}(A))$, we have $G \cap ((A \cup D_{ggrb}(A)) - \{x\}) \neq \emptyset$ for every $ggrb$-open set G containing x. Then either $G \cap (A - \{x\}) \neq \emptyset$ or $G \cap (D_{ggrb}(A) - \{x\}) \neq \emptyset$. If $G \cap (A - \{x\}) \neq \emptyset$, then $x \in D_{ggrb}(A)$. If $G \cap (D_{ggrb}(A) - \{x\}) \neq \emptyset$, then $x \in D_{ggrb}(D_{ggrb}(A))$. Since $x \notin A$, we have $x \in D_{ggrb}(D_{ggrb}(A)) - A$. Then by (v), $x \in D_{ggrb}(A)$. Hence $D_{ggrb}(A \cup D_{ggrb}(A)) \subseteq A \cup D_{ggrb}(A)$.

Theorem 5.4: For any subset A of X, $D_{ggrb}(A) \subseteq ggrbcl(A)$.

Proof: Let $x \in D_{ggrb}(A)$. Then $G \cap (A - \{x\}) \neq \emptyset$ for every $ggrb$-open set G containing x. Therefore $G \cap A \neq \emptyset$ for every $ggrb$-open set G containing x. Then by Theorem 4.16, $x \in ggrbcl(A)$. Hence $D_{ggrb}(A) \subseteq ggrbcl(A)$.

Theorem 5.5: Let A be any subset of X. Then $ggrbcl(A) = A \cup D_{ggrb}(A)$.

Proof: Let $x \in ggrbcl(A)$. If $x \in A$, then $ggrbcl(A) \subseteq A \cup D_{ggrb}(A)$. Suppose $x \notin A$. Since $x \in ggrbcl(A)$, by Theorem 4.16 $G \cap A \neq \emptyset$ for every $ggrb$-open set G containing x. Since $x \notin A$, we have $G \cap (A - \{x\}) \neq \emptyset$ for every $ggrb$-open set G containing x. Therefore $x \in D_{ggrb}(A)$. Hence $ggrbcl(A) \subseteq A \cup D_{ggrb}(A)$. Since $A \subseteq ggrbcl(A)$ and $D_{ggrb}(A) \subseteq ggrbcl(A)$, we have $A \cup D_{ggrb}(A) \subseteq ggrbcl(A)$. Hence $ggrbcl(A) = A \cup D_{ggrb}(A)$.

Theorem 5.6: For any subset A of a discrete topological space X, then $D_{ggrb}(A) = \emptyset$.

Proof: Let A be any subset of a discrete topological space X. Let $x \in X$. Since X is a discrete topological space, every subset of X is open. Since every open set is $ggrb$-open, every subset of X is $ggrb$-open. In particular, $G = \{x\}$ is $ggrb$-open and hence $G \cap (A - \{x\}) = \emptyset$. Then x is not a $ggrb$-limit point of A. Therefore $D_{ggrb}(A) = \emptyset$.

Theorem 5.7: For any subset A of X, $ggrbint(A) = A - D_{ggrb}(X - A)$.s

Proof: Let $x \in ggrbint(A)$. Then there exists a $ggrb$-open set G such that $x \in G \subseteq A$. Therefore $G \cap (X - A) = \emptyset$, where G is a $ggrb$-open set containing x and hence $x \notin D_{ggrb}(X - A)$ so $x \in A - D_{ggrb}(X - A)$. Hence $ggrbint(A) \subseteq A - D_{ggrb}(X - A)$. On the other hand, let $x \in A - D_{ggrb}(X - A)$. Then $x \notin D_{ggrb}(A)$. Therefore there exists a $ggrb$-open set G containing x such that $G \cap (X - A) = \emptyset$. Then $x \in G \subseteq A$. This implies x is a $ggrb$-interior point of A. Therefore $x \in ggrbint(A)$. Hence $A - D_{ggrb}(X - A) \subseteq ggrbint(A)$. Therefore $ggrbint(A) = A - D_{ggrb}(X - A)$.

VI. CONCLUSION

In this paper, we have introduced a new type of sets, namely, $ggrb$-open sets in topological spaces. Also we have introduced $ggrb$-interior, $ggrb$-closure, $ggrb$-neighborhood, $ggrb$-derived set and established some of its properties.

ACKNOWLEDGMENT

The authors would like to express their sincere thanks to the referees for their valuable suggestions and helpful comments in improving this paper.

REFERENCES

[1] D. Andrijivic, *On b-open sets*, Mat. Vesnik, 48(1-2), 59-64 (1996).

[2] W. Dunhum, *A new closure operator for non T_1-topologies*, Kyungpook Math, 22, 56-60 (1982).

[3] Md. Hanif PAGE, V. T. Hasamath, *Some Properties of θsgp- neighborhoods*, International Journal of Mathematics Trends and Technology, Vol. (20), No. 1, 47-54 (2015).

[4] N. Levine, *Generalized closed sets in topology*, Rend. Circ. Math. Palermo, 19(2), 89-96 (1996).

[5] A. Narmadha, N. Nagaveni and T. Noiri, *On regular b-open sets in topological spaces*, Int. Journal of Math. Analysis, Vol. 7, n0. 19, 937-948 (2013).

[6] Savithri D and Janaki C, *On regular ^generalized closed sets in topological spaces*, International Journal of Mathematical Archieve-

4(4), 162-169 (2013).

[7] B. Sorna Praba, M. J. Jeyanthi, *On generalization of generalized regular b-closed sets in topological spaces*, Proceedings of the Instructional School of Emerging Trends in Advanced Mathematics (ISETAM'19), 75-83, (2019).

[8] M. Stone, Application *of the Theory of Boolean rings to general topology*, Trans. Amer. Math. Soci, 41, 374-481 (1937).