Volatility Modelling using Arch and Garch Models (A Case Study of the Nigerian Stock Exchange)

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Abstract
ARCH and GARCH models have become important tools in the analysis of time series data, particularly in financial application. These models are especially useful when the goal of the study is to analyse and forecast volatility. This study investigates the volatility in equity prices of insurance stocks traded on the floor of the Nigerian Stock Exchange. The time series data covers almost five years starting from 4th of March 2011 to 31st of December 2015 excluding weekends and public holidays resulting to approximately 1,106 observations. This study shows that GARCH(0,3), which is the same as ARCH(3) and GARCH(1,1), is the best model that captures the volatility that exist in the insurance stocks through the information criteria of Akaike, Bayesian, Shibata and Hanna Quinn. Also, Value at Risk (VaR) was examined to determine the maximum expected loss in the insurance stocks on daily basis at 95% confidence level. Finally, Potential investors are thereby advised to invest in insurance stocks as they show calm tranquility, though their present stock prices are low but the future remains bright because their market is relatively stable going by the result of the analysis.

Keywords - *GARCH, Volatility, Value at Risk, Insurance Sector.*

I. INTRODUCTION

Over the last few years, modeling and forecasting volatility of a financial time series has become a fertile area for research; this is simply because volatility is considered as an important concept for many economic and financial applications, like portfolio optimization, risk management and asset pricing. In simple words, volatility means “the conditional variance of the underlying asset returns” which means the relative rate at which the prices of stocks move up and down, Ahmed and Suliman [1]. A number of models have been developed that are suited to estimate the volatility of financial time series data, of which the most well-known and frequently applied models for this volatility are the heteroscedastic models. The main objective of building these models is to make a good forecast of future volatility which will therefore be helpful in obtaining a more efficient portfolio allocation, having a better risk management and more accurate derivative prices of a certain financial instrument.

Among these models, the Autoregressive Conditional Heteroskedasticity (ARCH) model proposed by Engle [2] and its extension, the Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model byBollerslev [3], were found to be the first models introduced into the literature and they have become very popular in that they enable an analyst to estimate the variance of a series at a particular point in time. Since then there have been a great number of empirical applications of modeling the variance of a financial time series.

ARCH models were created to handle financial problems having to do with the rate that stock prices increased (or decreased) per time. A Generalized Autoregressive Conditional Heteroskedasticity (GARCH) model [3] extends the ARCH models and it uses values of the past squared returns and past variances to model the current variance of a financial data at time t.

A. The Nigerian Stock Exchange

The Nigerian Stock Exchange has grown tremendously over the years in an attempt to meet the ever increasing needs and sophistication of the domestic economy; it also plays a significant role in the daily capital formation and resource generation of the nation. The Nigerian Stock Exchange as an essential place for sourcing funds needed to power the financial needs of industries is highly dependent on the steadiness (stability) of prices of stock on the exchangeAdekunle et al [4]. The major factor that causes unsteadiness (instability) in stock prices is the effect of volatility. Thus, for us to have a strong stock exchange that will play a significant role in the growth of the economy of the nation there is need to examine volatility of stock prices.

The main thrust of this article is to undertake a careful study of volatility of daily closing prices of three insurance companies stocks, traded on the floor of the Nigerian Stock Exchange. These three insurance companies are American International Insurance Company (AIICO), Intercontinental Wapic Insurance Plc (WAPIC Plc) and Continental Reinsurance Plc. In this study, varying variances in daily returns of the three insurance stocks are investigated using Box and Jenkins procedure of ARCH and GARCH models.

II. LITERATURE REVIEW

There has been substantial literature on the effect of volatility on the volume of trade, most common of these studies focused on the argument that volatility of equity prices increases the risk and uncertainties in both local and international transaction and thus discourages trade. Engle and Ng [5] observed that volatility is an input used for the purpose of estimating value at risk. In other cases, volatility may be a causal variable in modeling expected returns.

Cambell et al [6] argued that "it is both logically inconsistent and statistically inefficient to use volatility measures that are based on the assumption of constant volatility over some period when the resulting series moves through time". In the case of financial data, for example, large and small errors occur in clusters, i.e. large returns are followed by more large returns and small returns are followed by more small returns. When dealing with non-linearities, [6] made a distinction between Linear time series: shocks are assumed to be uncorrelated but not necessarily independent and identically distributed (iid), and Non-linear time series: shocks are assumed to be independent and identically distributed but there is a non-linear function relating the observed time series,$\{Z_t\}_{t=0}^{\infty}$ and the underlying shocks $\{\varepsilon_t\}_{t=0}^{\infty}$.

Rydberg[7] observed that neither the ARCH nor the GARCH models consider asymmetry and leverage effect (the fact that past returns correlate with future volatility). Although GARCH (p,q) models give adequate fits for most equity-return dynamics, these models often fail to perform well in modeling the volatility of stock returns because GARCH models assume that there is a symmetric response between volatility and returns.

Floros[8] used various GARCH models to bootstrap out-of-sample period data and evaluate the performance of Minimum Capital Risk Requirement (MCRR) estimates. The models show that higher capital requirements are necessary for a short position, since a loss is then more likely. Other ARCH and GARCH based studies includeArinze et al [9], Alberg et al [10],Shamiri and Isa [11].

III. METHODOLOGY

The data for this study are from daily closing prices of three insurance companies, the time series data covers almost five years starting from 4th of March 2011 to 31st of December 2015 excluding weekends and public holidays resulting to approximately 1,106 observations.

In this research, the daily closing prices of the insurance stocks are positive, i.e. $P_i > 0$. The return for holding such asset (stock) on the ith day is given by

$R_i = \left(\frac{P_i}{P_{i-1}}\right)$, Cryer and Kung [12] (1)

Where R_i is the current day return, P_i is the current day closing price and P_{i-1} is the previous day closing price. The grand mean of the daily returns are given by

$$\overline{R} = \frac{\sum_{i=1}^{N} R_i}{N} \tag{2}$$

Let $Z_i = R_i - \overline{R}$

$$Z_i^2 = [R_i - \overline{R}]^2 \tag{3}$$

$Z_i^2$is a measure of volatility for $i = 1,2,3, \dots, N$ we are to modelZ_i^2.

A. Generalized Autoregressive Conditional Heteroskedastic (Garch) Model

ARCH model was extended to the GARCH model by [3], this generalization was achieved by allowing the conditional variance, σ_t^2, to further depend on previous conditional variances. In that case, the GARCH (p, q) model where p is the order of the GARCH terms of $\sigma^2$and q is the order of the ARCH terms ofZ^2 is given by

$$Z_t = \varepsilon_t\sqrt{\sigma_t^2}, \quad \varepsilon_t \sim N(0,1), \quad t = 1,2,\dots,n \tag{4}$$

$$\sigma_t^2 = \omega + \alpha_1 Z_{t-1}^2 + \alpha_2 Z_{t-2}^2 + \cdots + \alpha_q Z_{t-q}^2 + \beta_1\sigma_{t-1}^2 + \beta_2\sigma_{t-2}^2 + \cdots + \beta_p\sigma_{t-p}^2 \tag{5}$$

where $\omega > 0,\ \alpha_i \geq 0,\ \beta_j \geq 0,\ i = 1,2,\ldots,q,\ j = 1,2,\ldots,p,\ Z_t/\,I_t \sim N(0,\sigma_t^2)$. $Z_t$is the dependent variable, ε_t is the error terms and I_t is the information set available at time, t, andα_i and β_j are coefficients of unknown parameters. When$p = 0$, the process reduces to the ARCH (q) process and for$p = q = 0$, $\varepsilon_t$is simply a white noise.

B. Estimation Of Value At Risk (VAR)

Value at Risk (VaR) estimates the maximum expected loss on an investment, over a given period of time and given a specified degree of confidence. A value at risk statistic has three components: a time period, a confidence level and a loss amount (expressed either in currency or loss percentage). Value at risk answers question like what is the worst I can, with a 95% or 99% level of confidence, expect to lose in investment over the next day, month or year. According to [13] value at risk is given by

$$Var = \mu + Z_\alpha \sigma \tag{6}$$

In this research, value at risk (VaR) will be estimated using GARCH models identified by the three insurance firms to determine the daily maximum expected loss on the insurance stocks.

IV. EMPIRICAL RESULTS

In this research, time series plots were carried out to show the pattern of price movement of three insurance stocks for a period of 5 years on daily basis. The time series plots are presented in Figures 1 and 2 below for AIICO insurance Plc, Wapic insurance Plc and Continental Reinsurance Plc respectively.

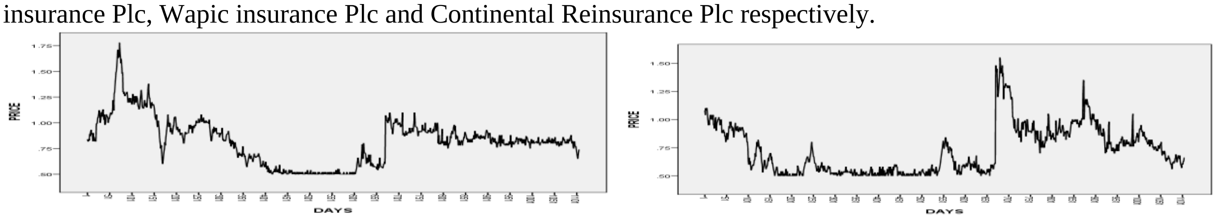

Figure 1: Time Series Plot of AIICO Insurance Plc and Wapic Insurance Plc Prices

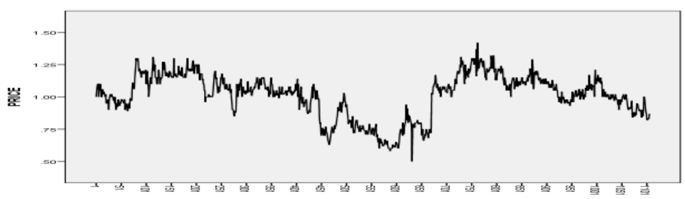

Figure 2: Time Series Plot of Continental Reinsurance Plc Price

A visual inspection of the plots of the three insurance stocks daily closing prices shows that the mean and variance are not constant, implying non-stationarity of the data. The series were transformed by taking the first difference of the logarithm of the values in each of the three insurance stocks traded on daily basis. The transformation was aimed at obtaining stationarity in the mean. The sequence plots for the daily returns are presented in Figures below.

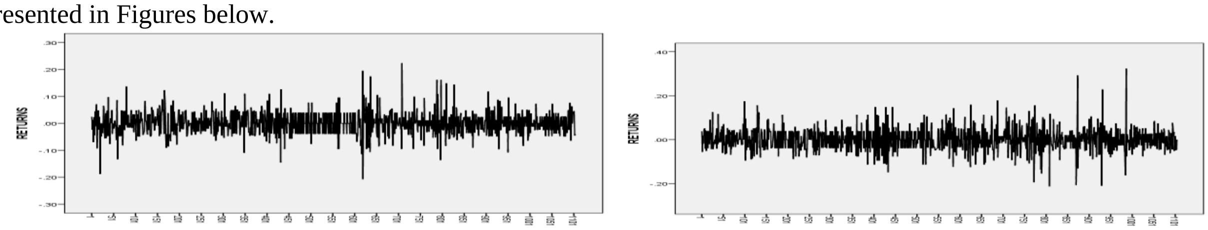

Figure 3: Time Series Plot of AIICO Insurance Plc and Wapic Returns

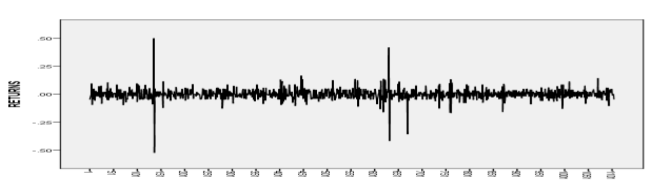

Figure 4: Time Series Plot of Continental Reinsurance Returns

The examination of the returns series plots reveals well known characteristics of high frequency data. It is easy to see that "large changes tend to be followed by large changes of either sign, and small changes tend to be followed by small changes of either sign. In general, all the series under study exhibit ARCH effects (also referred to as heteroskedasticity) prevalent in many financial time series data.

A. Descriptive Statistics

The basic statistical properties of the returns series data are presented in Table1 below. The mean returns are positive and close to zero a characteristic common in financial return series. Since the kurtosis of the insurance returns series are greater than 3, the returns series have very heavy tails showing a strong departure from the Gaussian assumption. Also, considering the variance (which is a measure of volatility) for the insurance stocks, Continental Reinsurance has the highest volatility followed by Wapic Insurance while AIICO Insurance has the least volatility.

Table 1: Basic Descriptive Statistics of Return Series of the Insurance Stocks

	N	Minimum	Maximum	Mean	Std. Dev	Skewness	Kurtosis
AIICO	1105	-0.57848	0.22314	0.000115	0.04508	-1.688	25.854
CONT	1105	-1.52444	1.49995	0.000126	0.07862	-0.391	24.643
WAPIC	1105	-.88634	0.32323	0.000446	0.58050	-2.922	50.574

Below are tables for the result of the Stationarity test using Augmented Dickey Fuller and Heteroskedasticity test for ARCH effect with the aid of R Statistical Package.

Table 2: Augmented Dickey–FullerandHeteroskedasticity Test Results

Insurance stocks	Dickey-fuller	Lag order	ADF p-value	Chi-square	df	Chi-square p-value
AIICO	-11.1557	10	0.01	93.4893	12	1.044e-14
WAPIC	-10.6641	10	0.01	41.3325	12	4.311e-05
Continental	-11.2082	10	0.01	317.7435	12	2.2e-16

Small p-values (less than 0.05) suggest that the data is stationary and doesn't need to be differenced, From Table 2 above, it is clearly seen that all the p-values of ADF test are less than 0.05, suggesting that the return series of the insurance stocks are stationary in mean but not in variance. Also, the three insurance returns series show a clear evidence of ARCH effect since the p-values of Chi-square test are less than$\alpha = 0.05$. This signifies that the variances of the returns series of the insurance stocks are non-constant for all the periods specified. The Box and Jenkins procedure is applied to the variance series(Z_i^2) obtained from difference logged series of the closing prices which will lead to the model building process.

B. Garch Model Building Procedures

The model building procedures begins with the identified GARCH processes for the three insurance stocks. The GARCH process that has the smallest information criteria for Akaike, Bayesian, Shibata and HQ is considered as the best volatility model for the insurance stocks.From Table 4, GARCH (1,1) process was identified for AIICO Insurance Plc, GARCH(0,3) for Continental Reinsurance Plc and GARCH(1,1) process for Wapic Insurance Plc.

Table 3: Models Identified and their Information Criterion

Insurance company	Models Identified	Information criteria			
	GARCH(p,q)	AIC	BIC	Shibata	HQ
AIICO	GARCH(0,1)	-3.6542	-3.6315	-3.6543	-3.6456
	GARCH(0,8)	-3.6689	-3.6339	-3.6585	-3.6637

	GARCH(1,1)	**-3.6749**	**-3.6477**	**-3.6650**	**-3.6647**
WAPIC	GARCH(1,0)	-3.1518	-3.1292	-3.1519	-3.1433
	GARCH (2,0)	-3.1433	-3.1229	-3.1503	-3.1398
	GARCH (1,1)	**-3.2229**	**-3.1958**	**-3.2230**	**-3.2127**
CONT	GARCH(O,1)	-3.3860	-3.3633	-3.3960	-3.3774
	GARCH(0,2)	-3.3960	-3.3688	-3.3960	-3.3857
	GARCH(0,3)	**-3.4538**	**-3.4221**	**-3.4539**	**-3.4418**
	GARCH(0,4)	-3.4518	-3.4156	-3.4519	-3.4381

Below are table of GARCH model fit summary for the three identified GARCH processes and their parameters for the insurance stocks, the analysis were done using R statistical package.

Table 4: Summary of GARCH model Fit for the three Insurance Firms

Firms	Model Identified	Parameters				
	GARCH(p,q)	omega	alpha1	alpha 2	alpha 3	beta1
AIICO	GARCH(1,1)	0.000312	0.249078	NIL	NIL	0.572918
WAPIC	GARCH(1,1)	0.00747	0.194452	NIL	NIL	0.505917
CONT	GARCH(O,3)	0.001114	0.399918	0.000000	0.195233	NIL

The volatility model identified for AIICO, WAPIC and Continental Reinsurance are:
$\sigma_t^2 = 0.000312 + 0{,}249078Z_{t-1}^2 + 0.572918\sigma_{t-1}^2.$
$\sigma_t^2 = 0.000747 + 0.194452Z_{t-1}^2 + 0.505917\sigma_{t-1}^2.$
$\sigma_t^2 = 0.001114 + 0.399918Z_{t-1}^2 + 0.000000Z_{t-2}^2 + 0.195233Z_{t-3}^2.$
In a worst case scenario, the daily maximum expected loss for the three insurance stocks discussed in this research work at 95% confidence level is given in table 5 below

Table 5: Forecast for 10 days and the Corresponding Value at Risk (VaR)

	1	2	3	4	5	6	7	8	9	10
AIICO	0.00212	0.00206	0.00200	0.00196	0.00192	0.00189	0.00187	0.00185	0.00183	0.00182
	9.02	8.90	8.77	8.67	8.57	8.52	8.45	8.42	8.32	8.35
Wapic	0.00192	0.0021	0.0023	0.00232	0.00237	0.00241	0.00243	0.00245	0.00246	0.00247
	8.74	9.06	9.29	9.44	9.55	9.62	9.66	9.71	9.73	9.75
Cont	0.00182	0.00190	0.00112	0.00111	0.00111	0.00111	0.00111	0.00111	0.00111	0.00111

	8.35	8.55	6.56	6.54	6.54	6.54	6.54	6.54	6.54	6.54

From Table 5 above, it can be seen that the loss that can be incurred by these insurance stocks is minimal in the sense that the loss cannot exceed 10% of the amount invested for the 10 forecasted periods. Also, WAPIC Insurance Plc stands greater chances of incurring high loss followed by AIICO Insurance Plc while the least is Continental Reinsurance Plc.

V. DISCUSSION OF RESULTS

This article investigates the volatility present in the daily closing prices of three insurance stocks traded on the floor of the Nigerian Stock Exchange (NSE) from March 2011 to December 2015. These insurance stocks show that the error term is heteroskedastic (non-constant variance) which implies that the conventional procedure based on ARMA estimation techniques may not produce optimal result; that is why the researchers resorted to the use of GARCH methodology.

The volatility experienced by the insurance returns series were modeled using univariate Generalized Autoregressive Conditional Heterskedastic (GARCH) model and the researcher observed that GARCH (1,1) was identified as the best model for AIICO Insurance Plc and WAPIC Insurance Plc while GARCH(0,3) which is the same as ARCH(3) model was selected for Continental Reinsurance Plc, based on having the smallest information criteria. Finally, the maximum expected loss that an investor can incur for these insurance stocks on daily basis with the aid of the models developed in this research cannot exceed 10% of the investment.

VI. CONCLUSION

The study has shown that GARCH models are better models for analyzing financial data because they give lower information criteria for AIC, BIC, Shibata and HQ. The only outlier observed is Continental Reinsurance Plc which selected ARCH (3) process. Potential investors are thereby advised to invest in insurance stocks as they exhibit calm tranquility, though their present stock prices are low but the future is bright because their market is relatively stable going by the result of the analyses. Also, from the estimated value at risk (VaR), the maximum expected loss for the insurance stocks considered in this research cannot exceed 10% of investment.

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