Intuitionistic Fuzzy $\widehat{\beta}$ Generalized Continuous Mappings

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Abstract

In this paper we have introduced intuitionistic fuzzy $\widehat{\beta}$ generalized continuous mappings and studied some of their basic properties.

Key words

Intuitionistic fuzzy topology, intuitionistic fuzzy $\widehat{\beta}$ generalized closed sets, intuitionistic fuzzy $\widehat{\beta}$ generalized continuous mappings, intuitionistic fuzzy $\widehat{\beta}$ $aT_{1/2}$ space and intuitionistic fuzzy $\widehat{\beta}$ $bT_{1/2}$.

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [11] and later Atanassov [1] generalized this idea to intuitionistic fuzzy sets using the notion of fuzzy sets. On the other hand Coker [4] introduced intuitionistic fuzzy topological spaces using the notion of intuitionistic fuzzy sets. In this paper, we introduced intuitionistic fuzzy $\widehat{\beta}$ generalized continuous mappings and studied some of their basic properties. We arrived at some characterizations of intuitionistic fuzzy $\widehat{\beta}$ generalized continuous mappings.

II. PRELIMINARIES

Definition 2.1: [1] Let X be a non empty fixed set. An *intuitionistic fuzzy set* (IFS in short) A in X is an object having the form

$$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X\}$$

where the functions $\mu_A(x): X \to [0, 1]$ and $\nu_A(x): X \to [0, 1]$ denote the degree of membership (namely $\mu_A(x)$) and the degree of non -membership (namely $\nu_A(x)$) of each element $x \in X$ to the set A, respectively, and $0 \leq \mu_A(x) + \nu_A(x) \leq 1$ for each $x \in X$. Denote the set of all intuitionistic fuzzy sets in X by IFS (X).

Definition 2.2: [1] Let A and B be IFSs of the form

$A = \{\langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$ and $B = \{ \langle x, \mu_B(x), \nu_B(x) \rangle / x \in X \}$. Then

(a) $A \subseteq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $\nu_A(x) \geq \nu_B(x)$ for all $x \in X$
(b) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(c) $A^c = \{ \langle x, \nu_A(x), \mu_A(x) \rangle / x \in X \}$
(d) $A \cap B = \{ \langle x, \mu_A(x) \wedge \mu_B(x), \nu_A(x) \vee \nu_B(x) \rangle / x \in X \}$
(e) $A \cup B = \{ \langle x, \mu_A(x) \vee \mu_B(x), \nu_A(x) \wedge \nu_B(x) \rangle / x \in X \}$

For the sake of simplicity, we shall use the notation $A = \langle x, \mu_A, \nu_A \rangle$ instead of $A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle / x \in X \}$. Also for the sake of simplicity, we shall use the notation $A = \{ \langle x, (\mu_A, \mu_B), (\nu_A, \nu_B) \rangle \}$ instead of $A = \langle x, (A/\mu_A, B/\mu_B), (A/\nu_A, B/\nu_B) \rangle$.

The intuitionistic fuzzy sets $0_\sim = \{ \langle x, 0, 1 \rangle / x \in X \}$ and $1_\sim = \{\langle x, 1, 0 \rangle / x \in X\}$ are respectively the empty set and the whole set of X.

Definition 2.3: [3] An *intuitionistic fuzzy topology* (IFT in short) on X is a family τ of IFSs in X satisfying the following axioms.

(i) $0_\sim, 1_\sim \in \tau$
(ii) $G_1 \cap G_2 \in \tau$ for any $G_1, G_2 \in \tau$

(iii) $\cup G_i \in \tau$ for any family $\{ G_i / i \in J \} \subseteq \tau$.
In this case the pair (X, τ) is called an *intuitionistic fuzzy topological space* (IFTS in short) and any IFS in τ is known as an intuitionistic fuzzy open set (IFOS in short) in X.
The complement A^c of an IFOS A in IFTS (X, τ) is called an intuitionistic fuzzy closed set (IFCS in short) in X.

Definition 2.4:[3] Let (X, τ) be an IFTS and $A = \langle x, \mu_A, \nu_A \rangle$ be an IFS in X. Then the intuitionistic fuzzy interior and intuitionistic fuzzy closure are defined by
$int(A) = \cup \{ G / G$ is an IFOS in X and $G \subseteq A \}$,
$cl(A) = \cap \{ K / K$ is an IFCS in X and $A \subseteq K \}$.

Note that for any IFS A in (X, τ), we have $cl(A^c) = [int(A)]^c$ and $int(A^c) = [cl(A)]^c$.

Definition 2.5: [4] An IFS $A = \langle x, \mu_A, \nu_A \rangle$ in an IFTS (X, τ) is said to be a
(i) intuitionistic fuzzy semi closed set (IFSCS for short) if $int(cl(A)) \subseteq A$,
(ii) intuitionistic fuzzy pre-closed set (IFPCS for short) if $cl(int(A)) \subseteq A$,
(iii) intuitionistic fuzzy α-closed set (IFαCS for short) if $cl(int(cl(A))) \subseteq A$,
(iv) intuitionistic fuzzy γ-closed set (IFγCS for short) if $cl(int(A)) \cap int(cl(A)) \subseteq A$
The respective complements of the above IFCSs are called their respective IFOSs.

The family of all IFSCSs, IFPCSs, IFαCSs and IFγCSs (respectively IFSOSs, IFPOSs, IFαOSs and IFγOSs) of an IFTS (X,τ) are respectively denoted by IFSC(X), IFPC(X), IFαC(X) and IFγC(X) (respectively IFSO(X), IFPO(X), IFαO(X) and IFγO(X)).
Definition 2.6:[12] Let A be an IFS in an IFTS (X, τ). Then
$sint(A) = \cup \{ G / G$ is an IFSOS in X and $G \subseteq A \}$,
$scl(A) = \cap \{ K / K$ is an IFSCS in X and $A \subseteq K \}$.
Note that for any IFS A in (X, τ), we have $scl(A^c)=(sint(A))^c$ and $sint(A^c) = (scl(A))^c$.

Definition 2.7:[9] An IFS A in an IFTS (X, τ) is an
(i) *intuitionistic fuzzy generalized closed set* (IFGCS in short) if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in X.

Definition 2.8:[9] An IFS A in an IFTS (X, τ) is said to be an *intuitionistic fuzzy generalized semi closed set* (IFGSCS in short) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is an IFOS in (X, τ).

Definition 2.9:[9] An IFS A is said to be an *intuitionistic fuzzy generalized semi open set* (IFGSOS in short) in X if the complement A^c is an IFGSCS in X.
The family of all IFGSCSs (IFGSOSs) of an IFTS (X, τ) is denoted by IFGSC(X) (IFGSO(X)).

Definition 2.10:[5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be *intuitionistic fuzzy continuous* (IF continuous in short) if $f^{-1}(B) \in IFO(X)$ for every $B \in \sigma$.

Definition 2.11: [5] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an
(i) intuitionistic fuzzy semi continuous mapping (IFS continuous mapping for short) if $f^{-1}(B) \in IFSO(X)$ for every $B \in \sigma$
(ii) intuitionistic fuzzy α-continuous mapping (IFα continuous mapping for short) if $f^{-1}(B) \in IF\alpha O(X)$ for every $B \in \sigma$
(iii) intuitionistic fuzzy pre continuous mapping (IFP continuous mapping for short) if $f^{-1}(B) \in IFPO(X)$ for every $B \in \sigma$
(iv) intuitionistic fuzzy β continuous mapping (IFγ continuous mapping for short) if $f^{-1}(B) \in IF\gamma O(X)$ for every $B \in \sigma$.

Definition 2.12: [10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy generalized continuous mapping (IFG continuous mapping for short) if $f^{-1}(B) \in$ IFGC(X) for every IFCS B in Y.

Definition 2.13: [10] Let f be a mapping from an IFTS (X, τ) into an IFTS (Y, σ). Then f is said to be an intuitionistic fuzzy semi-pre continuous mapping (IFSP continuous mapping for short) if $f^{-1}(B) \in$ IFSPO(X) for every $B \in \sigma$.

Result 2.14:[9] Every IF continuous mapping is an IFG continuous mapping.

Definition 2.15:[8] A mapping f: (X, τ) → (Y,σ) is called an *intuitionistic fuzzy generalized semi continuous* (IFGS continuous in short) if $f^{-1}(B)$ is an IFGSCS in (X, τ) for every IFCS B of (Y, σ).

Definition 2.16: [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\hat{\beta}$ **a**$T_{1/2}$ (IF$\hat{\beta}$ **a**$T_{1/2}$ in short) space if every IF$\hat{\beta}$ GCS in X is an IFCS in X.

Definition 2.17: [8] An IFTS (X, τ) is said to be an intuitionistic fuzzy $\hat{\beta}$ **b**$T_{1/2}$ (IF$\hat{\beta}$ **b**$T_{1/2}$ in short) space if every IF$\hat{\beta}$ GCS in X is an IFGCS in X.

III. INTUITIONISTIC FUZZY $\hat{\beta}$ GENERALIZED CONTINUOUS MAPPINGS

In this section we have introduced intuitionistic fuzzy $\hat{\beta}$ generalized continuous mappings and investigated some of their properties.

Definition 3.1: A mapping f : (X, τ) → (Y,σ) is called an intuitionistic fuzzy $\hat{\beta}$ generalized continuous (IF$\hat{\beta}$ G continuous in short) mapping if $f^{-1}(B)$ is an IF$\hat{\beta}$ GCS in (X, τ) for every IFCS B of (Y, σ).

Example 3.2: Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.1, 0.1), (0.5, 0.6) ⟩, G_2 = ⟨ y, (0.5, 0.6), (0.3, 0.1) ⟩. Then τ = { $0_\sim$, G_1, $1_\sim$ } and σ = { $0_\sim$, G_2, $1_\sim$ } are IFTs on X and Y respectively. Here μ_{G_1}(a) = 0.1, μ_{G_1}(b) = 0.1, ϑ_{G_1}(a) = 0.5, ϑ_{G_1}(b)= 0.6, μ_{G_2}(u)= 0.5, μ_{G_2}(v)= 0.6, ϑ_{G_2}(u)= 0.3, and ϑ_{G_2}(v)= 0.1. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then clearly for the IFCS $0_\sim$, $1_\sim$ in Y, $f^{-1}(0_\sim)$ and $f^{-1}(1_\sim)$ are IF$\hat{\beta}$ GCS in X. Let us consider the IFCS G_2^c in Y. Then $f^{-1}(G_2^c)$ = ⟨ x, (0.3, 0.1), (0.5, 0.6) ⟩ is an IF$\hat{\beta}$ GCS in X. Hence f is an IF$\hat{\beta}$ G continuous mapping.

Theorem 3.3: Every IF continuous mapping is an IF$\hat{\beta}$ G continuous mapping but not conversely.
Proof: Let f : (X, τ) → (Y, σ) be an IF continuous mapping. Let A be an IFCS in Y. Since f is an IF continuous mapping, $f^{-1}(A)$ is an IFCS in X. Since every IFCS is an IF$\hat{\beta}$ GCS, $f^{-1}(A)$ is an IF$\hat{\beta}$ GCS in X. Hence f is an IF$\hat{\beta}$ G continuous mapping.

Example 3.4: Let X = { a, b }, Y = { u, v } and G_1 = ⟨ x, (0.2, 0.1), (0.5, 0.6) ⟩, G_2 = ⟨ y, (0.4, 0.3), (0.3, 0.1) ⟩. Then τ = { $0_\sim$, G_1, $1_\sim$ } and σ = { $0_\sim$, G_2, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF$\hat{\beta}$ G continuous mapping. Now consider the IFCS G_2^c = ⟨ y, (0.3, 0.1), (0.4, 0.3) ⟩ is an IFCS in Y. Then $f^{-1}(G_2^c)$ = ⟨ x, (0.3, 0.1), (0.4, 0.3) ⟩ is not an IFCS in X. Hence f is not an IF continuous mapping.

Theorem 3.5: Every IFα continuous mapping is an IF$\hat{\beta}$ G continuous mapping but not conversely.
Proof: Let f : (X, τ) → (Y,σ) be an IFα continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IFαCS in X. Since every IFαCS is an IF$\hat{\beta}$ GCS, $f^{-1}(A)$ is an IF$\hat{\beta}$ GCS in X. Hence f is an IF$\hat{\beta}$ G continuous mapping.

Example 3.6: Let X = { a, b }, Y = { u, v } and let the IFS G_1 = ⟨ x, (0.3, 0.1), (0.5, 0.6) ⟩, G_2 = ⟨ x, (0.7, 0.7), (0.1, 0.1) ⟩ and G_3 = ⟨ y, (0.3, 0.3), (0.4, 0.5) ⟩. Then τ = { $0_\sim$, G_1, G_2, $1_\sim$ } and σ = { $0_\sim$, G_3, $1_\sim$} are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IF$\hat{\beta}$ G continuous mapping.

Let us consider the IFCS $G_3^c = \langle y, (0.4, 0.5), (0.3, 0.3) \rangle$ in Y. Then $f^{-1}(G_3^c)$ is not an IFαCS in X. Hence f is not an IFα continuous mapping.

Theorem 3.7: Every IFG continuous mapping is an IF$\hat{\beta}$G continuous mapping but not conversely.
Proof: Let $f : (X, \tau) \rightarrow (Y, \sigma)$ be an IFG continuous mapping. Let A be an IFCS in Y. Since f is an IFG continuous mapping, $f^{-1}(A)$ is an IFGCS in X. Since every IFGCS is an IF$\hat{\beta}$GCS, $f^{-1}(A)$ is an IF$\hat{\beta}$GCS in X. Hence f is an IF$\hat{\beta}$G continuous mapping.

Example 3.8: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.1, 0.7), (0.2, 0.1) \rangle$, $G_2 = \langle y, (0.3, 0.8), (0.1, 0) \rangle$. Then $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\hat{\beta}$G continuous mapping. Now consider the IFCS $G_2^c = \langle y, (0.1, 0), (0.3, 0.8) \rangle$ in Y. Then $f^{-1}(G_2^c) = \langle x, (0.1, 0), (0.3, 0.8) \rangle$ is not an IFGCS in X. Hence f is not an IFG continuous mapping.

Theorem 3.9: Every IF$\hat{\beta}$G continuous mapping is an IFGS continuous mapping but not conversely.
Proof: Let $f : (X, \tau) \rightarrow (Y,\sigma)$ be an IF$\hat{\beta}$G continuous mapping. Let A be an IFCS in Y. Then by hypothesis $f^{-1}(A)$ is an IF$\hat{\beta}$GCS in X. Since every IF$\hat{\beta}$GCS is an IFGSCS, $f^{-1}(A)$ is an IFGSCS in X. Hence f is an IFGS continuous mapping.

Example 3.10: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.1, 0.2), (0.3, 0.4) \rangle$, $G_2 = \langle y, (0.4, 0.5), (0.1, 0) \rangle$. Then $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFGS continuous mapping. Let us consider the IFCS $G_2^c = \langle y, (0.1, 0), (0.4, 0.5) \rangle$ in Y. Then $f^{-1}(G_2^c)$ is not an IF$\hat{\beta}$GCS in X. Hence f is not an IF$\hat{\beta}$G continuous mapping.

Remark 3.11: IFP continuous mapping and IF$\hat{\beta}$G continuous mapping are independent of each other.

Example 3.12: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0, 0.9), (0.5, 0.1) \rangle$, $G_2 = \langle y, (0.7, 0.7), (0, 0.3) \rangle$. Then $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFP continuous mapping. But f is not an IF$\hat{\beta}$G continuous mapping since $G_2^c = \langle y, (0, 0.3), (0.7, 0.7) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0, 0.3), (0.7, 0.7) \rangle$ is not an IF$\hat{\beta}$GCS in X.

Example 3.13: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.2, 0.2), (0.5, 0.6) \rangle$, $G_2 = \langle y, (0.4, 0.5), (0.3, 0.2) \rangle$. Then $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\hat{\beta}$G continuous mapping. But f is not an IFP continuous mapping since $G_2^c = \langle y, (0.3, 0.2), (0.4, 0.5) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.3, 0.2), (0.4, 0.5) \rangle$ is not an IFPCS in X.

Remark 3.14: IFγ continuous mapping and IF$\hat{\beta}$G continuous mapping are independent of each other.

Example 3.15: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.4, 0.6), (0.2, 0.2) \rangle$ $G_2 = \langle y, (0.6, 0.2), (0.4, 0.3) \rangle$. Then $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IFγ continuous mapping. But f is not an IF$\hat{\beta}$G continuous mapping since $G_2^c = \langle y, (0.4, 0.3), (0.6, 0.2) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.4, 0.3), (0.6, 0.2) \rangle$ is not an IF$\hat{\beta}$GCS in X.

Example 3.16: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.5, 0.1), (0.5, 0.9) \rangle$, $G_2 = \langle y, (0.2, 0.1), (0.7, 0.8) \rangle$. Then $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_2, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\hat{\beta}$G continuous mapping but f is not an IFγ continuous mapping since $G_2^c = \langle y, (0.7, 0.8), (0.2, 0.1) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.7, 0.8), (0.2, 0.1) \rangle$ is not an IFγCS in X.

Remark 3.17: IFS continuous mapping and IF$\hat{\beta}$G continuous mapping are independent of each other.

Example 3.18: Let X = { a, b }, Y = { u, v } and $G_1 = \langle x, (0.3, 0.5), (0.1, 0.1) \rangle$, $G_2 = \langle y, (0.1, 0), (0.8, 0.8) \rangle$. Then $\tau = \{ 0_\sim, G_1, 1_\sim \}$ and $\sigma = \{0_\sim, G_2, 1_\sim\}$ are IFTs on X and Y respectively. Define a mapping $f : (X, \tau) \rightarrow (Y, \sigma)$ by f(a) = u and f(b) = v. Then f is an IF$\hat{\beta}$G continuous mapping. But f is not an IFS continuous mapping since $G_2^c = \langle y, (0.8, 0.8), (0.1, 0) \rangle$ is an IFCS in Y but $f^{-1}(G_2^c) = \langle x, (0.8, 0.8), (0.1, 0) \rangle$ is not an IFSCS in X.

Example 3.19: Let X = { a, b }, Y = { u, v } and let $G_1 = \langle x, (0.1, 0.2), (0.4, 0.5) \rangle$, $G_2 = \langle x, (0.3, 0.3), (0.1, 0.2) \rangle$ and $G_3 = \langle y, (0.4, 0.4), (0.2, 0.3) \rangle$. Then $\tau = \{ 0_\sim, G_1, G_2, 1_\sim \}$ and $\sigma = \{ 0_\sim, G_3, 1_\sim \}$ are IFTs on X and Y respectively. Define a mapping f : (X, τ) → (Y, σ) by f(a) = u and f(b) = v. Then f is an IFS continuous mapping. But f is not an IF$\widehat{\beta}$ G continuous mapping since $G_3^c = \langle y, (0.2, 0.3), (0.4, 0.4) \rangle$ is an IFCS in Y but $f^{-1}(G_3^c) = \langle x, (0.2, 0.3), (0.4, 0.4) \rangle$ is not an IF$\widehat{\beta}$ GCS in X.

Theorem 3.20: A mapping f : X → Y is an IF$\widehat{\beta}$ G continuous if and only if the inverse image of each IFOS in (Y, σ) is an IF$\widehat{\beta}$ GOS in (X, τ).

Proof: Necessity: Let A be an IFOS in (Y, σ). This implies A^c is an IFCS in Y.Since f is an IF$\widehat{\beta}$ G continuous mapping, $f^{-1}(A^c)$ is an IF$\widehat{\beta}$ GCS in (X, τ). Since $f^{-1}(A^c) = (f^{-1}(A))^c$, $f^{-1}(A)$ is an IF$\widehat{\beta}$ GOS in X.

Sufficiency: Let A be an IFCS in (Y, σ). Then A^c is an IFOS in Y. By hypothesis, A^c is an IF$\widehat{\beta}$ GOS in (X, τ). Hence A is an IF$\widehat{\beta}$ GCS in X.

Theorem 3.21: Let f : (X, τ) → (Y,σ) be a mapping and let $f^{-1}(A)$ be an IFRCS in X for every IFCS A in Y. Then f is an IF$\widehat{\beta}$ G continuous mapping.

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IFRCS in X. Since every IFRCS is an IF$\widehat{\beta}$ GCS, $f^{-1}(A)$ is an IF$\widehat{\beta}$ GCS in X. Hence f is an IF$\widehat{\beta}$ G continuous mapping.

Theorem 3.22: Let f : (X, τ) → (Y, σ) be an IF$\widehat{\beta}$ G continuous mapping. Then f is an IF continuous mapping if X is an IF$\widehat{\beta}$ **a**$T_{1/2}$ space.

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IF$\widehat{\beta}$ GCS in X by hypothesis. Since X is an IF$\widehat{\beta}$ **a**$T_{1/2}$ space, $f^{-1}(A)$ is an IFCS in X. Hence f is an IF continuous mapping.

Theorem 3.23: Let f : (X, τ) → (Y, σ) be an IF$\widehat{\beta}$ G continuous mapping. Then f is an IFG continuous mapping if X is an IF$\widehat{\beta}$ **b**$T_{1/2}$ space.

Proof: Let A be an IFCS in Y. Then $f^{-1}(A)$ is an IF$\widehat{\beta}$ GCS in X, by hypothesis. Since X is an IF$\widehat{\beta}$ **b**$T_{1/2}$ space, $f^{-1}(A)$ is an IFGCS in X. Hence f is an IFG continuous mapping.

Theorem 3.24: Let f : (X, τ) → (Y, σ) be an IF$\widehat{\beta}$ G continuous mapping and g : (Y, σ) → (Z, η) is an IF continuous mapping, then g o f : (X, τ) → (Z, η) is an IF$\widehat{\beta}$ G continuous mapping.

Proof: Let A be an IFCS in Z. Then $g^{-1}(A)$ is an IFCS in Y, by hypothesis. Since f is an IF$\widehat{\beta}$ G continuous mapping, $f^{-1}(g^{-1}(A))$ is an IF$\widehat{\beta}$ GCS in X. That is $(g \circ f)^{-1}(A)$ is an IF$\widehat{\beta}$ GCS in X. Hence the mapping g o f is an IF$\widehat{\beta}$ G continuous mapping.

Theorem 3.25: Let f : (X, τ) → (Y, σ) be a mapping from an IFTS X into an IFTS Y. Then the following conditions are equivalent if X is an IF$\widehat{\beta}$ **a**$T_{1/2}$ space:

(i) f is an IF$\widehat{\beta}$ G continuous mapping

(ii) If B is an IFOS in Y then $f^{-1}(B)$ is an IF$\widehat{\beta}$ GOS in X

(iii) $f^{-1}(int(B)) \subseteq int(cl(int(f^{-1}(B))))$ for every IFS B in Y.

Proof: (i) ⇒ (ii): It is obviously true.

(ii) ⇒ (iii): Let B be any IFS in Y. Then int(B) is an IFOS in Y. Then $f^{-1}(int(B))$ is an IF$\widehat{\beta}$ GOS in X. Since X is an IF$\widehat{\beta}$ **a**$T_{1/2}$ space, $f^{-1}(int(B))$ is an IFOS in X. Therefore, $f^{-1}(int\ B) = int(f^{-1}(int(B))) \subseteq int(cl(int(f^{-1}(B))))$.

(iii) ⇒ (i): Let B be an IFCS in Y. Then B^c is an IFOS in Y. By hypothesis $f^{-1}(int(B^c)) \subseteq int(cl(int(f^{-1}(B^c))))$. This implies $f^{-1}(B^c) \subseteq int(cl(int(f^{-1}(B^c))))$. Hence $f^{-1}(B^c)$ is an IFαOS in X. Since every IFαOS is an IF$\widehat{\beta}$ GOS, $f^{-1}(B^c)$ is an IF$\widehat{\beta}$ GOS in X. Therefore, $f^{1}(B)$ is an IF$\widehat{\beta}$ GCS in X. Hence f is an IF$\widehat{\beta}$ G continuous mapping.

Theorem 3.26: Let f : (X, τ) → (Y, σ) be a mapping. Then the following conditions are equivalent if X is an IF$\widehat{\beta}$ **a**$T_{1/2}$ space:

(i) f is an IF$\widehat{\beta}$ G continuous mapping

(ii) $f^{-1}(B)$ is an IF$\widehat{\beta}$ GCS in X for every IFCS B in Y

(iii) $cl(int(cl(f^{-1}(A)))) \subseteq f^{-1}(cl(A))$ for every IFS B in Y.

Proof: (i) ⇒ (ii): is obviously true.
(ii) ⇒ (iii): Let A be an IFS in Y. Then cl(A) is an IFCS in Y. By hypothesis, f^{-1}(cl(A)) is an IF$\hat{\beta}$ GCS in X. Since X is an IF$\hat{\beta}$ **a**$T_{1/2}$ space, f^{-1}(cl(A)) is an IFCS in X. Therefore, cl(f^{-1}(cl(A))) = f^{-1}(cl(A)). Now cl(int(cl(f^{-1}(A)))) $\subseteq$ cl(int(cl(f^{-1}(cl(A))))) $\subseteq$ f^{-1}(cl(A)).
(iii) ⇒ (i): Let A be an IFCS in Y. By hypothesis cl(int(cl(f^{-1}(A)))) $\subseteq$ f^{-1}(cl(A)) = f^{-1}(A). This implies f^{-1}(A) is an IFαCS in X and hence it is an IF$\hat{\beta}$ GCS in X. Therefore, f is an IF$\hat{\beta}$ G continuous mapping.

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