On Neutrosophic Feebly Frontier in Neutrosophic Topological Spaces

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Abstract

In this paper, we introduce a new class of operator using neutrosophic feebly closed sets namely neutrosophic feebly frontier. Also we study some of their basic properties and their characterizations.

Keywords and Phrases - *Neutrosophic feebly closed sets, neutrosophic feebly frontier.*

I. INTRODUCTION

Neutrosophy, as a new branch of Philosophy has been introduced by Smrandache [6-9]and explained, neutrosophic set is a generalization of Intuitionistic fuzzy set [2]. In 2012, Salama, Alblowi[12-15], introduced the concept of neutrosophic topological spaces. They introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space and a neutrosophic set besides the degree of membership, the degree of indeterminacy and the degree of non-membership of each element. In 2014, Salama, Smarandache and Valeri[14] were introduced the concept of neutrosophic closed sets and neutrosophic continuous functions. The authors defined the concept of neutrosophic feebly open sets and neutrosophic feebly closed sets in neutrosophic topological spaces.

In this paper, we have to introduce neutrosophic feebly frontier of a neutrosophic set using neutrosophic feebly closed set. This paper consists of three sections. The introduction given in Section I. The Section II consists of the basic definitions of neutrosophic feebly open sets, neutrosophic feebly closed sets and their properties which are used in the later sections. The Section III deals with the concept of neutrosophic feebly frontier in neutrosophic topological space and their properties.

II. PRELIMINARIES

For basic notations and definitionsof Neutrosophic Theory is not given here, the reader can refer [1-16].

Definition 2.1.[11]A neutrosophic subset A of a neutrosophic topological space (X,τ) is neutrosophic feebly open if there is a neutrosophic open set U in X such that $U \leq A \leq NScl(U)$.

Lemma 2.2.[11]A neutrosophic subset A is neutrosophic feebly open if and only if

(i) $A \leq Nint\left(Ncl(Nint(A))\right)$.

(ii) $A \leq NScl(Nint(A))$.

Lemma 2.3. [11]Let (X,τ) and (Y,σ) be any two neutrosophic topological spaces such that X is product related to Y. Then the product $A_1{\times}A_2$ of a neutrosophic feebly open set A_1 of X and a neutrosophic feebly open set A_2 of Y is a neutrosophic feebly open set of the neutrosophic product space $X \times Y$.

Definition 2.4. [11]A neutrosophic subset A of a neutrosophic topological space (X,τ) is neutrosophic feebly closed if there is a neutrosophic closed set U in X such that $NSint(U) \leq A \leq U$.

Lemma 2.5. [11]A neutrosophic subset A of a neutrosophic topological space (X,τ) isneutrosophic feebly closed if and only if $Ncl\left(Nint(Ncl(A))\right) \leq A$.

(i) A neutrosophic subset A is neutrosophic feebly closed iff if $NSint(Ncl(A)) \leq A$.

(ii) A neutrosophic subset A is a neutrosophic feebly closed set if and only if A^c is neutrosophic feebly open.

Lemma 2.6. [11]Let (X, τ) and (Y, σ) be any two neutrosophic topological spaces such that X is product related to Y. Then the product $A_1 \times A_2$ of a neutrosophic feebly closed set A_1 of X and a neutrosophic feebly closed set A_2 of Y is a neutrosophic feebly closed set of the neutrosophic product space $X \times Y$.

Definition 2.7. [11]Let (X, τ) be neutrosophic topological space and $A=\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ be a neutrosophic set in X. Then neutrosophic feebly interior of A is defined by $NFint(A) = \vee\{G: G$is a neutrosophic feebly open set in X and $G \leq A\}$.

Lemma 2.8. [11]Let (X, τ) be neutrosophic topological space. Then for any neutrosophic feebly subsets A and B of a neutrosophic topological space$X$, we have

(i) $NFint(A) \leq A$

(ii) $A$is neutrosophic feebly open set in $X \Leftrightarrow NFint(A) = A$

(iii) $NFint(NFint(A)) = NFint(A)$

(iv) If $A \leq B$,$NFint(A) \leq NFint(B)$

(v) $NFint(A \wedge B) \leq NFint(A) \wedge NFint(B)$.

(vi) $NFint\ (A \vee B) \geq NFint(A) \vee NFint(B)$.

Definition 2.9. [11]Let (X, τ) be neutrosophic topological space and$A= \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ be a neutrosophic set in X. Then the neutrosophic feebly closure is defined by$NFcl(A)= \wedge\{K: K$is a neutrosophic feebly closed set in X and $A \leq K\}$.

Lemma 2.10.[11]Let (X, τ) be a neutrosophic topological space. Then for any neutrosophic subset $A$of X,

(i) $(NFint(A))^c = NFcl(A^c)$

(ii) $(NFcl(A))^c = NFint(A^c)$.

Lemma 2.11.[11]Let (X, τ) be a neutrosophic topological space. Then for any neutrosophic subsets A and B of a neutrosophic topological space X,

(i) $A \leq NFcl\ (A)$

(ii) $A$is a neutrosophic feebly closed set in $X \Leftrightarrow NFcl(A) = A$

(iii) $NFcl(NFcl\ (A)) = NFcl(A)$

(iv) If $A \leq B$then $NFcl(A) \leq NFcl(B)$.

(v) $NFcl(A \vee B) \geq NFcl(A) \vee NFcl(B)$and

(vi)$NFcl(A \wedge B) \leq NFcl(A) \wedge NFcl(B)$.

(vii) $NFcl(A) \times NFcl(B) \geq NFcl(A \times B)$,

(viii) $NFint(A) \times NFint(B) \leq NFint(A \times B)$.

Lemma2.12.[11]Let(X, τ) and (Y, σ) be two neutrosophic topological spaces such that X is neutrosophic product related to Y. Then for neutrosophic subsets A of X and B of Y,

(i) $NFcl(A \times B) = NFcl(A) \times NFcl(B)$,

(ii) $NFint(A \times B) = NFint(A) \times NFint(B)$.

III. NEUTROSOPHIC FEEBLYFRONTIER

In this section, we introduce the neutrosophic feebly frontier and their properties in neutrosophic topological spaces.

Definition 3.1.Let A be a neutrosophicsubset in the neutrosophic topological space X. Then the neutrosophic feebly frontier of a neutrosophicsubset A is defined asneutrosophicsubset$NFFr(A)= NFcl(A) \wedge NFcl(A^C)$.

Remark 3.2. For a neutrosophic subset A of X, $NFFr(A)$ is neutrosophic feeblyclosed.

Theorem 3.3.For a neutrosophicsubsetAin the neutrosophic topological space X, $NFFr(A) = NFFr(A^C)$.

Proof. Let A be the neutrosophicsubset in the neutrosophic topological space X. Then by Definition 3.1,$NFFr(A)=NFcl(A) \wedge NFcl(A^C) = NFcl(A^C) \wedge NFcl(A)=$
$NFcl(A^C) \wedge NFcl((A^C)^C)$. Again by Definition 3.1,this is equal to$NFFr(A^C)$. Hence $NFFr(A) = NFFr(A^C)$.

Theorem 3.4.Let Abe a neutrosophicsubset in the neutrosophic topological space X. Then $NFFr(A) = NFcl$(A) – $NFint(A)$.

Proof.Let A be the neutrosophicsubset in the neutrosophic topological space X. By Lemma 2.10(ii),$(NFcl(A^C))^C$=$NFint(A)$ and by Definition

3.1,$NFFr(A) = NFcl(A) \wedge (NFcl(A^C))$=$NFcl(A) \wedge (NFint(A^C))^C$ by using A–B =A∧B^C,$NFFr(A)$= $NFcl(A)$ – $NFint(A)$. Hence $NFFr(A)$=$NFcl(A)$ – $NFint(A)$.

Theorem 3.5. Aneutrosophicsubset Ais neutrosophic feebly closed set in X if and only if$NFFr(A) \leq$A.

Proof. Let A be the neutrosophic feebly closed set in the neutrosophic topological space X. Then by Definition 3.1,$NFFr(A) = NFcl(A) \wedge NFcl(A^C) \leq NFcl(A)$. By Lemma 2.11(ii),$NFcl(A)$= A.Hence $NFFr(A) \leq A$, if A is neutrosophicfeebly closedin X.

Conversely, Assume that,$NFFr(A) \leq A$. Then $NFcl(A) - NFint(A) \leq A$. Since $NFint(A) \leq A$, then we conclude that $NFcl(A) = A$ and hence A is neutrosophic feebly closed.

Theorem 3.6. If Ais neutrosophicfeeblyopenset in X, then $NFFr(A) \leq A^c$.

Proof. Let A be the neutrosophicfeeblyopen set in the neutrosophic topological space X. By Lemma 2.5,A^c is neutrosophicfeebly closed set in X. By Theorem 3.5, $NFFr(A^c) \leq A^c$and by Theorem 3.3, we get $NFFr$(A) ≤ A^c.

The converse of the above theorem is not true as shown by the following example.

Example 3.7. Let $X = \{a, b, c\}$andτ= $\{0_N, A, B, C, D, 1_N\}$. Then (X, τ) is a neutrosophic topological space. The neutrosophicclosed sets areτ^C= $\{1_N, F, G, H, I, 0_N\}$ where,

A = <(0.7, 0.2, 0.3), (0.1, 0.8, 0.4), (0.5, 0.6, 0.7)> ,
B = <(0.9, 0.6, 0.7), (0.5, 0.4, 0.2), (0.8, 0.8, 0.5)>,
C = <(0.9, 0.6, 0.3), (0.5, 0.8, 0.2), (0.8, 0.8, 0.5) >,
D = <(0.7, 0.2, 0.7), (0.1, 0.4, 0.4), (0.5, 0.6, 0.7)>,
E = <(0.9, 0.7, 0.2), (0.5, 0.8, 0.1), (0.8, 0.8, 0.4)>,
F = <(0.3, 0.8, 0.7), (0.4, 0.2, 0.1), (0.7, 0.4, 0.5)>,
G = <(0.7, 0.4, 0.9), (0.2, 0.6, 0.5), (0.5, 0.2, 0.8)>,
H = <(0.3, 0.4, 0.9), (0.2, 0.2, 0.5), (0.5, 0.2, 0.8)>,

I = <, (0.7, 0.8, 0.7), , (0.4, 0.6, 0.1)(0.7, 0.4, 0.5)>and
J = <(0.2, 0.3, 0.9), (0.1, 0.2, 0.5), (0.4, 0.2, 0.8) >. Here E and J are neutrosophicfeeblyopen and neutrosophicfeeblyclosed set respectively. Some of the neutrosophicfeeblyopen are 0_N, A, B, C, D, E, $1_N$and neutrosophic feebly-closed set are 1_N, F, G, H, I, J, 0_N. Therefore $NFFr(C)$=H≤C. But C is not a neutrosophic feeblyclosed set.$NFFr$(J)≤ J^c= E. But J is not a neutrosophic feebly open set.

Proposition3.8. Let A≤Band Bbe any neutrosophic feebly closed set in X. Then $NFFr(A) \leq$ B.

Proof. By Lemma 2.11(iv), A≤B, $NFcl(A) \leq NFcl(B)$.By Definition 3.1, $NFFr(A) = NFcl(A) \wedge NFcl(A^C) \leq NFcl(B) \wedge NFcl(A^C) \leq NFcl(B)$. Then this is equal to B. Hence $NFFr(A) \leq$B.

Theorem 3.9. Let Abe a neutrosophicsubset in the neutrosophic topological space X. Then$(NFFr(A))^c$= $NFint$ (A)∨$NFint$ (A^C).

Proof. Let A be the neutrosophicsubset in the neutrosophic topological space X. Then by Definition 3.1,$(NFFr(A))^c$= $(NFcl(A) \wedge NFcl(A^c))^c = ((NFcl(A))^c \vee (NFcl(A^c))^c$. By Lemma 2.10(ii), which is equal to$NFint(A^C) \vee NFint(A)$. Hence $(NFFr(A))^c$= $NFint$(A)∨$NFint(A^c)$.

Theorem 3.10. For a neutrosophic subsetA in the neutrosophic topological space X, then$NFFr(A) \leq NFr(A)$.

Proof. Let A be the neutrosophic subset in the neutrosophic topological space X. Then by Lemma 2.11, $NFcl(A) \leq Ncl(A)$and $NFcl(A^C) \leq Ncl(A^C)$. Now by Definition 3.1, $NFFr(A)$= $NFcl(A) \wedge NFcl(A^C) \leq Ncl(A) \wedge Ncl(A^C)$, this is equal to $NFr(A)$. Hence $NFFr(A) \leq NFr(A)$.

The converse of the above theorem is not true as shown by the following example.

Example 3.11. From Example 3.7, letA_1 = < (0.4, 0.1, 0.9), (0.1, 0.2, 0.6), (0.1, 0.3, 0.9) >.Then$A_1{}^C$ = <(0.9, 0.9, 0.4), (0.6, 0.8, 0.1) , (0.9, 0.7, 0.1) >. Therefore $NFFr\ (A_1) = H \nsubseteq J = NFr(A_1)$.

Theorem 3.12. For a neutrosophic subsetA in the neutrosophic topological space X, $NFcl(NFFr(A))$≤$NFFr(A)$.

Proof. Let A be the neutrosophic subset in the neutrosophic topological space X. Then by Definition 3.1,$NFcl(NFFr(A))$ =$NFcl(NFcl(A) \wedge (NFcl(A^C)))$ ≤$(NFcl(NFcl(A))) \wedge (NFcl(NFcl(A^C)))$. By Lemma 2.11 (iii),$NFcl(NFFr(A))$=$NFcl$ (A)∧$(NFcl(A^C))$. By Definition 3.1, this is equal to $NFFr(A)$.

The converse of the above theorem is not true as shown by the following example.

Example 3.13. From Example 3.7,$NFFr(A_1)$ = H ⊈ J = $NFcl(NFFr(A_1))$.

Theorem 3.14. For a neutrosophic subsetA in the neutrosophic topological space X, $NFFr\big(NFint(A)\big) \leq NFFr(A)$.

Proof. Let A be the neutrosophic subset in the neutrosophic topological space X. Then by Definition 3.1, $NFFr\big(NFint(A)\big) = NFcl(Nint(A)) \wedge (NFcl(NFint(A))^C)$.By Lemma 2.10(i),$NFFr\big(NFint(A)\big)$= $NFcl(NFint(A)) \wedge (NFcl\ (NFcl\ (A^C)))$. By Lemma 2.11(iii),

= $NFcl(NFint(A)) \wedge (NFcl(A^C)$. By Lemma 2.11(i),
≤ $NFcl$(A)∧$NFcl(A^C)$.By Definition 3.1,

= $NFFr(A)$.Hence $NFFr(NFint(A)) \leq (NFFr(A))$.

The converse of the above theorem is not true as shown by the following example.

Example 3.15.Let $X = \{a, b, c\}$ and neutrosophic feebly open sets are 0_N, $A, B, C, D, E, 1_N$ and neutrosophicfeebly closed sets are 1_N, $F, G, H, I, J, 0_N$ where

A = <(0.3, 0.4, 0.2), (0.5, 0.6, 0.7), (0.9, 0.5, 0.2)>,
B = <(0.3, 0.5, 0.1), (0.4, 0.3, 0.2), (0.8, 0.4, 0.6)>,
C = <(0.3, 0.5, 0.1), (0.5, 0.6, 0.2), (0.9, 0.5, 0.2)>,
D = <(0.3, 0.4, 0.2), (0.4, 0.3, 0.7), (0.8, 0.4, 0.6)>,
E = <(0.5, 0.6, 0.1), (0.6, 0.7, 0.1), (0.9, 0.5, 0.2)>,
F = <(0.2, 0.6, 0.3), (0.7, 0.4, 0.5), (0.2, 0.5, 0.9)>,
G = <(0.1, 0.5, 0.3), (0.2, 0.7, 0.4), (0.6, 0.6, 0.8)>,
H = <(0.1, 0.5, 0.3), (0.2,0.4, 0.5), (0.2, 0.5, 0.9)>,
I = <(0.2, 0.6, 0.3), (0.7, 0.7, 0.4), (0.6, 0.6, 0.8)>,
J = <(0.1, 0.4, 0.5), (0.1, 0.3, 0.6), (0.2, 0.5, 0.9)>

Define A_1= <(0.2, 0.3, 0.4), (0.4, 0.5, 0.6), (0.3, 0.4, 0.8)>.

Then $(A_1)^C$ = <(0.4, 0.7, 0.2), (0.6, 0.5, 0.4) , (0.8, 0.6, 0.3)>.

Therefore $NFFr(A_1)$= I $\nsubseteq 0_N$ = $NFFr((NFint(A_1))$.

Proposition3.16. For a neutrosophic subsetA in the neutrosophic topological space X, then $NFFr(NFcl(A)) \leq NFFr(A)$.

Proof. Let A be the neutrosophic subset in the neutrosophic topological space X. Then by Definition 3.1, $NFFr(NFcl$(A)) = $NFcl(NFcl$(A))$\wedge$($NFcl$(($NFcl$(A)^C)).

By Lemma 2.11(ii),(iii)&(iv) , = $NFcl$(A)$\wedge$($NFcl$($NFint$(A^C)). By Lemma 2.10,

$\leq$ $NFcl$(A)$\wedge NFcl(A^C)$. By Definition 3.1,

= $NFFr(A)$. Hence $NFFr(NFcl$(A))$\leq NFFr(A)$.

The converse of the above theorem is not true as shown by the following example.

Example 3.17.From Example 3.15, let A_2 = <(0.2, 0.6, 0.2), (0.3, 0.4, 0.6), (0.3, 0.4, 0.8)>. Then $(A_2)^C$ = <(0.2, 0.4, 0.2), (0.6, 0.6, 0.3), (0.8, 0.6, 0.3) >. Therefore $NFFr(A_2)$ = $1_N \nsubseteq 0_N$ = $NFFr(NFcl(A_2))$.

Proposition3.18.Let Abe the neutrosophic subset in the neutrosophic topological space X. Then $NFint(A) \leq$A – $NFFr(A)$.

Proof. Let A be the neutrosophic subset in the neutrosophic topological space X. Nowby Definition 3.1,A – $NFFr(A)$=$A\wedge(NFFr$(A))^c=$A\wedge[NFcl$(A)$\wedge NFcl(A^c)]^c$ = $A\wedge[NFint(A^c)\vee NFint(A)]$=$[A\wedge NFint(A^c)]\vee[A\wedge NFint(A)]$=$[A\wedge NFint(A^c)]\vee NFint(A) \geq NFint(A)$. Hence $NFint(A) \leq$A – $NFFr(A)$.

The converse of the above theorem is not true as shown by the following example.

Example 3.19.From Example 3.15,A_1 – $NFFr(A_1)$ = <(0.2, 0.3, 0.4), (0.4, 0.3, 0.7), (0.3, 0.4, 0.8)>$\neq 0_N$ = $NFint(A_1)$.

Remark 3.20.In general topology, the following conditions are hold:

$NFFr(A)\wedge NFint(A) = 0_N$,

$NFint(A)\vee NFFr(A) = NFcl$ (A),

$NFint(A)\vee NFint(A^C)\vee NFFr(A) = 1_N$.

But the neutrosophictopology, we give counter-examples to show that the conditionneutrosophic subset of the above remark may not be hold in general.

Example 3.21. From Example 3.15. Let us takeA_1 =<(0.4,0.6, 0.1), (0.5, 0.8, 0.3), (0.9, 0.6, 0.2)>.Then$(A_1)^C$= <(0.1, 0.4, 0.4), (0.3, 0.2, 0.5), (0.2, 0.4, 0.9) >.It can be shown that, $NFFr(A_1)\wedge(NFint(A_1)$ = F $\wedge$ D = <(0.2, 0.4, 0.3), (0.4, 0.3, 0.7), (0.2, 0.4,0.9) >$\neq 0_N$.

Now, $NFint$ (A_1) $\vee NFFr(A_1)$ = D $\vee$ F = <(0.3, 0.6, 0.2), (0.7, 0.4, 0.5), (0.8, 0.5, 0.6) >$\neq 1_N$ = $NFcl(A_1)$.Further $NFint$ (A_1) $\vee NFint((A_1)^C\vee(NFFr$ (A_1) = D $\vee$ $0_N\vee$F = (0.3, 0.6, 0.2), (0.7, 0.4, 0.5), (0.8, 0.5, 0.6) $\neq 1_N$.

Proposition3.22.Let A and Bbe neutrosophic subsets in the neutrosophic topological space X. Then $NFFr$ (A$\vee$B) $\leq NFFr$ (A) $\vee NFFr$ (B).

Proof. Let A and Bbeneutrosophic subsets in the neutrosophic topological space X. Then by Definition 3.1, $NFFr$(A$\vee$B) = $NFcl$ (A$\vee$B)$\wedge NFcl$(A$\vee$B)^C=$NFcl$ (A$\vee$B)$\wedge NFcl$ $(A^C\wedge B^C)$)

By Lemma 2.12 (i) and (ii),$\leq$ ($NFcl$ (A) $\vee NFcl$ (B))$\wedge$(($NFcl(A^C)\wedge(NFcl$ (B^C))

= [($NFcl$ (A)$\vee$($NFcl$ (B))$\wedge$($NFcl$ (A^C)] $\wedge$[($NFcl$(A)$\vee$($NFcl$ (B) $\wedge$($NFcl$ (B^C)]

= [($NFcl$(A) $\wedge NFcl(A^C)$)$\vee$(($NFcl$(B) $\wedge$($NFcl(A^C)$)]$\wedge$

[($NFcl$(A) ∧($NFcl(B^C$) ∨(($NFcl$(B) ∧($NFcl(B^C$)]
By Definition 3.1,= [$NFFr(A)$∨($NFcl(B)$∧($NFcl(A^C$))]∧
[($NFcl$(A)∧($NFcl(B^C$))∨($NFFr$(B)]
= ($NFFr$(A)∨($NFFr$(B)) ∧[($NFcl$(B)∧($NFcl(A^C$) ∨(($NFcl$(A) ∧$NFcl(B^C$))]
≤ $NFFr$(A) ∨$NFFr$(B).Hence $NFFr$(A∨B)≤ $NFFr$(A) ∨$NFFr$(B).

The converse of the above theorem is need not be true as shown by the following example.

Example 3.23.Let X={a} with some neutrosophic feebly open sets are 0_N, $A, B, C, D, 1_N$ and neutrosophic feebly closed sets are 1_N, E, F, G, H, 0_N where A = <(0.6, 0.8, 0.4)>, B = <(0.4, 0.9, 0.7)>, C = <(0.6, 0.9, 0.4)>, D = <(0.4, 0.8, 0.7)>, E = <(0.4, 0.2, 0.6)>,
F = <(0.7, 0.1, 0.4)>, G = <(0.4, 0.1, 0.6) >and H = <(0.7, 0.2, 0.4)>. Now we define B_1 = <(0.7, 0.6, 0.5) >, B_2 = <(0.6, 0.8, 0.2)>,B_1∨ B_2 = B_3= <(0.7, 0.8, 0.2) > andB_1∧ B_2 = B_4 = <(0.6, 0.6, 0.5) >. Then$B_1{}^C$ = < (0.5, 0.4, 0.7) >,$B_2{}^C$ = < (0.2, 0.2, 0.6) >,$B_3{}^C$ = <(0.2, 0.2, 0.7) > and$B_4{}^C$ = <(0.5, 0.4, 0.6) >.Therefore $NFFr$(B_1) ∨$NFFr$(B_2) = 1_N∨ E = 1_N ⊈ E = $NFFr$(B_3) = $NFFr$(B_1∨ B_2).

Note 3.24.The following example shows that $NFFr$ (A∧B) ⊈$NFFr$(A)∧$NFFr$(B) and $NFFr$(A)∧$NFFr$ (B) ⊈$NFFr$ (A∧B).

Example 3.25.From Example 3.23, we defineA_1 = <(0.5, 0.1, 0.9) >, A_2 = <(0.3, 0.5, 0.6) >, A_1∨A_2 = A_3 = <(0.5, 0.5, 0.6) > and A_1∧A_2 = A_4 = <(0.3, 0.1, 0.9) >. Then$(A_1)^C$= <(0.9, 0.9, 0.5) >,$(A_2)^C$ = <(0.6, 0.5, 0.3) >, $(A_3)^C$ = <(0.6, 0.5, 0.5)> and$(A_4)^C$=<(0.9, 0.9, 0.3)>.Therefore $NFFr$ (A_1) ∧$NFFr$ (A_2) = F ∧ 1_N = F ⊈ G = $NFFr$ (A_4) = $NFFr(A_1∧A_2)$.

Theorem 3.26.For any neutrosophic subsetsA and Bin the neutrosophic topological space X, $NFFr$ (A∧B)≤($NFFr$ (A) ∧($NFcl$ (B)) ∨ ($NFFr$(B) ∧$NFcl$(A)).

Proof. Let A and Bbe neutrosophic subsets in the neutrosophic topological space X. Then by Definition 3.1, $NFFr(A∧B)$ = $NFcl$ $(A∧B)$∧($NFcl$(A∧B)^C).
By Demorgan Law,= $NFcl$(A∧B)∧($NFcl(A^C∨B^C$)
By Lemma 2.12(ii) and (i),
≤($NFcl(A)$∧($NFcl$(B))∧(($NFcl(A^C$) ∨$NFcl(B^C$))
=[$NFcl$ (A) ∧($NFcl$(B))∧($NFcl$ (A^C)]∨[($NFcl$(A) ∧($NFcl$ (B))($NFcl$ (B^C)]
By Definition 3.1,
= ((A) ∧($NFcl$(B))∨ ($NFFr$(B) ∧($NFcl$(A))
Hence $NFFr$ (A∧B) ≤(($NFFr$ (A) ∧($NFcl$ (B))∨ ($NFFr$ (B) ∧($NFcl$ (A)).
The converse of the above theorem is not true as shown by the following example.

Example 3.27. From Example 3.23, ($NFFr(A_1$) ∧($NFcl(A_2$))∨ ($NFFr$ (A_2) ∧($NFcl$ (A_1)) = (F ∧ 1_N) ∨ (1_N∧ F) = F∨ F = F⊈ G = $NFFr$ (A_1∧A_2).

Corollary 3.28.For any neutrosophic subsetsAand Bin the neutrosophic topological space X, $NFFr$(A∧B) ≤ $NFFr$(A) ∨$NFFr$(B).

Proof. Let A and Bbe neutrosophic subsets in the neutrosophic topological space X. Then by Definition 3.1, $NFFr$(A∧B) = $NFcl$(A∧B)∧($NFcl$((A∧B)^C).By Lemma1.1.10,
= $NFcl$(A∧B)∧($NFcl(A^C∨B^C$). By Proposition 2.3.11 (ii) and (i),
≤ ($NFcl$(A)∧$NFcl$(B))∧($NFcl(A^C$) ∨$NFcl(B^C$))
=($NFcl$(A)∧$NFcl$(B))∧($NFcl(A^C$)∨ ($NFcl$(A) ∧($NFcl$(B) ∧($NFcl(B^C$)). By Definition 3.1,
= ($NFFr$(A)∧($NFcl$(B) ∨(($NFcl$(A)∧($NFFr$ (B))≤ $NFFr$ (A) ∨($NFFr$ (B).
Hence $NFFr$ (A∧B) ≤ $NFFr$ (A) ∨$NFFr$ (B).
The equality in the above theorem may not hold as seen in the following example.

Example 3.29.From Example 3.23, $NFFr(A_1$) ∨$NFFr$ (A_2) = F ∨ 1_N = 1_N⊈ G =$NFFr$ (A_4) = $NFFr$ (A_1∧A_2).

Theorem 3.30. For any neutrosophic subsetA in the neutrosophic topological space X,
(1) $NFFr$ ($NFFr$ (A))≤ $NFFr$(A),
(2) $NFFr$ ($NFFr$ ($NFFr$ (A)))≤ $NFFr(NFFr$ (A)).

Proof. (1) Let A be the neutrosophic subset in the neutrosophic topological space X. Then by Definition 3.1,$NFFr$ ($NFFr$(A)) = $NFcl(NFFr$(A))∧$NFcl$ ($NFFr$ (A))^C))
By Definition 3.1,= $NFcl(NFcl$(A)∧($NFcl(A^C$))∧($NFcl$(($NFcl$(A)∧($NFcl(A^C$))^C)
By Lemma 2.10(iii) and by Lemma 2.12 (ii),
≤ ($NFcl(NFcl$(A))∧($NFcl(NFcl$ (A^C)) ∧($NFcl$($NFint(A^C$)∨$NFint(A)$)
By Lemma 2.11(iii), =($NFcl$(A)∧($NFcl(A^C$))∧(($NFcl(NFint(A)$ (A^C))∨($NFcl(NFint(A)$) ≤ $NFcl$(A)∧($NFcl(A^C$).By Definition 3.1,

$=NFFr(A)$. Therefore $NFFr(NFFr(A)) \leq NFFr(A)$.
Again, $NFFr(NFFr(NFFr(A))) \leq NFFr(NFFr(A))$.

Remark 3.31. From the above theorem, the converse of (1) is need not be true as shown by the following example and no counter-example could be found to establish the irreversibility of inequality in (2).

Example 3.32. From Example 3.15, $NFFr(A_2) = 1_N \nsubseteq 0_N = NFFr(NFFr(A_2))$.

Theorem 3.33. Let $X_i = 1, 2, \ldots, n$ be a family of neutrosophicproduct related neutrosophic topological spaces. If each A_i is a neutrosophic subset in X_i, then $NFFr(\prod_{i=1}^{n} A_i) = [NFFr(A_1) \times (NFcl(A_2)) \times \cdots \times (NFcl(A_n)] \vee [NFcl(A_1) \times (NFFr(A_2) \times (NFcl(A_3) \times \cdots \times (NFcl(A_n)] \vee \cdots ([NFcl(A_1) \times (NFcl(A_2) \times \cdots \times (NFFr(A_n)]$.

Proof. It suffices to prove this for $i = 2$. Let A_i be the neutrosophic subset in the neutrosophic topological space X_i. Then by Definition 3.1, $NFFr(A_1 \times A_2) = NFcl(A_1 \times A_2) \wedge (NFcl((A_1 \times A_2)^c)$

By Lemma 2.10 (i), $= NFcl(A_1 \times A_2) \wedge (NFint(A_1 \times A_2))^c$

By Theorem 2.14(i) and (ii),

$= (NFcl(A_1) \times (NFcl(A_2)) \wedge ((NFint(A_1) \times (NFint(A_2))^c$

$= (NFcl(A_1) \times NFcl(A_2)) \wedge [(NFint(A_1) \wedge (NFcl(A_1))) \times ((NFint(A_2) \wedge (NFcl(A_2))]^C = (NFcl(A_1) \times (NFcl(A_2)) \wedge [(NFint(A_1) \wedge (NFcl(A_1)^C \times 1_N \vee 1_N \times (NFint(A_2) \wedge (NFcl(A_2)^C] = (NFcl(A_1) \times (NFcl(A_2)) \wedge [(NFcl(A_1{}^C) \vee (NFint(A_1{}^C)) \vee 1_N \vee 1_N \vee (NFcl(A_2{}^C) \vee (NFint(A_2{}^C))]$

$= (NFcl(A_1) \times (NFcl(A_2)] \wedge [(NFcl(A_1{}^C)) \times 1_N) \vee (1_N \times NFcl(A_2{}^C))]$

$= [(NFcl(A_1) \times (NFcl(A_2)) \wedge ((NFcl((A_1)^C) \times 1_N)]$

$\vee [(NFcl(A_1) \times (NFcl(A_2)) \wedge (1_N \times NFcl((A_2)^C))]$

$= [(NFcl(A_1) \wedge (NFcl((A_1)^C)) \times (1_N \wedge (NFcl(A_2))]$

$\vee [(NFcl(A_1) \wedge 1_N) \times (NFcl(A_2) \wedge (NFcl((A_2)^C))]$

$= (NFFr(A_1) \times (NFcl(A_2)) \vee (NFcl(A_1) \times NFFr(A_2))$

Hence $NFFr(A_1 \times A_2) = (NFFr(A_1) \times (NFcl(A_2)) \vee ((NFcl(A_1) \times (NFFr(A_2))$.

V. CONCLUSION

So far, we have studied some new operator called neutrosophic feebly frontier with the help of neutrosophic feebly open sets and neutrosophic feebly closed sets in neutrosophic space. We discussed the important properties of them and the relations between them.

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