Difference Labelling of Jewel Graph

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Abstract

Let G be a (V,E) graph. G is said to be Square, Cube and Quad difference labeling if there exist a bijection $\boldsymbol{f}:\boldsymbol{V}(\boldsymbol{G}) \to \{\mathbf{1},\mathbf{2},\ldots,\boldsymbol{p}\}$ *such that the induced function* $\boldsymbol{f}^*:\boldsymbol{E}(\boldsymbol{G}) \to \boldsymbol{N}$ *given by*

$$f^*(uv) = |f(u)^2 - f(v)^2|, f^*(uv) = |f(u)^3 - f(v)^3| \;\&\; f^*(uv) = |f(u)^4 - f(v)^4|, uv \in E(G)$$

respectively are all distinct. In this paper, we investigate that the quad difference labeling and prove that the Jewel graph J_n *admits a square difference, cube difference and quad difference labeling.*

Keywords - *Square difference, Cube difference, Quad difference, Jewel graph, Difference labeling,*

MSC2010 - 05C78, 05C22

I. INTRODUCTION

Graph theory is one of the branches of mathematics with many applications in different disciplines. Labelling of graph is the assignment of values to vertices or edges or both subject in certain conditions. In 1960's Rosa initiated the concept of labeling in the name of $\boldsymbol{\beta}$-valuation.

Graph labeling can be applied in the areas such as communication and network channel assignment and medical field. A dynamic survey on graph labeling is regularly updated by Gallian [2].

The concept of cube difference labeling was introduced by J.Shiama[6]. J.Shiama proved that the following graphs paths, cycle, stars and trees admits cube difference labeling.

Sharon Philomena.V [8] proved that the Square and cube difference labeling of graphs like cycle cactus graph $\boldsymbol{C_k^3}$, tree and newly defined key graph.

II. PRELIMINARIES

Definition-2.1: A *graph* $G = (V(G), E(G))$ consists of two sets, a non empty vertex set $V(G)$ and edge set $E(G)$. The elements of $V(G)$ and $E(G)$ are called *vertices* and *edges* respectively. The members of $V(G)$ and $E(G)$ are commonly termed as *graph elements*. The number of vertices in $V(G)$ is denoted by $|V(G)|$ and the number of edges in $E(G)$ is denoted by $|E(G)|$. Throughout this thesis we consider a graph G with $|V(G)| = p$ and $|E(G)| = q$.

Definition -2.2: The *comb* (P_n Ⓢ K_1) is obtained by joining a pendant edge to each vertex of path P_n.

Definition -2.3: The *crown* (C_n Ⓢ K_1) is obtained by joining a pendant edge to each vertex of cycle C_n.

Definition -2.4: Bistar $B_{n,n}$ is the graph obtained by joining the center(apex) vertices of two copies of $K_{1,n}$ by an edge.

Definition- 2.5: A *graph labeling* is an assignment of integers to the vertices or edges or both subject to certain condition(s). If the domain of mapping is the set of vertices (edges) then the labeling is called *a vertex(an edge) labeling*.

Definition- 2.6: Let G = (V (G), E (G)) be a graph. G is said to be *square difference labeling* if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f*: E(G)→N given by $f^*(uv) = \left|f(u)^2 - f(v)^2\right|$ is injective.

Definition- 2.7: Let G = (V (G), E (G)) be a graph. G is said to be *cube difference labeling* if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f*: E(G)→N given by $f^*(uv) = \left|f(u)^3 - f(v)^3\right|$ is injective.

***Definition- 2.8*:** Let G = (V (G), E (G)) be a graph. G is said to be *quad difference labeling* if there exist a bijection f: V(G)→{0,1,2,….p-1} such that the induced function f*: E(G)→N given by $f^*(uv) = \left|f(u)^4 - f(v)^4\right|$ is injective.

***Definition- 2.9*:** The *jewel graph* J_n is the graph with the vertex set $V(J_n) = \{u, v, x, y, u_i : 1 \leq i \leq n\}$ and the edge set $E(J_n) = \{ux, uy, xy, xv, yv, uu_i, vu_i : 1 \leq i \leq n\}$.

III. MAIN RESULT

***Theorem-3.1*:** *The Jewel graph J_r admits square difference labelling.*
Proof: Let J_r be the jewel graph.

Let $V(J_r) = \{u_1, u_2, u_{3,} u_4, v_i : 1 \leq i \leq r$ be the vertices of the graph.

Let $E(J_r) = \{u_i u_{i+1} | 1 \leq i \leq 2\} \cup \{u_i u_4 | 1 \leq i \leq 3\} \cup \{v_i u_1 | 1 \leq i \leq r\} \cup \{v_i u_3 | 1 \leq i \leq r\}$

Here $V(J_r) = r + 4$, $E(J_r) = 2r + 5$

Define the vertex labeling f: V→ {0,1,2,…r+4} by
f(u_i) = i-1, 1≤ i ≤ 4
f(v_i) = i+3, 1≤ i ≤ r
and the induced edge labeling function f^* : E→N defined by $f^*(uv) = \left|f(u)^2 - f(v)^2\right|$ for every $uv \in E(J_r)$ are all distinct such that f(e_i) ≠ f(e_j) for every e_i ≠e_j
The edge sets are

$E_1 = \{u_i u_{i+1} | 1 \leq i \leq 2\}$

$E_2 = \{u_i u_4 | 1 \leq i \leq 3\}$

$E_3 = \{v_i u_1 | 1 \leq i \leq r\}$

$E_4 = \{v_i u_3 | 1 \leq i \leq r\}$

and the edge labelling are

In E_1

$$f^*(u_i u_{i+1}) = \bigcup_{i=1}^{2} \left|f(u_i)^2 - f(u_{i+1})^2\right|$$

$$= \bigcup_{i=2}^{2} |1 - 2i| = \{1,3\}$$

In E_2

$$f^*(u_i u_4) = \bigcup_{i=1}^{3} \left|f(u_i)^2 - f(u_4)^2\right|$$

$$= \bigcup_{i=1}^{3} \left|i^2 - 2(i+4)\right| = \{9,8,5\}$$

In E_3

$$f^*(v_i u_1) = \left|f(v_i)^2 - f(u_1)^2\right|$$

$$= \left|i^2 + 3(2i+3)\right| = \{16,25,\ldots.\}$$

In E_4

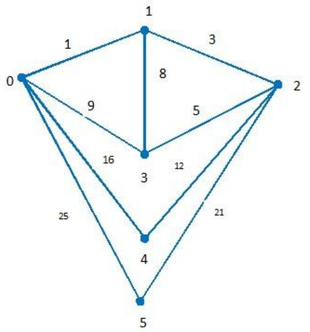
$$f^{*}(v_i u_3) = \left| f(v_i)^2 - f(u_3)^2 \right|$$

$$= \left| i^2 + 5 + 6i \right| \quad = \{12,21,\ldots\}$$

Here all the edge labelling are distinct.

Hence the Jewel graph J_r admits a square difference labelling.

Example-3.1:



Theorem-3.2: *The Jewel graph J_r is a cube difference labelling*.

Proof: By theorem 3.1, define the vertex and edge labelling.

Define the vertex labeling f: V→ {0,1,2,…r+4} by
$f(u_i) = i-1, \quad 1 \le i \le 4$
$f(v_i) = i+3, \quad 1 \le i \le r$
and the induced edge labeling function f^{*} : E→N defined by $f^{*}(uv) = \left| f(u)^3 - f(v)^3 \right|$ for every $uv \in E(J_r)$ are all distinct such that $f(e_i) \neq f(e_j)$ for every $e_i \neq e_j$

In E_1

$$f^{*}(u_i u_{i+1}) = \bigcup_{i=1}^{2} \left| f(u_i)^3 - f(u_{i+1})^3 \right|$$

$$= \bigcup_{i=2}^{2} \left| 3i(1-i) - 1 \right|$$

In E_2

$$f^{*}(u_i u_4) = \bigcup_{i=1}^{3} \left| f(u_i)^3 - f(u_4)^3 \right|$$

$$= \bigcup_{i=1}^{3} \left| i(i^2 - 3i + 3) - 28 \right|$$

In E_3

$$f^{*}(v_i u_1) = \left| f(v_i)^3 - f(u_1)^3 \right|$$

$$= \left| i^3 + 27 + 9i^2 + 27i \right|$$

In E_4

$$f^*(v_i u_3) = \left| f(v_i)^3 - f(u_3)^3 \right|$$

$$= \left| i^3 + 9i^2 + 27i + 19 \right|$$

Here all the edge labelling are distinct.

Hence the Jewel graph J_r admits a cube difference labelling.

Theorem-3.3: *The Jewel graph J_r admits quad difference labelling.*

Proof: By theorem, 3.1 & 3.2

Define the vertex labeling f: V→ {0,1,2,…r+4} by
$f(u_i) = i-1, \quad 1 \leq i \leq 4$
$f(v_i) = i+3, \quad 1 \leq i \leq r$
and the induced edge labeling function f^* : E→N defined by $f^*(uv) = \left| f(u)^4 - f(v)^4 \right|$ for every $uv \in E(J_r)$ are all distinct such that $f(e_i) \neq f(e_j)$ for every $e_i \neq e_j$

In E_1

$$f^*(u_i u_{i+1}) = \bigcup_{i=1}^{2} \left| f(u_i)^4 - f(u_{i+1})^4 \right|$$

$$= \bigcup_{i=2}^{2} \left| 2i(3i - 2i^2 - 2) + 1 \right|$$

In E_2

$$f^*(u_i u_4) = \bigcup_{i=1}^{3} \left| f(u_i)^4 - f(u_4)^4 \right|$$

$$= \bigcup_{i=1}^{3} \left| i(i^3 - 4i^2 + 6i - 4) - 80 \right|$$

In E_3

$$f^*(v_i u_1) = \left| f(v_i)^4 - f(u_1)^4 \right|$$

$$= \left| i^4 + 3i(4i^2 + 18i + 36) + 81 \right|$$

In E_4

$$f^*(v_i u_3) = \left| f(v_i)^4 - f(u_3)^4 \right|$$

$$= \left| i^4 + 3i(4i^2 + 18i + 36) + 65 \right|$$

Here all the edge labelling are distinct.

Hence the Jewel graph J_r admits a quad difference labelling.

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