Some Properties of the F-Structure Satisfy $F^{2k-F}=0$

Sandeep Kuamr Mogha[1] and Alok Kumar[2]

[1] *Department of Mathematics, Faculty of Sciences, SGT University Gurugram – 122505, India*
[2]*Department of Mathematics, Swami Vivekanand Subharti University Meerut, 250005, India*

Abstract
The purpose of this paper is to study various properties of F - structure satisfying $F^{2k}-F=0$, where $k\geq 2$ is a positive integer. The metric F - structure, the kernel and tangent vector have also been discussed.

Keywords - *Differentiable manifold, complementary projections operator, metric kernel and tangent vector.*

I. INTRODUCTION

Let V_n be a C^{∞} differentiable manifold and $F\neq 0$ be a $C^{\infty}(1,1)$ tensor defined on V_n, such that

$$F^{2k}-F=0 \tag{1.1}$$

We define the projection operator l and m on V_n by

$$l=F^{2k-1},\ m=I-F^{2k-1} \tag{1.2}$$

where I is the identity operator [1]

From (1.1) and (1.2) we have

$$l+m=I,\ l^2=l,\ m^2=m,\ lm=ml=0,\ lF=Fl=F,\ mF=Fm=0 \tag{1.3}$$

THEOREM 1.1

Let rank $((F))=n=\dim V_n$, then $l=I,\ m=0.$ (1.4)

Proof: From the fact $\text{rank}((F))+\text{nulity}((F))=\dim V_n=n$ (1.5)

We have, $\text{nulity}((F))=0\Rightarrow Ker(F)=\{0\}$ or $X=0\Rightarrow X=0$

Let

$FX_1=FX_2$

$\Rightarrow F(X_1-X_2)=0$

$\Rightarrow X_1=X_2$

$\Rightarrow F$ is one-one.

Also, V_n being finite dimensional F is onto and thus F^{-1} exists.

Operating F^{-1} on $Fl=F$ and $mF=0$ we get the result (1.4).

THEOREM 1.2:

Let M and F satisfy $m^2=m,\ mF=Fm=0,\ (m+F^k)(m+F^{k-1})=I$ then F satisfies (1.1).

Proof: We have

$(m+F^k)(m+F^{k-1})=I$

$m^2+mF^{k-1}+F^k m+F^{2k-1}=I$

$m+0+0+F^{2k-1}=I$

$mF+F^{2k}=F$

$0+F^{2k}=F$

$\Rightarrow F^{2k}-F=0$

DEFINITION 1.1: $Ker(F)=\{X:FX=0\}, Tan\,F=\{X:FX\,\|\,X\}$ [2].

THEOREM 1.3:

For the F - structure satisfying (1.1) we have

$$Ker(F) = Ker(F^2) = ... = Ker(F^{2k}) \tag{1.7}$$

$$Tan(F) = Tan(F^2) = ... = Tan(F^{2k}) \tag{1.8}$$

Proof: Let $X \in Kar\,(F) \Rightarrow FX = 0,$

$\Rightarrow F^2 X = 0, \Rightarrow X \in Kar\,(F^2)$

Thus, $Kar\,F \subseteq Kar\,F^2$ (1.9)

Let $X \in Kar\,F^2 \Rightarrow F^2 X = 0,$

$\Rightarrow F^3 X = 0, \Rightarrow X \in Kar\,F^3$

Thus, $Kar\,F^2 \subseteq Kar\,F^3$

Let $X \in Kar\,F^{2k-1} \Rightarrow F^{2k-1} X = 0,$

$\Rightarrow F^{2k} X = 0, \Rightarrow X \in Kar\,F^{2k}$

Thus, $Kar\,F^{2k-1} \subseteq Kar\,F^{2k}$ (1.10)

Let $X \in Kar\,F^{2k} \Rightarrow F^{2k} X = 0,$

$\Rightarrow FX = 0, \Rightarrow X \in Kar\,F$

Thus, $Kar\,F^{2k} \subseteq Kar\,F$ (1.11)

In all, $Ker\,F \subseteq Ker\,F^2 \subseteq .... \subseteq Ker\,F^{2k} \subseteq Ker\,F$

Thus we get (1.7).

Following the same we get (1.8).

1. **Metric F - Structure:**

Let us define $F(X, Y) = g(FX, Y)$ (2.1)

is skew symmetric then $g(FX, Y) = -g(X, FY)$ (2.2)

$\{F, g\}$ is called metric F - structure [3,4].

THEOREM 2.1: g satisfying (2.2) and (1.1), (1.2), (1.3), we have

$$g(F^k X, F^{k-1} Y) = (-1)^k [g(X, Y) - m(X, Y)] \tag{2.3}$$

Where, $m(X, Y) = g(mX, Y) = f(X, mY)$ (2.4)

Proof: We have

$$\begin{aligned} g(F^k X, F^{k-1} Y) &= (-1)^k g(X, F^{2k-1} Y) \\ &= (-1)^k (g, lY) \\ &= (-1)^k [g, (I - m)Y] \\ &= (-1)^k [g(X, Y) - g(X, mY)] \\ &= (-1)^k [g(X, Y) - m(X, Y)] \end{aligned}$$

THEOREM 2.2: $\{F, g\}$ is not unique [5].

Proof: Let μ be a non – singular one-one tensor such that

$$\mu F' = F\mu, \ g'(X, Y) = g(\mu X, \mu Y) \tag{2.5}$$

Then $\mu F'^{2k} = F^{2k} \mu$

$= F\mu$

$= \mu F'$

Thus,

$F'^{2k} = F'$ or $F'^{2k} = F'^{2k} - F' = 0$

Also,

$g'(F' X, F'^{k-1} Y) = g(\mu F'^k X, \mu F'^{k-1} Y)$

$$= g(F^k \mu X, F\mu Y)$$
$$= (-1)^k g(\mu X, F^{2k-1}\mu Y)$$
$$= (-1)^k g(\mu X, l\mu Y)$$
$$= (-1)^k g[\mu X, (I-m)\mu Y]$$
$$= (-1)^k [g(\mu X, \mu Y) - g(\mu X - m\mu Y]$$
$$= (-1)^k [g^{'}(X, Y) - m(X, Y)]$$

THEOREM 2.3: With the notations (2.5), we have $\mu l^{'} = l\mu, \; \mu m^{'} = m\mu$. (2.6)

Proof: We have

$$\mu l^{'} = \mu F^{'2k-1}$$
$$= F^{2k-1}\mu$$
$$= l\mu$$

$$\mu m^{'} = \mu(I - F^{'2k-1})$$
$$= \mu - \mu F^{'2k-1}$$
$$= \mu - F^{2k-1}\mu$$
$$= (I - F^{2k-1})\mu$$
$$= m\mu$$

REFERENCES

[1]. Bejancu, "CR Submanifolds of a Kaeher Manifold, Proceedings of the American Mathematical Society, Vol. 69, No. 1, pp. 135-142, 1978.

[2]. Prasad, "Semi-invariant submanifolds of a Lorentzian Para-sasakian manifold, Bull Malaysian Math. Soc. (Second Series) Vol. 21, pp. 21-26, 1988.

[3]. K. Yano, "On a structure defined by a tensor field f of the type (1,1) satisfying $F^3 + f = 0$, Tensor N.S.", 14, pp. 99-109, 1963.

[4]. E. Hiroshi, "On invariant submanifolds of connect metric manifolds", Indian J. Pure Applied Math 22, 6, pp. 449-453, 1991.

[5]. F. Careres, "Linear Invairant of Riemannian product manifold", Math Proc. Cambridge Phil. Soc. 91, pp. 99-106, 1982.

[6]. Rakesh Yadav & Sandeep Mogha, "A Mathematical Model Related to a Family Growth" IJMTT, Vol. 50, No. 03, pp. 186-193, 2017