A Note on G(γ_{tss}) of Some Graphs

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Abstract

A total dominating set D of graph G = (V, E) is a total strong split dominating set if the induced sub graph < V−D > is totally disconnected with at least two vertices. The total strong split domination number γ_{tss}(G) is the minimum cardinality of a total strong split dominating set. In this paper, we introduce the concept γ_{tss} − graph of a graph G and define the graph $G(\gamma_{tss}) = (V(\gamma_{tss}), E(\gamma_{tss}))$ of G to be the graph whose vertices $V(\gamma_{tss})$ corresponds injectively with the γ_{tss} −sets of a graph G and two γ_{tss} −sets D_1 and D_2 form an edge in $G(\gamma_{tss})$ if there exists a vertex v ∈ D_1 and w ∈ D_2 such that v is adjacent to w and $D_1 = D_2 - \{w\} \cup \{v\}$ or equivalently $D_2 = D_1 - \{v\} \cup \{w\}$. With this definition, two γ_{tss} −sets are said to be adjacent if they differ by one vertex, and the two vertices defining this difference are adjacent in G. We also determine $G(\gamma_{tss})$ of some graphs.

Keywords - *Domination number, total strong split domination number, γ_{tss} − graph of a graph.*

I. INTRODUCTION

The graphs considered here are finite, undirected, without loops, multiple edges. For all graph theoretic terminology not defined here, the reader is referred to [2]. A set of vertices D in a graph G is a dominating set, if every vertex in V−D is adjacent to some vertex in D. The domination number γ(G) is the minimum cardinality of a dominating set. A total dominating set D of a connected graph G is a total split dominating set if the induced sub graph < V−D > is disconnected. The total split domination number γ_{ts}(G) is the minimum cardinality of a total split dominating set. This concept was introduced by B. Janakiram, Soner and Chaluvaraju in [3]. Strong split domination was introduced by V. R. Kulli and B. Janakiram in [4]. A dominating set D of a graph G = (V, E) is a strong split dominating set if the induced sub graph < V – D > is totally disconnected with at least two vertices. The strong split domination number γ_{ss} (G) is the minimum cardinality of a strong split dominating set. We have introduced a new concept namely total strong split domination number in [5]. A total dominating set D of a connected graph G is a total strong split dominating set if the induced sub graph < V−D > is totally disconnected with at least two vertices. The total strong split domination number γ_{tss}(G) is the minimum cardinality of a total strong split dominating set. Gerd H. Fricke et al. [1] introduced γ −graph of a graph. Consider the family of all γ-sets of a graph G and define the γ −graph $G(\gamma) = (V(\gamma), E(\gamma))$ of G to be the graph whose vertices $V(\gamma)$ correspond 1−1 with the γ −sets of a graph G, and two γ −sets, say D_1 and D_2, form an edge in $E(\gamma)$ if there exists a vertex v ∈ D_1 and w ∈ D_2 such that v is adjacent to w and $D_1 = D_2 - \{w\} \cup \{v\}$ or equivalently $D_2 = D_1 - \{v\} \cup \{w\}$. With this definition, two γ −sets are said to be adjacent if they differ by one vertex, and the two vertices defining this difference are adjacent in G. We introduce the concept γ_{tss} − graph of a graph G and define the graph $G(\gamma_{tss}) = (V(\gamma_{tss}), E(\gamma_{tss}))$ of G to be the graph whose vertices $V(\gamma_{tss})$ corresponds injectively with the γ_{tss} −sets of a graph G and two γ_{tss} −sets D_1 and D_2 form an edge in $G(\gamma_{tss})$ if there exists a vertex v ∈ D_1 and w ∈ D_2 such that v is adjacent to w and $D_1 = D_2 - \{w\} \cup \{v\}$ or equivalently $D_2 = D_1 - \{v\} \cup \{w\}$. With this definition, two γ_{tss} −sets are said to be adjacent if they differ by one vertex, and the two vertices defining this difference are adjacent in G and we determine $G(\gamma_{tss})$ of some graphs.

Definition 1.1[5] A total dominating set D of a connected graph G is a total strong split dominating set if the induced sub graph < V−D > is totally disconnected with at least two vertices. The total strong split domination number γ_{tss}(G) is the minimum cardinality of a total strong split dominating set.

Definition 1.2[1] Consider the family of all γ-sets of a graph G and define the γ −graph $G(\gamma) = (V(\gamma), E(\gamma))$ of G to be the graph whose vertices $V(\gamma)$ correspond 1−1 with the γ −sets of a graph G, and two γ −sets, say D_1 and D_2, form an edge in $E(\gamma)$ if there exists a vertex v ∈ D_1 and w ∈ D_2 such that v is adjacent to w and $D_1 = D_2 - \{w\} \cup \{v\}$ or equivalently $D_2 = D_1 - \{v\} \cup \{w\}$. With this definition, two γ −sets are said to be adjacent if they differ by one vertex, and the two vertices defining this difference are adjacent in G.

Definition 1.3. Consider the family of all γ_{tss} −sets of a graph G and define the graph G(γ_{tss}) = (V(γ_{tss}), E(γ_{tss})) of G to be the graph whose vertices V(γ_{tss}) corresponds injectively with the γ_{tss}-sets of a graph G and two γ_{tss} −sets D_1 and D_2 form an edge in G(γ_{tss}) if there exists a vertex v ∈ D_1 and w ∈ D_2 such that v is adjacent to w and $D_1 = D_2 - \{w\} \cup \{v\}$ or equivalently $D_2 = D_1 - \{v\} \cup \{w\}$.

Example 1.4.

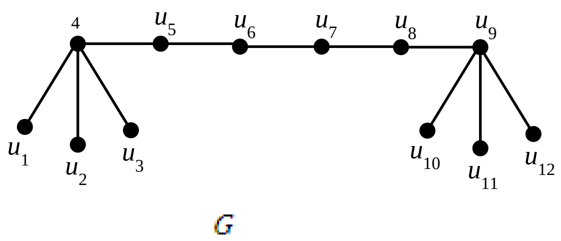

Figure 1.1

For the given graph in Figure 1.1 the total strong split dominating sets are

$D_1 = \{u_4, u_5, u_7, u_8, u_9\}$, $D_2 = \{u_4, u_5, u_6, u_8, u_9\}$, $v = u_7$ and $w = u_6$

Then $D_1 - \{v\} \cup \{w\} = \{u_4, u_5, u_7, u_8, u_9\} - \{u_7\} \cup \{u_6\} = \{u_4, u_5, u_6, u_8, u_9\} = D_2$

$D_2 - \{w\} \cup \{v\} = \{u_4, u_5, u_6, u_8, u_9\} - \{u_6\} \cup \{u_7\} = \{u_4, u_5, u_7, u_8, u_9\} = D_1$

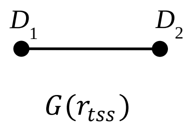

Figure 1.2

Definition 1.5. A vertex v in a graph G = (V, E) is a γ_{tss}–indispensable vertex if it is an element of every γ_{tss}–set of G. In a caterpillar, every vertex of degree ≥ 3 is a γ_{tss}– indispensable vertex.

II. RESULTS

Theorem 2.1. Let T be a caterpillar with exactly 2 support vertices v_1 and $v_2$which are γ_{tss}–indispensable vertices. If the number of vertices in between v_1 and v_2 of T is 3k+1 then G(γ_{tss}) is a path of length k.

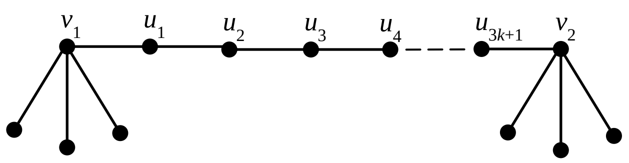

Figure 2.1

Proof: Let T be a caterpillar with exactly 2 support vertices v_1, v_2 which are γ_{tss}– indispensable vertices and u_1, u_2, u_3, u_4, . . . , u_{3k+1} be the vertices between v_1 and v_2. Then the γ_{tss}–sets of T can be listed as follows. $D_1 = \{v_1, u_1, u_3, u_4, u_6, u_7, \ldots, u_{3k+1}, v_2\}$, $D_2 = \{v_1, u_1, u_3, u_4, u_6, u_7, \ldots, u_{3k-3}, u_{3k-2}, u_{3k-1}, u_{3k+1}, v_2\}$, $D_3 = \{v_1, u_1, u_3, u_4, u_6, u_7, \ldots, u_{3k-6}, u_{3k-5}, u_{3k-4}, u_{3k-2}, u_{3k-1}, u_{3k+1}, v_2\}$, . . . , $D_k = \{v_1, u_1, u_3, u_4, u_6, u_7, \ldots, u_{3k-9}, u_{3k-8}, u_{3k-7}, u_{3k-5}, u_{3k-4}, u_{3k-2}, u_{3k-1}, u_{3k+1}, v_2\}$, $D_{k+1} = \{v_1, u_1, u_3, u_4, u_6, u_7, \ldots, u_{3k-8}, u_{3k-7}, \ldots, u_{3k+1}, v_2\}$. Here γ_{tss}–sets $D_2, D_3, D_4, \ldots$

, D_k are adjacent to both the preceding and succeeding γ_{tss}–sets and hence get degree 2. The γ_{tss}–set D_1 is adjacent to D_2 alone and D_{k+1} is adjacent to D_k alone. So both D_1 and D_{k+1} get degree 1. Thus we get a path containing vertices $D_1, D_2, D_3, D_4, \ldots, D_{k+1}$ of length k. ■

Theorem 2.2. Let T be a caterpillar as shown in the figure. If the number of vertices in between v_1 and v_2 of T is 3k, $k \geq 1$ then $T(\gamma_{tss})$ is K_1.

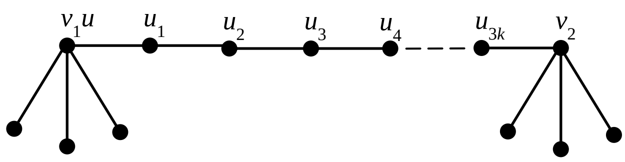

Figure 2.2

Proof: Let T be a caterpillar with exactly 2 support vertices v_1, v_2 which are γ_{tss}– indispensable vertices and $u_1, u_2, u_3, u_4, \ldots, u_{3k}$ be the vertices between v_1 and v_2. Then $D = \{v_1, u_1, u_3, u_4, u_6, u_7, \ldots, u_{3k}, v_2\}$is the only γ_{tss}–set of T. Hence we get $T(\gamma_{tss})$ to be K_1. ■

Proposition 2.3. $C_{3k}(\gamma_{tss}) \cong \overline{K_3}$, for $k \geq 2$.

Proof: Let $\{v_1, v_2, \ldots, v_{3k}\}$ be the vertex set of C_{3k}, for $k \geq 2$. Let D be the minimal total strong split domination set of C_{3k}. $D_1 = \{v_1, v_2, v_4, v_5, \ldots, v_{3k-2}, v_{3k-1}\}$, $D_2 = \{v_2, v_3, v_5, \ldots, v_{3k-2}, v_{3k}\}$, $D_3 = \{v_1, v_3, v_4, v_6, \ldots, v_{3k-2}, v_{3k}\}$ are the γ_{tss}–sets of C_{3k}. Since each C_{3k}, for $k \geq 2$ has 3 disjoint γ_{tss}–sets $C_{3k}(\gamma_{tss}) \cong \overline{K_3}$.
■

Theorem 2.4. $C_{3k+1}(\gamma_{tss}) \cong C_{3k+1}$, for $k \geq 2$.

Proof: Let $\{v_1, v_2, \ldots, v_{3k+1}\}$ be the vertex set of C_{3k+1}, for $k \geq 2$. We arrange the vertices of γ_{tss}–sets of C_{3k+1} in the ascending order of the suffixes of the vertices. Let D be the minimal total strong split domination set of C_{3k}. $D_1 = \{v_1, v_2, v_4, v_5, \ldots, v_{3k-1}, v_{3k+1}\}$, $D_2 = \{v_1, v_2, v_3, v_5, v_6, \ldots, v_{3k-2}, v_{3k-1}\}$, $D_3 = \{v_2, v_3, v_4, v_6, v_7, \ldots, v_{3k}, v_{3k+1}\}$, $D_4 = \{v_1, v_3, v_4, v_5, \ldots, v_{3k-1}, v_{3k+1}\}$, $D_5 = \{v_1, v_2, v_4, v_5, v_6, \ldots, v_{3k-1}, v_{3k}\}, \ldots, D_{k+5} = \{v_2, v_3, v_5, \ldots, v_{3k}, v_{3k+1}\}, \ldots, D_{2k+2} = \{v_1, v_3, v_4, v_6, v_7, \ldots, v_{3k-1}, v_{3k}\}$, $D_{2k+3} = \{v_1, v_2, v_4, \ldots, v_{3k-1}, v_{3k}\}$ $D_{2k+4} = \{v_1, v_3, v_4, v_6, v_7, \ldots, v_{3k}, v_{3k+1}\}, \ldots, D_{3k+1} = \{v_1, v_3, v_4, v_6, v_7, \ldots, v_{3k}, v_{3k+1}\}$. We have 3k+1 γ_{tss}–sets of C_{3k+1}. The γ_{tss}–sets D_1 is adjacent to D_k and D_{2k+3}. D_2 is adjacent to D_{k+1} and D_{3k}. D_3 is adjacent to D_{k+2} and $D_{2k+1}, \ldots,$ D_k is adjacent to D_1 and D_{k+3}. D_{k+1} is adjacent to D_2 and D_{2k}. D_{k+2} is adjacent to D_3 and D_{2k+1}. D_{k+3} is adjacent to D_k and $D_{2k+2}, \ldots$, D_{2k} is adjacent to D_{k+1} and D_{2k+3}. D_{2k+1} is adjacent to D_{k+2} and D_{3k}. D_{2k+2} is adjacent to D_{k+3} and D_{3k+1}. D_{2k+3} is adjacent to D_1 and $D_{2k}, \ldots, D_{3k}$ is adjacent to D_2 and D_{2k+1}. D_{3k+1} is adjacent to D_3 and D_{2k+2}. Thus we get a cycle $D_1, D_k, D_{2k+3}, D_{k+1}, D_{3k}, D_{k+2}, D_{2k+1}, \ldots, D_{k+3}, D_{2k}, D_3, D_k, D_{2k+2}, \ldots, D_{k+1}, D_{2k+3}, D_{k+3}, D_{3k+1}, D_1, D_{2k}, \ldots, D_2, D_{2k+1}, D_3, D_{2k+2}$. Thus the degree of each γ_{tss}–set D_i is 2. Then we get a cycle of 3k+1 vertices. Hence it is proved that $C_{3k+1}(\gamma_{tss}) \cong C_{3k+1}$, for $k \geq 2$.
■

Proposition 2.5. $P_{3k+1}(\gamma_{tss}) \cong K_1$ where k = 1, 2, 3, . . .
Proof: Let $\{v_1, v_2, v_3, \ldots, v_{3k+1}\}$ be the vertex set of the path P_{3k+1}.

Case (i) k =1.
The path obtained is P_4. The γ_{tss}–set of P_4 is $D = \{v_2, v_3\}$.The order of $P_4(\gamma_{tss})$ *is* 1. Hence $P_4(\gamma_{tss}) \cong K_1$.
Case (ii) k =2.
The path obtained is P_7. The γ_{tss}–set of P_7 is $D = \{v_2, v_3, v_5, v_6\}$.The order of $P_7(\gamma_{tss})$ *is* 1. Hence $P_7(\gamma_{tss}) \cong K_1$.
Case (ii) $k \geq 3$.
The γ_{tss}–set of P_{3k+1} is $D = \{v_2, v_3, v_5, v_6, \ldots, v_{3k-3}, v_{3k-1}, v_{3k}\}$. The order of $P_{3k+1}(\gamma_{tss})$ *is* 1. Hence $P_{3k+1}(\gamma_{tss}) \cong K_1$. ■

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