Complete Anti Fuzzy Graphs

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Abstract
In this paper, we define three new operations on anti fuzzy graphs, namely direct product, semi – strong product and strong product. Moreover, we introduce and study the notion of balanced graph.

Keywords - *anti fuzzy graph, direct product, semi – strong product, strong product, balanced anti fuzzy graph.*

I. INTRODUCTION

The concept of fuzzy graph was first introduced by Kaufmann [1] from the fuzzy relation introduced by Zedah [7]. Although Rosenfield [4] introduced another elaborated definition, including fuzzy vertex and fuzzy edge, and also introduced the notion of fuzzy graph .J.N. Mordeson and P.S. Nair [2] introduced the concept of operations on fuzzy graphs but this concept was extended by M.S. Sunitha and A.Vijaya kumar[6]. R.Muthuraj and A. Sasireka [3] introduced the concept of anti fuzzy graph. R. Seethalakshmi and R.B. Gnanajothi[5] discussed the concept some operations such as anti union and anti join on anti fuzzy graph.

In this paper, we introduce the new operations on anti fuzzy graphs, namely direct product, semi – strong product and strong product and balanced anti fuzzy graph. We derived some theorems and results on them.

II. PRELIMINARIES

Definition 2.1 : A fuzzy graph $G:(\sigma,\mu)$ is said to be an anti fuzzy graph with a pair of functions $\sigma : V\to[0,1]$ and $\mu:V\times V\to[0,1]$ where for all u, v $\in$V. We have $\mu(u,v)\geq\sigma(u) \vee \sigma(v)$ for all u, v $\in$V and it is denoted by

$G_A:(\sigma,\mu)$ where `$\vee$` stands for maximum.

Definition 2.2: An anti fuzzy graph $H_A : (\tau,\rho)$ is called an anti fuzzy subgraph of $G_A:(\sigma,\mu)$ if $\tau(u) \leq \sigma(u)$ for all $u \in V$ and $\rho(u,v) \leq \mu(u,v)$ for all $u, v \in V$

Definition 2.3: The underlying crisp graph of an anti fuzzy graph $G_A:(\sigma,\mu)$ is denoted by $G_A^*:(\sigma^*,\mu^*)$ where $\sigma^*=\{u\in V:\sigma(u)>0\}$ and $\mu^*=\{(u,v) \in V\times V:\mu(u,v)>0\}$.

Definition 2.4: The complement of an anti fuzzy graph $G_A:(\sigma,\mu)$ is an anti fuzzy sub graph $\overline{G_A} : (\bar{\sigma},\bar{\mu})$ where $\bar{\sigma}\equiv\sigma$ and $\bar{\mu}(u,v)=\mu(u,v) - (\sigma(u)\vee\sigma(v))$ for all u, v in V.

III. COMPLETE ANTI FUZZY GRAPH

Definition 3.1: An anti fuzzy graph $G_A : (\sigma,\mu)$ is said to be complete anti fuzzy graph if $\mu(u,v)\geq \sigma(u)\vee\sigma(v)$ for all u, v in V.

Definition 3.2: The direct product of two anti fuzzy graphs $G_{A_1}:(\sigma_1,\mu_1)$ with crisp graph $G_{A_1}^*: (V_1, E_1)$ and

$G_{A_2}: (\sigma_2, \mu_2)$ with crisp graph $G_{A_2}^* : (V_2, E_2)$, where we assume that $V_1 \cap V_2 = \emptyset$ is defined to be the anti fuzzy graph $G_{A_1} \sqcap G_{A_2}: (\sigma_1\sqcap \sigma_2, \mu_1 \sqcap \mu_2)$ with crisp graph $G_A^*: (V_1\times V_2, E)$

where $E = \{(u_1, v_1)(u_2, v_2) : (u_1, u_2)\in E_1, (v_1, v_2) \in E_2\}$,

$(\sigma_1 \sqcap \sigma_2)(u, v) = \sigma_1(u) \vee\sigma_2(v)$, for all $(u, v) \in V_1\times V_2$ and

$\mu_1 \sqcap \mu_2((u_1, v_1)(u_2, v_2))= \mu_1(u_1, u_2) \vee\mu_2(v_1, v_2)$.

Definition 3.3: The semi-strong product of two anti fuzzy graphs $G_{A_1}: (\sigma_1, \mu_1)$ with crisp graph $G_{A_1}^*: (V_1, E_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ with crisp graph $G_{A_2}^* : (V_2, E_2)$; where we assume that $V_1 \cap V_2 = \emptyset$ is defined to be the anti fuzzy graph $G_{A_1}. G_{A_2}: (\sigma_1.\sigma_2, \mu_1.\mu_2)$ with crisp graph $G_A^*: (V_1\times V_2, E)$

where $E = \{(u, v_1)(u, v_2) : u \in V_1, (v_1, v_2) \in E_2\} \cup \{(u_1,v_1)(u_2, v_2) : (u_1,u_2) \in E_1, (v_1, v_2) \in E_2\}$,

$(\sigma_1 . \sigma_2)(u, v) = \sigma_1(u) \vee \sigma_2(v)$, for all $(u, v) \in V_1 \times V_2$ and

$(\mu_1 . \mu_2)((u, v_1)(u, v_2)) = \sigma_1(u) \vee \mu_2(v_1, v_2)$ and

$(\mu_1 . \mu_2)((u_1, v_1)(u_2, v_2)) = \mu_1(u_1, u_2) \vee \mu_2(v_1, v_2)$.

Definition 3.4: The strong product of two anti fuzzy graphs $G_{A_1}: (\sigma_1, \mu_1)$ with crisp graph $G_{A_1}{}^*: (V_1, E_1)$ and G_{A_2}: (σ_2, μ_2) with crisp graph $G_{A_2}{}^*: (V_2, E_2)$ where we assume that $V_1 \cap V_2 = \emptyset$ is defined to be the anti fuzzy graph

$G_{A_1} \otimes G_{A_2}: (\sigma_1 \otimes \sigma_2, \mu_1 \otimes \mu_2)$ with crisp graph $G_A{}^*: (V_1 \times V_2, E)$

where $E = \{(u, v_1)(u, v_2) : u \in V_1, (v_1, v_2) \in E_2\} \cup \{(u_1, w)(u_2, w) : w \in V_2,$

$(u_1, u_2) \in E_1\} \cup \{(u_1,v_1)(u_2, v_2) : (u_1,u_2) \in E_1, (v_1, v_2) \in E_2\}$.

$(\sigma_1 \otimes \sigma_2)(u, v) = \sigma_1(u) \vee \sigma_2(v)$, for all $(u, v) \in V_1 \times V_2$ and

$(\mu_1 \otimes \mu_2)((u, v_1)(u, v_2)) = \sigma_1(u) \vee \mu_2(v_1, v_2)$

$(\mu_1 \otimes \mu_2)((u_1, w)(u_2, w)) = \sigma_2(w) \vee \mu_1(u_1, u_2)$.

$(\mu_1 \otimes \mu_2)((u_1, v_1)(u_2, v_2)) = \mu_1(u_1, u_2) \vee \mu_2(v_1, v_2)$.

Theorem 3.5: If $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ are anti fuzzy complete graphs, then $G_{A_1} \sqcap G_{A_2}$ is complete.

Proof: Let $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ be two anti fuzzy complete graphs.

Let $(u_1, u_2) \in E_1$, $(v_1, v_2) \in E_2$.

Then $\mu_1(u_1, u_2) = \sigma_1(u_1) \vee \sigma_1(u_2)$ and $\mu_2(v_1, v_2) = \sigma_2(v_1) \vee \sigma_2(v_2)$ $\longrightarrow$ (1)

Now, $(\mu_1 \sqcap \mu_2)((u_1,v_1)(u_2,v_2)) = (\sigma_1 \sqcap \sigma_2)(u_1,v_1) \vee (\sigma_1 \sqcap \sigma_2)(u_2,v_2)$.

If $(u_1, v_1)(u_2, v_2) \in E$.

Then $(\mu_1 \sqcap \mu_2)((u_1,v_1)(u_2,v_2)) = \mu_1(u_1,u_2) \vee \mu_2(v_1,v_2)$

$= \sigma_1(u_1) \vee \sigma_1(u_2) \vee \sigma_2(v_1) \vee \sigma_2(v_2)$ (by (1))

$= (\sigma_1 \sqcap \sigma_2)(u_1,v_1) \vee (\sigma_1 \sqcap \sigma_2)(u_2,v_2)$.

$G_{A_1} \sqcap G_{A_2}$ is complete.

Theorem 3.6: If $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ are anti fuzzy complete graphs, then $G_{A_1} . G_{A_2}$ is complete.

Proof: Let $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ be two anti fuzzy complete graphs.

If $((u, v_1), (u,v_2)) \in E$, then

i) $(\mu_1 . \mu_2)((u, v_1)(u, v_2)) = \sigma_1(u) \vee \mu_2(v_1,v_2)$

$= \sigma_1(u) \vee \sigma_2(v_1) \vee \sigma_2(v_2)$

$= (\sigma_1 . \sigma_2)(u,v_1) \vee (\sigma_1 . \sigma_2)(u,v_2)$.

ii) If $((u_1, v_1), (u_2,v_2)) \in E$, then

$(\mu_1 . \mu_2)((u_1,v_1)(u_2,v_2)) = \mu_1(u_1,u_2) \vee \mu_2(v_1,v_2)$.

$= \sigma_1(u_1) \vee \sigma_1(u_2) \vee \sigma_2(v_1) \vee \sigma_2(v_2)$.

$= (\sigma_1 . \sigma_2)(u_1,v_1) \vee (\sigma_1 . \sigma_2)(u_2,v_2)$.

$G_{A_1} . G_{A_2}$ is complete.

Theorem 3.7: If $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ are anti fuzzy complete graphs, then $G_{A_1} \otimes G_{A_2}$ is complete.

Proof: Let $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ be two anti fuzzy complete graphs.

i) If $((u,v_1), (u,v_2)) \in E$, then

$$(\mu_1 \otimes \mu_2)((u, v_1), (u, v_2)) = \sigma_1(u) \vee \mu_2(v_1,v_2)$$

$$= \sigma_1(u) \vee \sigma_2(v_1) \vee \sigma_2(v_2) \quad (\text{since } G_{A_2} \text{ is complete})$$

$$= (\sigma_1 \otimes \sigma_2)(u,v_1) \vee (\sigma_1 \otimes \sigma_2)(u,v_2).$$

ii) If $((u_1, w), (u_2, w)) \in E$, then

$$(\mu_1 \otimes \mu_2)((u_1, w), (u_2, w)) = \mu_1(u_1, u_2) \vee \sigma_2(w)$$

$$= \sigma_1(u_1) \vee \sigma\ \sigma_1(u_2) \vee \sigma_2(w) \quad (\text{since } G_{A_1} \text{ is complete})$$

$$= (\sigma_1 \otimes \sigma_2)(u_1,w) \vee (\sigma_1 \otimes \sigma_2)(u_2,w).$$

iii) If $((u_1, v_1), (u_2, v_2)) \in E$, then

$$(\mu_1 \otimes \mu_2)((u_1, v_1),(u_2,v_2)) = \mu_1(u_1, u_2) \vee \mu_2(u_2, v_2)$$

$$= \sigma_1(u_1) \vee \sigma_1(u_2) \vee \sigma_2(v_1) \vee \sigma_2(v_2)$$

$$= (\sigma_1 \otimes \sigma_2)(u_1,v_1) \vee (\sigma_1 \otimes \sigma_2)(u_2,v_2).$$

$G_{A_1} \otimes G_{A_2}$ is complete.

Result 3.8: 1) If $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ are anti fuzzy complete graphs, then $\overline{G_{A_1} \otimes G_{A_2}} \simeq \overline{G_{A_1}} \otimes \overline{G_{A_2}}$

2) If $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ are anti fuzzy complete graphs, then $\overline{G_{A_1} . G_{A_2}} \simeq \overline{G_{A_1}} . \overline{G_{A_2}}$.

3) If $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ are anti fuzzy complete graphs, then $\overline{G_{A_1} \sqcap G_{A_2}} \simeq \overline{G_{A_1}} \sqcap \overline{G_{A_2}}$.

Note 3.9: The direct product, the semi –strong product (or) the strong product of two anti fuzzy graphs is complete, then atleast one of the two anti fuzzy graphs must be complete.

Theorem 3.10: If $G_{A_1}: (\sigma_1, \mu_1)$ and $G_{A_2}: (\sigma_2, \mu_2)$ are anti fuzzy graphs such that $G_{A_1} \sqcap G_{A_2}$ is complete , then at least G_{A_1} or G_{A_2} must be complete.

Proof: Suppose that G_{A_1} and G_{A_2} are not complete.

Then there exists at least one $(u_1, u_2) \in E_1$ and $(v_1, v_2) \in E_2$ such that

$$\mu_1(u_1, u_2) < \sigma_1(u_1) \vee \sigma_1(u_2)$$

$$\mu_2(v_1, v_2) < \sigma_2(v_1) \vee \sigma_2(v_2)$$

If $G_{A_1} \sqcap G_{A_2}$, $(\sigma_1 \sqcap \sigma_2)(u_1, v_1) = \sigma_1(u_1) \vee \sigma_2(v_1)$

$$(\sigma_1 \sqcap \sigma_2)(u_2, v_2) = \sigma_2(u_2) \vee \sigma_2(v_2)$$

Now, $(\mu_1 \sqcap \mu_2)((u_1, v_1)(u_2, v_2)) = \mu_1(u_1, u_2) \vee \mu_2(v_1, v_2)$

$$< \sigma_1(u_1) \vee \sigma_1(u_2) \vee \sigma_2(v_1) \vee \sigma_2(v_2)$$

$$= (\sigma_1 \sqcap \sigma_2) (u_1, v_1) \vee (\sigma_1 \sqcap \sigma_2) (u_2, v_2)$$

$$(\mu_1 \sqcap \mu_2)((u_1, v_1)(u_2,v_2)) < (\sigma_1 \sqcap \sigma_2) (u_1, v_1) \vee (\sigma_1 \sqcap \sigma_2) (u_2, v_2)$$

Therefore $G_{A_1} \sqcap G_{A_2}$ is not complete, which is a contradiction to our assumption.

Either G_{A_1} or G_{A_2} is complete.

Result 3.11:

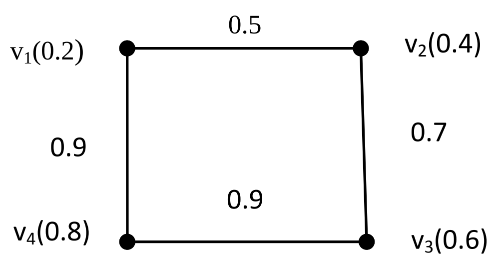
1) If G_{A_1}: (σ_1, μ_1) and G_{A_2}: (σ_2, μ_2) are two anti fuzzy graphs such that $G_{A_1} . G_{A_2}$ is complete, then at least G_{A_1} (or) G_{A_2} must be complete.

2) If G_{A_1}: (σ_1, μ_1) and G_{A_2}: (σ_2, μ_2) are two anti fuzzy graphs such that $G_{A_1} \otimes G_{A_2}$ is complete, then at least G_{A_1} (or) G_{A_2} must be complete.

IV. BALANCED ANTI FUZZY GRAPHS

Definition 4.1: The density of a anti fuzzy graph G_A:(σ, μ) is $D(G_A)=2(\sum_{u,v\in V} \mu(u,v))/(\sum_{u,v\in V} \sigma(u) \vee \sigma(v))$: G_A is balanced if $D(H_A) \leq D(G_A)$ for all anti fuzzy non-empty subgraphs H_A of G_A .

Example 4.2:



Theorem 4.3: Any complete anti fuzzy graph is balanced.

Proof: Let G be a complete anti fuzzy graph.

Then $D(G_A) = 2[(\sum_{u,v\in V} \mu(u,v))/(\sum_{u,v\in V} \sigma(u) \vee \sigma(v))]$

$=2[\sum_{u,v\in V} \sigma(u) \vee \sigma(v) / \sum_{u,v\in V} \sigma(u) \vee \sigma(v)]$

$D(G_A) =2$ (since G_A is complete).

If H_A is a non-empty anti fuzzy subgraph of G_A, then

$D(H_A) = 2[(\sum_{u,v\in V(H)} \mu(u,v))/(\sum_{u,v\in V(H)} \sigma(u) \vee \sigma(v))]$

$\leq 2[(\sum_{u,v\in V(H)} \sigma(u) \vee \sigma(v))/ (\sum_{u,v\in V(H)} \sigma(u) \vee \sigma(v))]$

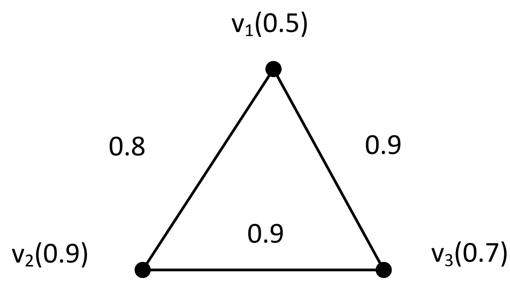
$\leq 2[(\sum_{u,v\in V(G)} \sigma(u) \vee \sigma(v))/ (\sum_{u,v\in V(G)} \sigma(u) \vee \sigma(v))]$

$=2 =D (G_A)$

$D(H_A) \leq D(G_A)$.

Note 4.4: The converse of the above theorem need not be true.

Example 4.5: The following anti fuzzy graph G_A: (σ, μ) is a balanced graph that is not complete.



Note 4.6: we use to give necessary and sufficient conditions for the direct product, semi-strong product and strong product of two anti fuzzy balanced graphs to be balanced.

Lemma 4.7: Let G_{A_1} and G_{A_2} be anti fuzzy graphs.

Then D (G_{Ai}) ≤ D($G_{A_1} \sqcap G_{A_2}$) for i = 1, 2 if and only if D(G_{A_1}) = D(G_{A_2}) = D($G_{A_1} \sqcap G_{A_2}$).

Proof: Let G_1 and G_2 be two anti fuzzy graphs.

Assume that D (G_{Ai}) ≤ D($G_{A_1} \sqcap G_{A_2}$) for i = 1, 2.

WKT, D(G_{A_1})= $2(\sum_{u_1,u_2 \in V_1} \mu_1(u_1,u_2)\,)/(\sum_{u_1,u_2 \in V_1} \sigma_1(u_1) \vee \sigma_1(u_2))$

$\leq 2(\sum_{u_1,u_2 \in V_1} \mu_1(u_1,u_2) \vee \sigma_2(v_1) \vee \sigma_2(v_2))/$

$(\sum_{u_1,u_2 \in V_1\, v_1,v_2 \in V_2} \sigma_1(u_1) \vee \sigma_1(u_2) \vee \sigma_2(v_1) \vee \sigma_2(v_2))$

$=2[(\sum_{u_1,u_2 \in V_1\ ,v_1,v_2 \in V} \mu_1(u_1,u_2) \vee \mu_2(v_1,v_2)]/[(\sum_{u_1,u_2 \in V_1\, v_1,v_2 \in V_2} \sigma_1(u_1) \vee \sigma_1(u_2) \vee \sigma_2(v_1) \vee \sigma_2(v_2))]$

$=2[\sum_{u_1,u_2 \in V_1\ ,v_1,v_2 \in V} \mu_1 \sqcap \mu_2(u_1,v_1)\,(u_2,v_2)]\,/\sum_{u_1,u_2 \in V_1\ ,v_1,v_2 \in V} (\sigma_1 \sqcap \sigma_2)((u_1,v_1)(u_2,v_2))]$

=D($G_{A_1} \sqcap G_{A_2}$)

∴D(G_{A_1})≤D($G_{A_1} \sqcap G_{A_2}$) and hence D(G_{A_1})=D($G_{A_1} \sqcap G_{A_2}$)

Similarly, D(G_{A_2}) ≤ D($G_{A_1} \sqcap G_{A_2}$) and thus D(G_{A_2})=D($G_{A_1} \sqcap G_{A_2}$).

Hence D(G_{A_1})= D(G_{A_2})=D($G_{A_1} \sqcap G_{A_2}$).

Theorem 4.8: Let G_{A_1} and G_{A_2} be anti fuzzy balanced graphs. Then $G_{A1} \sqcap G_{A2}$ is balanced if and only if D(G_{A_1}) = D(G_{A_2}) = D($G_{A_1} \sqcap G_{A_2}$).

Proof: If $G_{A_1} \sqcap G_{A_2}$ is balanced.

Then D(G_{Ai}) ≤ D($G_{A_1} \sqcap G_{A_2}$) for i = 1,2.

By lemma 4.7, D(G_{A_1}) = D(G_{A_2}) = D($G_{A_1} \sqcap G_{A_2}$).

Conversely, assume that D(G_{A_1}) = D(G_{A_2}) = D($G_{A_1} \sqcap G_{A_2}$).

Let H be a anti fuzzy subgraph of $G_{A_1} \sqcap G_{A_2}$.

Then there exist anti fuzzy subgraphs H_{A_1} of G_{A_1} and H_{A_2} of G_{A_2}

Let the edge set of G_{A_1} and G_{A_2} is n_1 and vertex set of G_{A1} and G_{A_2} is r_1.

Let the edge set of H_{A_1} and H_{A_2} is a_1 and a_2 , vertex set of H_{A_1} and H_{A_2} is b_1 and b_2.

Since G_{A_1} and G_{A_2} 2 are balanced and D(G_{A_1})=D(G_{A_2})=n_1/r_1, then D(H_{A_1})= $a_1/b_1 \leq n_1/r_1$ and D(H_{A_2})= $a_2/b_2 \leq n_1/r_1$

Now, $a_1 \leq n_1b_1/r_1$ and $a_2 \leq n_1b_2/r_1$.

$\Rightarrow a_1+a_2 \leq (n_1b_1+n_2b_2)/r_1$

$\Rightarrow a_1+a_2 \leq n_1 (b_1+b_2)/r_1$

$\Rightarrow a_1r_1+ a_2r_2 \leq n_1b_1+n_2b_2$.

Hence, $D(H_{A_1}) \leq (a_1+a_2)\backslash (b1+b_2) \leq n_1/r_1= D(G_{A1} \sqcap G_{A_2})$

$D(H_{A_1}) \leq D(G_{A_1} \sqcap G_{A_2})$

$(G_{A_1} \sqcap G_{A_2})$ is balanced.

Note 4.9: Let G_{A_1} and G_{A_2} be anti fuzzy balanced graphs.

Then i) $G_{A_1}. G_{A2}$ is balanced if and only if $D(G_{A_1}) = D(G_{A_2}) = D(G_{A_1}. G_{A_2})$.

ii) $G_{A_1} \otimes G_{A2}$ is balanced if and only if $D(G_{A_1}) = D(G_{A_2}) = D(G_{A_1} \otimes G_{A_2})$

Theorem 4.10: Let G_{A_1} and G_{A_2} be isomorphic anti fuzzy graphs. If G_{A_2} is balanced, then G_{A_1} is balanced.

Proof: Let $h: V_1 \to V_2$ be a bijection such that $\sigma_1(x) = \sigma_2(h(x))$ and $\mu_1(x, y) = \mu_2(h(x), h(y))$ for all $x, y \in V_1$

We have $\sum_{x\in V1} \sigma_1(x) = \sum_{x\in V2} \sigma_2(x)$ and $\sum_{x,y\in V1} \mu_1(x,y) = \sum_{x,y\in V2} \mu_2(x,y)$

Let $H_{A_1} = (\sigma'_{1,} \mu'_1)$ be a anti fuzzy subgraph of G_{A_1} with underlying set W.

Let $H_{A_2} = (\sigma'_{2,} \mu'_2)$ be a anti fuzzy subgraph of G_{A_2} with underlying set h(W),where

$\sigma'_2(h(x)) = \sigma'_1(x)$ and $\mu'_2 (h(x),h(y)) = \mu'_1(x,y)$ for all $x,y\in W$.

Since G_{A_2} is balanced, $D(H_{A_2}) \leq D(G_{A_2})$

Now, $2(\sum_{x,y\in W} \mu'(h(x), h(y)) / \sum_{x,y\in W} \sigma'_2(x) \vee \sigma'_2(y)$

$$\leq 2(\sum_{x,y\in V_2} \mu_2(x, y)) /(\sum_{x,y\in V_2} \sigma_2(x) \vee \sigma_2(y))$$

$$2(\sum_{x,y\in W} \mu'_1(x, y) / \sum_{x,y\in W} \sigma'_2(x) \vee \sigma'_2(y)$$

$$\leq 2(\sum_{x,y\in V_1} \mu_1(x, y)) /(\sum_{x,y\in V_1} \sigma_2(x) \vee \sigma(y))$$

$$\leq D(G_1).$$

Therefore G_{A_1} is balanced.

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