International Journal of Mathematics Trends and Technology (IJMTT) - Volume 65 Issue 4 - April 2019

Fractional-Order Model for Fingerprint Image Processing in ATM Banking

[1,2]Thomas Wetere Tulu and [1]Tiande Guo
[1]*University of Chinese Academy of Sciences, School of Mathematical Sciences, Beijing, China*
[2]*Addis Ababa University,Computational Science Program, P.o.Box 1176, Addis Ababa, Ethiopia*

Abstract

Optimization modeling and algorithms of dynamical systems using the classical integer order system of differential equations is becoming an interesting research are now a days. However due to the effective nature of fractional derivatives and integrals, several optimization models and other models in science and engineering have successful being formulated and analyzed. Fractional order derivatives has an important characteristics called memory effect and this special property do not exist in the classical derivatives. These derivatives are non local opposed to the local behavior of integer derivatives. It implies the next state of a fractional system depends not only upon its current state but also upon all of its historical states. In this article we discuss the definition of Caputo derivative and present the fractional-order EUWD model with interventions V and Q in the sense of the Caputo derivative of order $\alpha \in (0,1]$. The analysis part then follows accordingly. The main objective of this chapter is therefore to formulate an Optimization model using fractional order derivatives which has an advantage over the classical integer order models discussed in the previous literatures due to its memory effect property. We will consider qualitative stability analysis for the model and finally perform numerical simulations.

Key words: Fingerprints, ATM, Optimization problem, Caputo derivative

1 Introduction

Biometric provides automated method to identify a person based on physiological or behavioral characteristics. The unique features measured are face, fingerprints, hand geometry, handwriting, iris, retina, vein, and voice. Biometric technologies are playing vital role to provide highly secure identification and personal verification methods. As the level of security breaches and transaction fraud increases, the role of highly secure identification is becoming apparent. Every biometric method uses some aspect of an individual's physical or behavioral attributes as a means of authenticating the individual's identity. Today the most pervasive biometric in use is fingerprint, a physical biometric. Fingerprints have a unique pattern of ridges and furrows. One can find that the finger prints rarely use the full print for identification. This pattern is stored in a database either in a remote computer or in the device itself. When a person scans a print, this device compares the pattern generated by the print with one in the database to make a positive identification. There are various phases of fingerprint identification as image loading, image enhancement, normalization, thinning, minutia marking and minutia extraction. Fingerprint image quality is a vital issue to measure the performance of Fingerprint identification system[1-5]. So quality assessment of fingerprint data leads to identify the fingerprints in a better way. The main purpose of such procedure is to enhance the image by improving the clarity of ridge structure or increasing the consistence of the ridge orientation. In noisy regions, it is difficult to define a common orientation of the ridges. The process of enhancing the image before the feature extraction is also called pre-processing. According to Hong, for identification purpose generally two methods are used. First is normalization; this is a method to improve the image quality by eliminating noise and correcting the deformations of the image intensity caused by non-uniform finger pressure during the image capture. The idea of normalization consists of changing the intensity of each pixel. The normalization preserves the clarity and contrast of the ridges; however it is not able to connect broken ridges or improve the separation of the parallel ridges. The second is transformation. In particular, for detection of high or low frequencies. As the ridges have structure of repeated and parallel lines, it is possible to determine the frequency and the ridge orientation using transformation. Thinning is to eliminate the redundant pixels of ridges till the ridges are just one pixel wide. After the fingerprint ridge thinning, marking minutia points is relatively easy. In general, for each 3x3 window, if the central pixel is 1 and has exactly 3 one-value neighbors, then the central pixel is a ridge branch. If the central pixel is 1 and has only 1 one-value neighbor,then the central pixel is a ridge ending[4-10].

2 Fractional-order model

In this section we discuss the definition of Caputo derivative and present the fractional-order SIRD model with vaccination and quarantine interventions in the sense of the Caputo derivative of order $\alpha \in(0,1]$ in the next section.

Definition 1.1 The fractional integral of order $\alpha > 0$ of a function g:$R^{+} \rightarrow$ R is defined by: $I^{\alpha}g(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-x)^{\alpha-1}g(x)\,dx$ where $\Gamma(.)$ is a gamma function.
Definition 1.2 The Caputo fractional derivative of order $\alpha > 0$, $n-1<\alpha< n$, n$\in\mathbb{N}$ is defined as: $D^{\alpha}g(t) = \frac{1}{\Gamma(n-\alpha)}\int_0^t \frac{g(x)^{(n)}}{(t-x)^{\alpha+n-1}}g(x)\,dx$, where the function *g(t)* have absolutely continuous derivatives up to order *(n-1)*.

3 Mathematical model formulation and description

In this article we study the EUWD model using Caputo fractional derivatives. Hence, in our next new mathematical model , the population is divided into four sub populations: the Expected ATM Users(S), non ATM users(U), those withdraw(R), and Removed(D)because of different factors with intervention. In this case, it is assumed that the removed and the newly ATM users occur with the same constant rate μ and newly ATM users are exposed to our new security using fingerprint with the rate $c \in [0,1]$ when opening an account for the first time. The classical EUWD model with the intervention V and Q is given below:

$$\frac{dE}{dt} = (1-c)\mu N + \gamma_1 W - \frac{\beta_1(1-\beta)E(U)}{N} - \gamma E - \mu E \quad (3.1)$$
$$\frac{dU}{dt} = \frac{\beta_1(1-\beta)E(U)}{N} - \alpha_1\beta U - \alpha_2(1-\beta)U - \delta_1 U - \delta_2 I - \mu U \quad (3.2)$$
$$\frac{dW}{dt} = \alpha_1\beta U + \alpha_2(1-\beta)U + \gamma E - \gamma_1 W - \mu W \quad (3.3)$$
$$\frac{dD}{dt} = \delta_1 U + \delta_2 U + \mu S + \mu U + \mu W \quad (3.4)$$

Now, by replacing integer-order derivatives of the above sytem with fractional derivatives of order $\alpha \in [0,1]$ in the sense of Caputo fractional derivatives. We consider the fractional-order SIRD model with vaccination and quarantine as follow:

$$D_t^{\alpha}E(t) = (1-c)\mu N + \gamma_1 W - \frac{\beta_1(1-\beta)E(U)}{N} - \gamma E - \mu E \quad (3.5)$$
$$D_t^{\alpha}U(t) = \frac{\beta_1(1-\beta)E(U)}{N} - \alpha_1\beta U - \alpha_2(1-\beta)U - \delta_1 U - \delta_2 U - \mu U \quad (3.6)$$
$$D_t^{\alpha}W(t) = \alpha_1\beta U + \alpha_2(1-\beta)U + \gamma E - \gamma_1 W - \mu W \quad (3.7)$$
$$D_t^{\alpha}D(t) = \delta_1 U + \delta_2 U + \mu E + \mu U + \mu W \quad (3.8)$$

As the population is closed, $D(t) = N(t) - E(t) - U(t) - W(t)$. Hence, we can consider the first three equations of our system above.

$$D_t^\alpha E(t) = (1-c)\mu N + \gamma_1 W - \frac{\beta_1(1-\beta)E(U)}{N} - \gamma E - \mu E \tag{3.9}$$
$$D_t^\alpha U(t) = \frac{\beta_1(1-\beta)E(U)}{N} - \alpha_1\beta U - \alpha_2(1-\beta)U - \delta_1 U - \delta_2 U - \mu U \tag{3.10}$$
$$D_t^\alpha W(t) = \alpha_1\beta U + \alpha_2(1-\beta)U + \gamma E - \gamma_1 W - \mu W \tag{3.11}$$

4 Basic properties

Since the model monitors changes in the human population ATM usage, all the variables and parameters are assumed to be positive for all t≥0.
The model is therefore be analyzed in a suitable feasible region: $D = \{E(t), U(t), W(t), D(t) \in R_+^4\}$ which is positively invariant for the system (3.9) to (3.8) above.

Proof. Assume that the initial conditions E(0)≥ 0, $U(0)$≥0, $W(0)$≥0 and D(0)≥0.
The second equation of our model can be written as:

$$D_t^\alpha U(t) = \frac{\beta^\alpha(1-q^\alpha)EU}{N} - \alpha_1^\alpha q^\alpha U - \alpha_2^\alpha(1-q^\alpha)U - \delta_1^\alpha I - \delta_2^\alpha I \tag{4.1}$$
$$D_t^\alpha U(t) = [\frac{\beta^\alpha(1-q^\alpha)E}{N} - \alpha_1^\alpha q^\alpha - \alpha_2^\alpha(1-q^\alpha) - \delta_1^\alpha - \delta_2^\alpha]U \tag{4.2}$$

Rearranging we get:

$$D_t^\alpha U(t) - [\frac{\beta^\alpha(1-q^\alpha)E}{N} - \alpha_1^\alpha q^\alpha - \alpha_2^\alpha(1-q^\alpha) - \delta_1^\alpha - \delta_2^\alpha]U = 0 \tag{4.3}$$

This is a linear first-order equation in I and its solution is:
$U(t) = U(0)exp(\int_0^t -A(z)\,dz)$ where $A(z) = \frac{\beta^\alpha(1-q^\alpha)S(z)}{N} - \alpha_1^\alpha q^\alpha - \alpha_2^\alpha(1-q^\alpha) - \delta_1^\alpha - \delta_2^\alpha$ which implies U(t) ≥ 0 for all $t \geq 0$.

To show the non -negativity of the remaining variables, consider the subsystem:

$$D_t^\alpha E(t) = \gamma^\alpha W - \frac{\beta^\alpha(1-q^\alpha)EU}{N} - v^\alpha E \tag{4.4}$$
$$D_t^\alpha W(t) = \alpha_1^\alpha q^\alpha U + \alpha_2^\alpha(1-q^\alpha)U + v^\alpha E - \gamma^\alpha W \tag{4.5}$$
$$D_t^\alpha D(t) = \delta_1^\alpha U + \delta_2^\alpha U \tag{4.6}$$

which can be rewritten in a matrix form as: $\frac{dY(t)}{d(t)} = MY(t) + H(t)$
where

$$Y(t) = \begin{pmatrix} E(t) \\ W(t) \\ D(t) \end{pmatrix} \tag{4.7}$$

$$M = \begin{pmatrix} \frac{\beta(1-q)U}{N} - v & \gamma & 0 \\ v & -\gamma & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{4.8}$$

and

$$H(t) = \begin{pmatrix} 0 \\ \alpha_1 qU + \alpha_2(1-q)U \\ \delta_1 U + \delta_2 U \end{pmatrix} \tag{4.9}$$

Clearly, M is a Metzler matrix in a view of already established non negativity of the parameter U. Thus, the equation $\frac{dY(t)}{dt} = MY(t) + B(t)$ is a monotone system. Hence, $D = \{E(t), U(t), W(t), D(t) \in R_+^4\}$ is positively invariant for the system (3.9) to (3.8). □

5 Existence and stability of non ATM users free equilibrium point

At the non ATM users free equilibrium state all our population use ATM . Thus, all the non ATM users classes will be zero and the entire population will comprise of ATM users. A non ATM users free equilibrium state of the model above is given by E_0=(E^*, U^*, W^*).
Equating the model to zero and solving we get:
E_0=(E^*, U^*, W^*)=($\frac{(\gamma_1+\mu)(1-c)bN}{(\gamma+\mu)(\gamma_1+\mu)-\gamma_1\gamma}$, 0, $\frac{(1-c)\gamma bN}{(\gamma+\mu)(\gamma_1+\mu)-\gamma_1\gamma}$).

Theorem 5.1. *If:*

$$R_0 \quad = \quad \frac{\beta_1(1-\beta)E_0^*}{k_1N} < 1, \tag{5.1}$$

then the non ATM users free equilibrium of our system is locally asymptotically stable.
where $k_1 = \alpha_1\beta + \alpha_2(1-\beta) + \gamma_1 + \gamma_2 + \mu$
and $E_0^* = \frac{(\gamma_1+\mu)(1-c)bN}{(\gamma+\mu)(\gamma_1+\mu)-\gamma_1\gamma}$

Proof. According to Theorem 4.1, to prove the stability of the non ATM users free equilibrium , it is suffice to show that all eigenvalues of Jacobian matrix of our system evaluated at the non ATM users free equilibrium have negative real parts. This Jacobian matrix is derived as follow: As our population is closed, let $X = (U, W)^T$ then $\frac{dX}{dt} = f(x) - v(x)$ where:

$$f(x) = \begin{pmatrix} \frac{\beta_1(1-\beta)EU}{N} \\ 0 \end{pmatrix} \tag{5.2}$$

and

$$v(x) = \begin{pmatrix} \alpha_1\beta U + \alpha_2(1-\beta)U + \delta_1 U + \delta_2 U + \mu U \\ \gamma_1 W + \mu W - \alpha_1\beta U - \alpha_2(1-\beta)U - \gamma E \end{pmatrix} \tag{5.3}$$

The Jacobian matrices of *f(x)* and *v(x)* evaluated at the non ATM users free equilibrium, E_0 are:

$$Df(E_0) = F = \begin{pmatrix} \frac{\beta_1(1-\beta)E_0^*}{N} & 0 \\ 0 & 0 \end{pmatrix} \tag{5.4}$$

and

$$Dv(E_0) = V = \begin{pmatrix} \alpha_1\beta + \alpha_2(1-\beta) + \delta_1 + \delta_2 + \mu & 0 \\ -\alpha_1\beta - \alpha_2(1-\beta) & \gamma_1 + \mu \end{pmatrix} \tag{5.5}$$

where $E_0^* = \frac{(\gamma_1+\mu)(1-c)bN}{(\gamma+\mu)(\gamma_1+\mu)-\gamma_1\gamma}$.
The Jacobian matrix is therefore given by :

$$F - V = \begin{pmatrix} \frac{\beta_1(1-\beta)E_0^*}{N} - k_1 & 0 \\ k_2 & -\gamma_1 - \mu \end{pmatrix} \tag{5.6}$$

The eigenvalues of this matrix are as follows:
$\lambda_1 = -(\gamma_1 + \mu)$
$\lambda_2 = (k_1 - \frac{\beta_1(1-\beta)E_0^*}{N}) < 0$ for $W_0 < 1$.
where:
$k_1 = \alpha_1\beta + \alpha_2(1-\beta) + \delta_1 + \delta_2 + \mu$
$k_2 = -(\alpha_1\beta - \alpha_2(1-\beta))$

□

6 Existence and stability of non ATM users(endemic) equilibrium point

Existence and stability of non ATM users(endemic) equilibrium point This equilibrium points are characterized by the existence of non ATM users in the population and is given by:

E_1=(E^*, U^*,W^*)
$=(\frac{k_1N}{\beta_1(1-\beta)},(\frac{\gamma_1N(\gamma_1+\mu)}{k_3\gamma_1N-\beta_1(\gamma-1+\mu)(1-\beta)E^*})(\frac{-\gamma}{\gamma+\mu}-\frac{\gamma+\mu}{\gamma_1})E^*-\frac{(1-c)bN}{\gamma_1}),(\frac{\gamma\beta_1k_1+k_3(1-c)bN-k_3(\gamma+\mu)}{(\alpha_1+\mu)(\beta_1+k_1)-\gamma_1K_3})E^*) where$
$E^*=\frac{k_1N}{\beta_1(1-\beta)}$
$k_1=\alpha_1\beta+\alpha_2(1-\beta)+\gamma_1+\gamma_2+\mu$
$k_3=\alpha_1\beta+\alpha_2(1-\beta)$
By a similar technique it is not difficult to show the existence of the non ATM users(endemic) equilibrium and by evaluating the jacobian matrices at the equilibrium it can be easily shown (similar to the method for local stability of non ATM users free equilibrium) that the characteristics roots of the matrix have negative real parts. Therefore, we say the equilibrium is locally asymptotically stable.

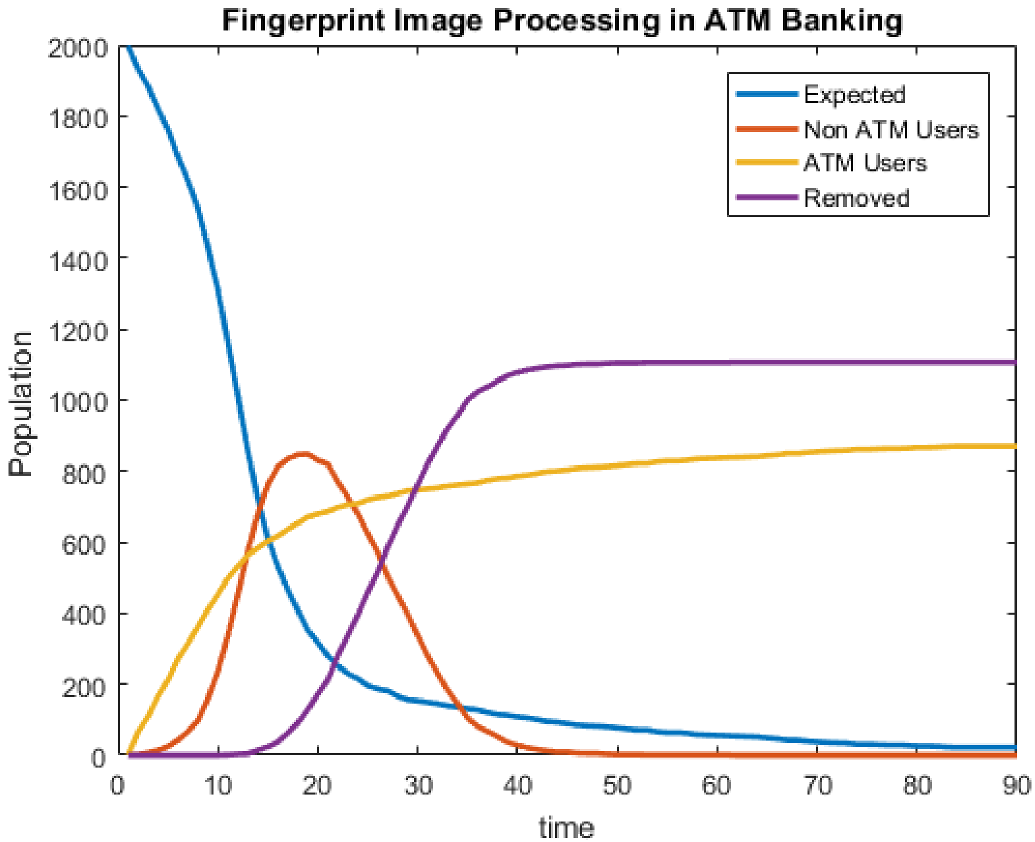

Figure 1.1 Numerical solution of the model.

7 Conclusion

This article considers the extension and analysis of the classical EUWD model to study the dynamics of ATM frauds in banking industry. Due to the extensive applications of fractional ordinary differential equations in in real world problems, research in this area has grown significantly all around the world. In this study the dynamics of the ATM frauds in the banking industry is discussed and analyzed intensively. For our fractional-order model the basic reproduction number, R_0 is determined and proved that if R_0 <1, the non ATM users free equilibrium is locally asymptotically stable. Finally simulation is done. From our result it is seen that the number of cases in ATM fraud is booming from time to time and hence a great loss in banking industry. In overall the we claim that as the ATM frauds are becoming a challenging problem all over the world, studying the dynamics of it is really crucial and we recommend the application of different interventions strategy to overcome this problem as per our study.

Authors' contributions

The authors declare that the study was conducted in collaboration with the same responsibility. All authors read and approved the final manuscript.

Acknowledgments

This research was supported by CAS-TWAS Presidents PhD Fellowship Programme. Besides, the authors declare no conflicts of interest. Finally, the authors thank the journal editors and anonymous reviewers for their comments and recommendations which greatly improve our manuscript and made it more suitable for for readers of the journal.

References

[1] RouquetP,etal.*Wild animal mortality monitoring and human Ebola outbreaks, Gabon and Republic of Congo*,2001-2003,*Emerging infectious disease 2005*;11:283-290

[2] Selvaraju, N. and Sekar, G., (2010), *A Method to Improve the Security Level of ATM Banking Systems Using AES Algorithm*, *International Journal of Computer Applications* (0975 8887) Volume 3 No.6.

[3] Feng, J.,(2008),*Combining minutiae descriptors for fingerprint matching, Pattern Recognition* 41, 342 352.

[4] Jain, A.K., Prabhakar, S., Hong, L., (1999), *A multichannel approach to fingerprint classification*, *IEEE Trans. Pattern Anal. Mach. Intell.* 21 (4),348359.

[5] FVC2002, *Second international fingerprint verification competition*

[6] Maltoni, D., Maio, D., Jain, A. K., and Prabhakar, S.,(2009), *Handbook of Fingerprint Recognition (2nd Edition). Springer.*

[7] Jiang, X. and Yau, W. Y.,(2000),*Fingerprint Minutiae Matching Based on the Local and Global Structures*, *In Proc. of ICPR*, volume 2, pages 60386041, Barcelona, Spain.

[8] Delac, K. and Grgic, M.,(2004), *A Survey of Biometric Recognition Methods, 46th International Symposium Electronics in Marine, ELMAR.*

[9] Jain, A.K.; Bolle, R.; Pankanti, S., eds. (1999). *Biometrics: Personal Identification in Networked Society, Kluwer Academic Publications*, ISBN 978-0-7923-8345-1.

[10] Bansal, R., Sehgal, P. and Bedi, P.,(2011), *Minutiae extraction from fingerprint images- a review*, IJCSI, Vol. 8, Issue 5.