Natural Convection Through Vertical Porous Plate

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Abstract

Natural convection through a vertical porous plate is studied analytically and numerically it is found that larger values of buoyancy parameter N with increasing values of temperature difference between the plates and permeability parameter. It is also noticed that temperature distributions, velocity, mass flow rate and rate of heat transfer have significantly changed at higher temperature differences, to achieve the mass flow rate in a porous medium. We also observed that the velocity and temperature distributions increased with increase in 'N' near the hotter plate than at the cooler plate and we also studied that the ratio of friction factor increases with 'N' and also observed that variation of skin friction increases with respect to σ and N.

Keywords: *Vertical porous plate, Natural convection, Buoyancy parameter, Reynolds Number.*

I. INTRODUCTION

The study of motion of ground water is usually based on the Darcy law [2] in which the macroscopic length scale of a system is so large that the diffusion effects are neglected. However, in zones of mixing between fluids at different temperatures the diffusion effects are significant since the gradients of fluid properties are large. In these zones, the nature of fluid motion is such that boundary layer approximations are valid and the usual potential nature of Darcy equations is not valid. To a first approximation, however, we can use the Brinkman model (Rudraiah and Naagaraj [5]). Flows obeying Brinkman model might be found in geothermal areas or might arise from the heat generated by deep explosions in saturated ground. Rudraiah and Gebhart [3] as studied viscous case extensively but not made attention to the porous medium.F.E.Lock Wood, Z.G.Zhang [4] have studied thermal characteristics of a new and used Diesel engine oil.M.LeBars and M.H.Worsters [6] have studied interfacial conditions between pure fluids and porous medium implications.

The present paper studied the Natural convection through vertical porous plate using Runge-Kutta-Grill and R programming for wide range of values of N. The effect of permeability and buoyancy parameter N on the skin friction and rate of heat transfer are studied and shown graphically.

II. MATHEMATICAL MODELLING

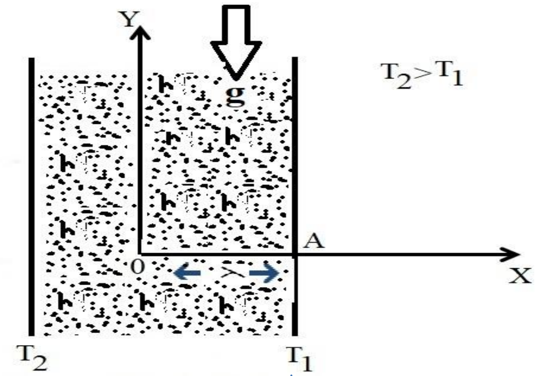


Figure 1.Physical Model

Nomenclature

u =Velocity of flow in y-direction cm/sec

T =Temperature in ℃

T_0 = Ambient temperature ℃

ρ =Density of the fluid gm/cm^3

ν =Kinetic viscositym^2/sec

K =Thermal conductivity $watt/cm\ kelvin$

ϵ =Permability of porous medium

β =Coefficient of thermal conductivity$watt/mt^2\ kelvin$

σ =Permeability parameter $Darcy\ unit$ $(9.8683\ X\ 10^{13})$

N =Buoyancy parameter $Dyne$

The physical model, as shown in figure 1, comprises of vertical porous plate bounded by two rigid walls at $x = \pm \lambda$ with y-axis in the axial direction and x-axis perpendicular to the walls. We assume that steady Boussinesq fluid percolating through a homogeneous isotropic porous medium in the y-direction and physical quantities vary with respect to x. The Buoyancy forces, due to density difference, cause the fluid to flow upwards through the channel. For this flow, the basic equations of motion, following Rudraish and Nagarj[5], are

$$\frac{d^2u}{dx^2} - \frac{u}{\epsilon} + \frac{g\beta(T-T_0)}{\nu} = 0 \quad -------------- (1)$$

$$\frac{d^2T}{dx^2} + \frac{\rho_0 \nu}{K}\left(\frac{du}{dx}\right)^2 + \frac{\rho_0}{\epsilon}\frac{\nu}{K}u^2 = 0 \quad ------------- (2)$$

$$\rho = \rho_0[1 - \beta(T - T_0)] \quad ----------------- (3)$$

Where u is the velocity component in the y-direction, T the temperature, T_0 the ambient temperature at $\rho = \rho_0$ ρ the density of the fluid, ν the kinematic viscosity, K the thermal conductivity, ϵ the permeability of a porous medium and β the coefficient of thermal expansion. These equations are solved using the boundary conditions.

$$u = 0 \;\; at \;\; x \; = \; \pm \curlywedge -------------(4)$$

$$T - T_1 \; at \; x = + \curlywedge -------------- (5)$$

$$T - T_2 \; at \; x = - \curlywedge -------------- (6)$$

The boundary condition on velocity represents the noslip condition and that on temperature points to the fact that the plates are isothermally maintained at different temperatures T_1 and T_2 $(T_2 > T_1)$. Equations (1) and (2) using the dimension less quantities

$$x^* = \frac{x}{\curlywedge}, \quad \theta = \frac{T - T_0}{T_1 - T_0}, \qquad u^* = \frac{\nu}{g\beta \curlywedge^2 (T_1 - T_0)} u --------- (7)$$

And for simplicity neglecting the asterisk $(*)$, become

$$\frac{d^2u}{dx^2} - \sigma^2 u + \theta = 0 ---------------(8)$$

$$\frac{d^2\theta}{dx^2} + N\left(\frac{du}{dx}\right)^2 + N\sigma^2 u^2 = 0 ----------- (9)$$

Where $\sigma = \curlywedge/\sqrt{\epsilon}$ is the permeability parameter

$N = \frac{\rho_0 g^2 \curlywedge^4 \beta^2 (T_1 - T_0)}{\nu K}$ is the buoyancy parameter.

The above equations corresponding boundary conditions are

$$u = 0 \;\; at \;\; x \; = \; \pm 1 ------------- (10)$$

$$\theta = 1 \; at \; x = 1 --------------- (11)$$

$$\theta = 1 + \bar{\theta} \; at \; x = -1 ------------ (12)$$

Where $\bar{\theta} = \frac{T_2 - T_1}{T_1 - T_0}$ --------------(13)

Since equations (8) and (9) are coupled non-linear equations because of the dissipation term which must be solved simultaneously to get the desired velocity and temperature profiles. Due to the non-linearity, analytical solutions of these equations are difficult for larger values of N. However, we find the solutions for smaller values for N and also effects of larger values of N by using Runge-Kutta Grill Method and R-Programming are shown in graphically.

III. NUMERICAL SOLUTIONS

The numerical method involves solving a non-linear two point boundary value problem using an iterative scheme. The velocity and temperature distributions are obtained for a wide range of values of N and are shown in figures. We find that the analytical solutions are matching with the numerical solutions when N is very small. For the sake of comparison we have given the values of velocity and temperature at $x = 0$for $\bar{\theta} = 2.0$ and $\sigma = 3.0$shown in tables.

Let M denote the mass flow rate per unit channel with in the presence of dissipations then

$$M_v = \int_{-\lambda}^{+\lambda} \rho_0 \, udy$$

$$= -\rho_0 G \lambda^3 \int_{-1}^{+1} udy ----------------(14)$$

In the presence of dissipation

Let M_l denote the mass flow rate per unit channel with in the presence of dissipation for viscous flow, then

$$M_l = -\rho_0 G \lambda^3 \int_{-1}^{+1} u_l dy ----------------(15)$$

In the presence of dissipation of viscous flow

The ratio of (14) to (15) is

$$\frac{M_v}{M_l} = \frac{\int_{-\lambda}^{+\lambda} udy}{\int_{-1}^{+1} u_l dy} = \frac{3}{\sigma^2}\left[1 - \frac{\tan h\sigma}{\sigma}\right] ------------------ (16)$$

The friction factor c_f is defined as

$$c_f = -\frac{\nu GD}{\frac{1}{2}\bar{u}^2} --------------------(17)$$

Where $G = \frac{g\beta(T - T_0)}{\nu}$

$$D = 4\lambda$$

Where $\bar{u} = \frac{1}{2\lambda}\int_{-\lambda}^{+\lambda} udy = -\frac{G\lambda^2}{2}\int_{-1}^{+1} udy -------------- (18)$

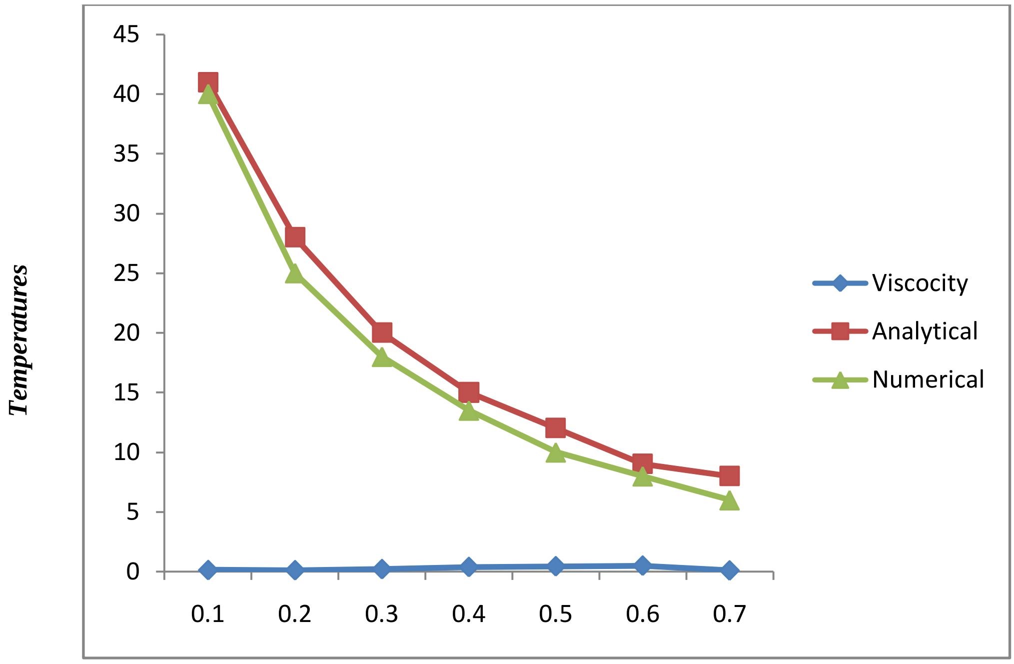
We get the result from above equations as

$$c_l = \frac{64}{Re \int_{-1}^{+1} u \, dy} --------------------------(19)$$

Velocity at $x = 0$, $\bar{\theta} = 2.0$, $\sigma = 3.0$

Viscosity	*Analytical*	*Numerical*
0.13	41	40
0.12	28	25
0.21	20	18
0.387	15	13.5
0.45	12	10
0.5	9	8
0.1	8	6

Table 1



Viscosity

Figure 2

Temperature at $x = 0$, $\overline{\theta} = 2.0$, $\sigma = 3.0$

Buoyancy Forces	Analytical	Numerical
0.0046	5	3
0.065	10	8
0.008	15	13
0.01	20	18
0.12	25	23.5
0.1	30	28
0.2	35	33.5
0.3	40	39
0.4	45	44.5

Table 2

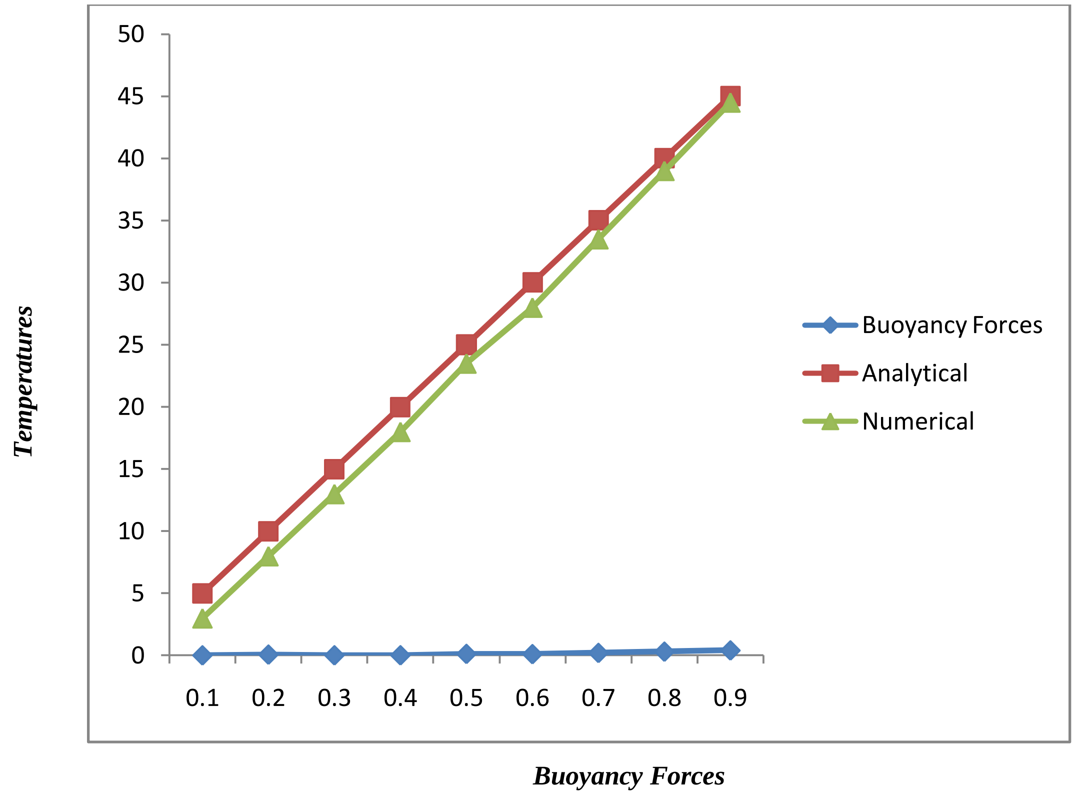


Figure 3

Parameter of Porous Media (σ)	*Ratio of Mass flow rate with and without porous medium* $\left(\frac{M_v}{M_l}\right)$
8	0.8012
10	0.4012
15	0.2503
20	0.2023
25	0.1902
30	0.1899
35	0.1798
40	0.17543

Table 3

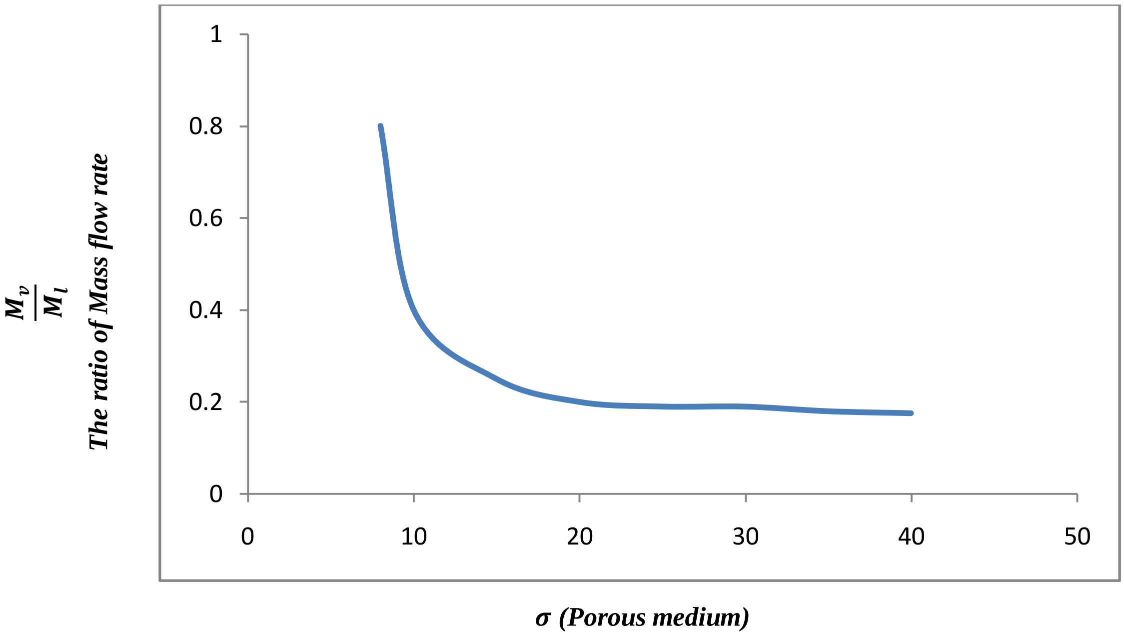
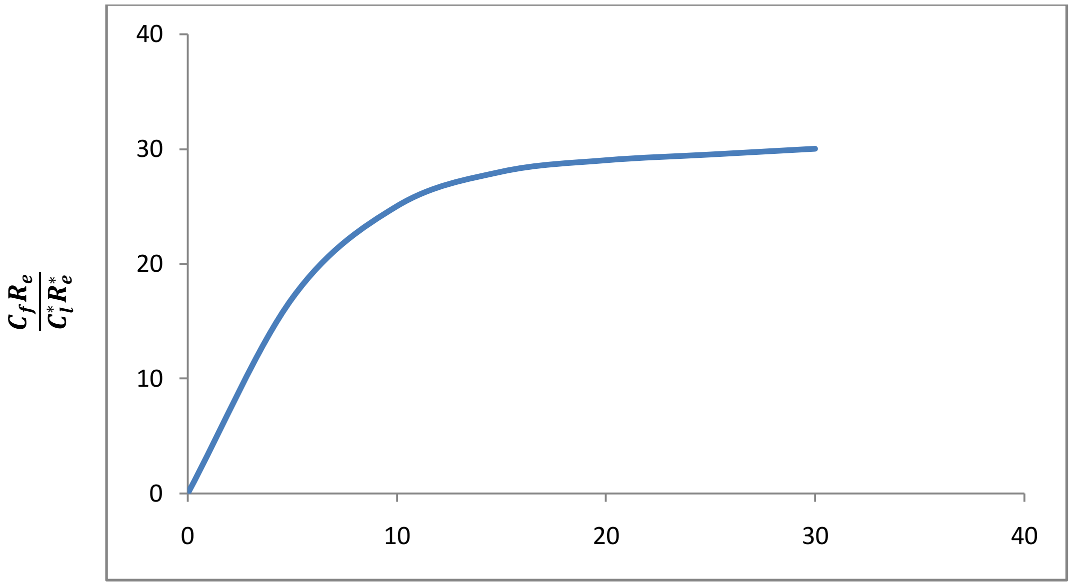


Figure 4

Friction Factor and Reynold Number	*Porous Medium*
0	0
5	17
10	25
15	28
20	29
25	29.5
30	30

Table 4



σ (Porous Medium)

Ratio of the product of friction factor and Reynolds number with and without porous medium

Figure 5

Where $R_e = \frac{uD}{\nu}$ is the Reynolds Number

Thus $C_f R_e = \frac{64}{\nu} \int_{-1}^{+1} u dy$ ------------------------(20)

The ratio of (19) to (20) is

$$\frac{C_f R_e}{C_l^* R_e^*} = \frac{\int_{-1}^{+1} u_f dy}{\int_{-1}^{+1} u\, dy} = \frac{\sigma^2}{3\left[1 - \frac{\tan h\sigma}{\sigma}\right]} \quad \text{--------------------(21)}$$

IV. DISCUSSION

1. The effect of increase in the temperature difference between the walls of the vertical plate with respect to velocity and temperature distributions have been studied and shown in the figure 2which are matching with the analytical solutions.
2. We observed that temperature increases proportionally to the Buoyancy force increases whereas viscosity decreases as shown in figure 3.
3. We observed that porous medium as σ increases the mass flow rate decreases as shown in the figure 4 and also have shown that friction factor increasesproportionally with the increase of porous medium as shown in figure 5.

V. REFERENCES

[1]. Brinkman H C 1947 Appl.Sci.Res.A1 27.

[2]. Darcy H 1956 Les Fontainespubliques de la ville de Dijon.

[3]. Gebhart B 1979 ASME J.Fluid Eng.101 5.

[4]. F.E.Lockwood, Z.G.Zhang, Thermal characterstics of new and used diesel engine oils, The Valvoline company, Lexington, Ky 40512, USA

[5]. Rudraiah N and Nagaraj S T 1977 Int.J.Eng.Sci.15.589.

[6]. M.LeBars and M.H.Worsters, Journal of Fluid Mechanics, Vol.550 pp 149-173, 2006

[7]. M.L.Mathur and F.S.Mehta, Thermal Engineering, Jain Brothers publications, Karl Bagh, New Delhi-110005