International Journal of Mathematics Trends and Technology (IJMTT) - Volume 65 Issue 4 - April 2019

# A New Approach In Determining Solution Of The Differential Equations And The First Order Partial Differential Equations

M. Murali Krishna Rao
Department of Mathematics,GIT,GITAM University
Visakhapatnam- 530 045,A.P.
India.
Email ID: mmarapureddy@gmail.com

**Abstract**

The main objective of this paper is to study Murali Krishna's method for solving homogeneous ( non-homogeneous) first order differential equations and formation of differential equations in short methods and solving the second order linear differential equations with constant coefficients of the form $f(D)y = X$, where $X$ is a function of $x$ in a short method without using differentiation.

## 1 Preliminaries

**Definition 1.1.** An equation which involves differential coefficients is called a differential equation.

The differential equation can be formed by differentiating and eliminating the arbitrary constants from a relation in the variables and constants. Differential equation represents a family of curves.
The study of a differential equation consists of three stages.
1) Formation of differential equation from the given physical situation, called modeling.
2) Solution of the differential equation.
3) Physical interpretation of the solution of differential equation.

Non-homogeneous equation of the first order is of the form
$\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}$

Case(i).When $\frac{a}{a'} \neq \frac{b}{b'}$

Put x=X+h, y=Y+k

Cae(ii) When $\frac{a}{a'} = \frac{b}{b'}$

Then put ax+by=t

Cae(iii) When b=-a' then the given equation is exact.
In this paper, we study the following methods. Murali Krishna's method[1,2,3] for Non-Homogeneous First Order Differential Equations and formation of the differential equation by eliminating parameter in short methods. We consider $z$ as dependent variables $x$ and $y$ are independent variables. The first order partial derivatives of $z$ with respect to $x$ and $y$ are $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$ which are denoted by $p, q$ respectively. The equation $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2y = X$ is called linear second order linear differential equation.

**Definition 1.2.** A partial differential equation in which $p$ and $q$ occur other than in the first degree is called the non-linear partial differential equation of first order. Otherwise it is called linear partial differential equation.

Standard types of the first ordered partial differential equations.

(i) $f(p, q) = 0$

(ii) $f(z, p, q) = 0$

(iii) $f(x, p) = g(y, q)$

(iv) $z = px + qy + f(p, q)$.

# 2 Formation of the differential equations

In this section, we form the differential equation by eliminating arbitraray constants in easier method.

1) Form the differential equation by eliminating A and B from $Ax^2 + By^2 = 1$
Solution: Given differential equation is $Ax^2 + By^2 = 1$, then

$$\frac{A}{B}x^2 + y^2 = \frac{1}{B}$$

Differentiating with respect to x we get

$$\frac{A}{B}2x + 2y\frac{dy}{dx} = 0$$
$$\Rightarrow \frac{A}{B} = -\frac{y}{x}\frac{dy}{dx}$$

Again differentiating with respect to x we get

$$-y\left(\frac{-1}{x^2}\right)\frac{dy}{dx} + \frac{1}{x}\left[-y\frac{d^2y}{dx^2} - (\frac{dy}{dx})^2\right] = 0$$
$$\Rightarrow xy_1^2 + xyy_2 - yy_1 = 0$$

2) Form the differential equation by eliminating A and B from $y = Ae^{-3x} + Be^{2x}$.
Solution:

$$y = Ae^{-3x} + Be^{2x}$$
$$ye^{3x} = A + Be^{5x}$$

Differentiating with respect to x we get

$$3ye^{3x} + e^{3x}\frac{dy}{dx} = 5e^{5x}B$$
$$\Rightarrow e^{-2x}\left[3y + \frac{dy}{dx}\right] = 5B$$

Again differentiating both sides with respect to x we get

$$e^{-2x}\left(3\frac{dy}{dx} + \frac{d^2y}{dx^2}\right) + \left(3y + \frac{dy}{dx}\right)\left(-2e^{-2x}\right) = 0$$
$$\Rightarrow \frac{d^2y}{dx^2} + \frac{dy}{dx} - 6y = 0$$

3) Show the system of confocal conics

$$\frac{x^2}{a^2+\lambda} + \frac{y^2}{b^2+\lambda} = 1,$$

where $\lambda$ is a parameter, is self orthogonal.
Solution: Given equation can be written in the form

$$x^2 + \frac{a^2+\lambda}{b^2+\lambda}y^2 = a^2 + \lambda \qquad ---(1)$$

differentiating both sides with respect to x we get

$$2x + \frac{a^2+\lambda}{b^2+\lambda} 2y \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{b^2+\lambda}{a^2+\lambda} = -\frac{yy_1}{x} \qquad ---(2)$$

$$\Rightarrow \frac{b^2+\lambda}{a^2+\lambda} - 1 = -\frac{yy_1}{x} - 1$$

$$\Rightarrow \frac{b^2-a^2}{a^2+\lambda} = -\left(\frac{x+yy_1}{x}\right)$$

$$\Rightarrow a^2+\lambda = \frac{(a^2-b^2)\,x}{x+yy_1} \qquad ---(3)$$

from (1),(2)and (3) we get

$$(x+yy_1)\left(x - \frac{y}{y_1}\right) = a^2 - b^2 \qquad ---(4)$$

Replacing $y_1$ by $-y_1$ we get

$$\left(x - \frac{y}{y_1}\right)(x+yy_1) = a^2 - b^2 \qquad ---(5)$$

The equations (4) and (5) are same. Therefore the given family of curves is orthogonal to itself.
Hence the given system of curves is self orthogonal.

5) Find the orthogonal trajectories of the family of circles

$$x^2 + y^2 + 2gx + c = 0$$

where g is a parameter and c is constant.
Solution: Given equation can be written in the form

$$x + \frac{y^2}{x} + 2g + \frac{c}{x} = 0$$

differentiating both sides with respect to x we get

$$1 + \frac{2y}{x}\frac{dy}{dx} - \frac{y^2}{x^2} - \frac{c}{x^2} = 0$$

Replacing $\frac{dy}{dx}$ by $-\frac{dx}{dy}$, we get

$$\Rightarrow -2xydx + \left(x^2 - y^2 - c\right) dy = 0$$
$$\Rightarrow -\left(y^2 + c\right) dy + \left(x^2dy - 2xydx\right) = 0$$
$$\Rightarrow -\left(1 + \frac{c}{y^2}\right) dy - d\left(\frac{x^2}{y}\right) = 0$$
$$\Rightarrow -y + \frac{c}{y} - \frac{x^2}{y} = x.$$

Therefore

$$x^2 + y^2 - xy + c = 0.$$

Hence orthogonal system is

$$x^2 + y^2 - xy + c = 0.$$

## 3 Non-homogeneous first order and first degree diferential equation

In this section, we solve non-homogeneous first order and first degree diferential equation in short method.

**Definition 3.1.** A differential equation of the form $\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}$is callled a non- homogenous linear differential equation.

Procedure for solving non-homogeneous first order liner differential equation

$\frac{dy}{dx} = \frac{ax+by+c}{a'x+b'y+c'}$

$\Rightarrow \frac{dy}{dx} + k = \frac{ax+by+c}{a'x+b'y+c'} + k$ ....(1)

Let $\frac{a+a'k}{b+b'k} = k$

.

If k=m,n where m,n are real numbers then substitute the value in (1)

Dividing and integrating,we get the solution.

If k values are imaginary then the solution is of the form A $\log(X^2+Y^2)$+B$tan^{-1}Y/X = c$,

where X and Y are functions of x and y respectively.

Problem 1. Solve $\frac{dy}{dx} = \frac{2x+9y-10}{6x+2y-10}$

Solution: $\frac{2+6k}{9+2k} = k \Rightarrow k = -2, 1/2$

Then $\frac{dy}{dx} - 2 = \frac{2x+9y-10}{6x+2y-10} - 2$

And $\frac{dy}{dx} + 1/2 = \frac{2x+9y-10}{6x+2y-10} + 1/2$

$\frac{dy-2dx}{5(y-2x} = \frac{2dy+dx}{10(x+2y-5)}$

Integrating

$(y-2x)^2$(x+2y-5)=c

Probelm.2. $\frac{dy}{dx} = \frac{-x-y}{3x+3y-4}$

Solution: $\frac{-1+3k}{1+3k} = k \Rightarrow k = 1, 1/3$

$\frac{dy}{dx} + 1 = \frac{-x-y}{3x+3y-4} + 1$

$\frac{dy}{dx} + 1/3 = \frac{-x-y}{3x+3y-4} + 1/3$

Then $\frac{dy+dx}{2x+2y-4} + 1 = \frac{3dy+dx}{1}$

Integrating

1/2 log(x+y-2)=x+3y+c

Problem.3. Solve (2x+y+3)dx=(2y+x+1)dy

Solution: $\frac{dy}{dx} = \frac{2x+y+3}{2y+x+1}$

Then

$\frac{2+k}{1+2k} = k \Rightarrow k = 1, -1$

$\frac{dy}{dx}+1=\frac{2x+y+3}{2y+x+1}+1$ —-(1)

$\frac{dy}{dx}+1=\frac{2x+y+3}{2y+x+1}-1$ —–(2)

Dividing (1) by (2), we get

$\frac{dy+dx}{3(x+y)+4}=\frac{-(dx+dy)}{x-y+2}$

Integrating

1/3 log(x+y+4/3)=-log(x-y+2)+log c

$\Rightarrow (x+y+4/3)^{1/3}$(x-y+2)=c

therefore (x+y+4/3) $(x-y+2)^3$=k

# 4 Homogenous linear differential equation

In this section, we solve homogeneous first order and first degree diferential equation in short method.

**Definition 4.1.** A differential equation of the form $\frac{dy}{dx}=\frac{f(x,y)}{g(x,y)}$ is callled a homogenous linear differential equation, if f(x,y),g(x,y) are homogenous functions of the same degree in x and y.

Procedure for solving non-homogeneous first order liner differential equation

Suppose Mdx+Ndy=0 is a homogenous linear differentail equation of first order

Step-1 Finding integrating factor(I.F)

I.F=$\frac{1}{Mx+Ny}$

Step-2 Multiplying the differential equation with I.F

Step-3 Integrating the differential equation both sides using the following ormulae

1.ydx+xdy=d(xy)

2.$\frac{ydx-xdy}{y^2}$=d(x/y)

3.$\frac{xdy-ydx}{xy} = d(log(\frac{y}{x}))$

4. $\frac{xdx+ydy}{x^2+y^2} = \frac{1}{2}d(log(x^2 + y^2)$

5. $\frac{xdx-ydy}{x^2+y^2} = d(Tan^-1(\frac{x}{y}))$

6.$\frac{ydx+xdy}{xy} = d(log(xy))$

7.$\frac{xdx-ydy}{x^2-y^2}=\frac{1}{2}d(log(\frac{x-y}{x+y}))$

8.$\frac{y}{x}(\frac{xdy-ydx}{x^2-y^2}) = \frac{1}{2}d(log\frac{x^2-y^2}{x^2}))$

1.Solve $\frac{dy}{dx} = \frac{x^2+y^2}{2xy}$

Solution: The given differential equation can be written in the form

$(x^2 + y^2)dx - 2xydy = 0$

Here M=$(x^2 + y^2)$, N=2xy

Therefore I.F= $\frac{1}{Mx+Ny} = \frac{1}{x(x^2-y^2)}$

Multiplying the differential equation with I.F.

We get

$\frac{(x^2+y^2)}{x(x^2-y^2)} - \frac{2xydx}{x(x^2-y^2)} = 0$

$\frac{x^2dx}{x(x^2-y^2)} - \frac{xydy}{x(x^2-y^2)} + \frac{y^2dy}{x(x^2-y^2)} - \frac{xydy}{x(x^2-y^2)} = 0$

$\Rightarrow \frac{xdx-ydy}{x^2-y^2} + \frac{y}{x}(\frac{xdy-ydx}{x^2-y^2}) = 0$

Integrating , we get

$\frac{1}{2}(log((x^2 + y^2)) - \frac{1}{2}(log(\frac{x^2-y^2}{x^2})) = \frac{1}{2}logC$by 3.4 & 3.4

$logx^2 + y^2 - log(\frac{x^2-y^2}{x^2}) = logC$

Therefore $x^2 - y^2 = Cx$

2.Solve $\frac{dy}{dx} = \frac{x^2y-2xy^2}{x^3-3x^2y}$
Solution:The given equation can be written in the form

$(x^2y - 2xy^2)dx + (3x^2y - x^3)dy = 0$

Integrating Factor $I.F = \frac{1}{Mx+Ny} = \frac{1}{x^2y^2}$

Multiplying the differential equation with I.F

,We get

$\frac{(x^2y-+2xy^2)dx}{x^2y^2} + \frac{(3x^2y-x^3)dy}{x^2y^2} = 0$

$\Rightarrow (\frac{1}{y} - \frac{2}{x})dx + (\frac{3}{y} - \frac{x}{y^2})dy = 0$

$\Rightarrow (\frac{1}{y}dx - \frac{x}{y^2dy}) - \frac{2}{x}dx + \frac{3}{y}dy = 0$

$\Rightarrow \frac{Ydx-Xdy}{Y^2} - \frac{2}{x}dx + \frac{3}{y}dy = 0$

Integrating,we get

$\frac{x}{y} - 2logx + 3logy + logC = 0$

$\frac{x}{y} = log(c\frac{y^3}{x^2})$

Therefore $\frac{cy^3}{x^2} = e^{\frac{x}{y}}$.

# 5 Linear second order linear differential equation.

The equation $\frac{d^2y}{dx^2} + a_1\frac{dy}{dx} + a_2y = X$ is called linear second order linear differential equation.

Important results of linear second order linear differential equation.

(i) $\frac{1}{f(D)}\ e^{ax}V = e^{ax}\ \frac{1}{f(D+a)}\ V$

(ii) If $c \neq 0$ then $\frac{1}{D^2+bD+c}\ x = \frac{x}{c} - \frac{b}{c^2}$

(iii) If $c \neq 0$ then $\frac{1}{D^2+bD+c}\ x^2 = \frac{x^2}{c} - \frac{2bx}{c^2} + \frac{2(b^2-c)}{c^3}$

(iv) If $c \neq 0$ then $\frac{1}{D^2+bD+c}\ x^3 = \frac{x^3}{c} - \frac{3bx^2}{c^2} + \frac{6(b^2-c)x}{c^3} - \frac{6b(b^2-c^2)}{c^4}$

(v) $\frac{1}{D+a}x = \frac{1}{a}\left(x - \frac{1}{a}\right)$

(vi) $\frac{1}{D+a}x^2 = \frac{1}{a}\left(x^2 - \frac{2x}{a} + \frac{2}{a^2}\right)$

Problem.1.

Solve $(D^2+4)y = xsinx$.

Solution: Auxiliary Equation is $m^2+4=0$. Then $m = \pm 2i$.
Therefore Complimentary function $y_c = c_1cos2x + c_2sin2x$.

$$\begin{aligned} P.I. = y_p =& \frac{1}{D^2+4}\ xsinx \\ =& I.P.\ of\ \frac{1}{D^2+4}\ xe^{ix} \\ =& I.P.\ of\ e^{ix}\ \frac{1}{(D+i)^2+4}\ x \\ =& I.P.\ of\ e^{ix}\ \frac{1}{D^2+2iD+3}\ x \\ =& I.P.\ of\ e^{ix}\left(\frac{x}{3} - \frac{2i}{9}\right) \\ =& sinx\frac{x}{3} - \frac{2}{9}cosx. \end{aligned}$$

Hence the complete solution of given differential equation is
$y = y_c + y_p = c_1cos2x + c_2sin2x + sinx\frac{x}{3} - \frac{2}{9}cosx$. Problem.2. Solve $(D^2+1)y = x^2coshx$. Solution:

Auxiliary Equation is $m^2+1=0$. Then $m = \pm i$.
Therefore Complimentary function $y_c = c_1cosx + c_2sinx$.

$$\begin{aligned} P.I. = y_p =& \frac{1}{2}\frac{1}{D^2+1}\ (e^x + e^{-x})x^2 \\ =& \frac{1}{2}e^x\ \frac{1}{(D+1)^2+1}\ x^2 + \frac{1}{2}e^{-x}\ \frac{1}{(D-1)^2+1}\ x^2 \\ =& \frac{1}{2}e^x\ \frac{1}{D^2+2D+2}\ x^2 + \frac{1}{2}e^{-x}\ \frac{1}{D^2-2D+2}\ x^2 \\ =& \frac{1}{2}e^x\left(\frac{x^2}{2} - \frac{2(2)x}{4} + \frac{2(4-2)}{8}\right) + \frac{1}{2}e^{-x}\left(\frac{x^2}{2} + \frac{2(2)x}{4} + \frac{2(4-2)}{8}\right) \\ =& \frac{x^2}{2}coshx - xsinhx + 2coshx. \end{aligned}$$

Hence the complete solution of given differential equation is $y = y_c + y_p$.
Problem.3. Solve $(D^2+1)y = x^2 sin2x$. Solution:

Auxiliary Equation is $m^2+1=0$. Then $m=\pm i$.
Therefore Complimentary function $y_c = c_1 cosx + c_2 sinx$.

$$\begin{aligned}
P.I. = y_p =& \frac{1}{D^2+1}\ x^2 sin2x \\
=& I.P.\ of\ \frac{1}{D^2+1}\ x^2 e^{2ix} \\
=& I.P.\ of\ e^{2ix}\ \frac{1}{(D+2i)^2+1}\ x^2 \\
=& I.P.\ of\ e^{2ix}\ \frac{1}{D^2+4iD-3}\ x^2 \\
=& I.P.\ of\ e^{2ix}\ \left(\frac{x^2}{-3} - \frac{2(4i)x}{(-3)^2} - \frac{26}{(-3)^3}\right) \\
=& I.P.\ of\ (cos2x + isin2x)\ \left(-\frac{x^2}{3} - \frac{8ix}{9} + \frac{26}{27}\right) \\
=& -\frac{x^2}{3} sin2x - \frac{8x}{9} cos2x + \frac{26}{27} sin2x
\end{aligned}$$

Hence the complete solution of given differential equation is $y = y_c + y_p$.
Problem.3. Solve $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = cos3x - 3x^3$. Solution: The given equation can be written as $(D^2+4D+3)y = cos3x - 3x^3$. Auxiliary Equation is $m^2+4m+3=0$. Then $m=-3,-1$.
Therefore Complimentary function $y_c = c_1 e^{-x} + c_2 e^{-3x}$.

$$\begin{aligned}
P.I. = y_p =& \frac{1}{D^2+4D+3}\ cos3x - 3x^3 \\
=& R.P.\ of\ \frac{1}{D^2+4D+3}\ e^{3ix} - 3\ \frac{1}{D^2+4D+3} x^3 \\
=& R.P.\ of\ \frac{1}{(3i)^2+4(3i)+3}\ e^{3ix} - 3\left(\frac{x^3}{3} - \frac{3(4)x^2}{3^2} + \frac{6(16-3)x}{3^3} - \frac{6(4)(16-6)}{3^4}\right) \\
=& R.P.\ of\ \frac{1+2i}{-6(1+2i)(1-2i)}\ (cos3x + sin3x) + \left(-x^3 + \ 4x^2 - \frac{26x}{3} + \frac{80}{9}\right) \\
=& R.P.\ of\ \frac{1+2i}{-30}\ (cos3x + sin3x) + \left(-x^3 + \ 4x^2 - \frac{26x}{3} + \frac{80}{9}\right) \\
=& \frac{cos3x}{30} + \frac{2sin2x}{30} - x^3 + \ 4x^2 - \frac{26x}{3} + \frac{80}{9}
\end{aligned}$$

Hence the complete solution of given differential equation is $y = y_c + y_p$.

# 6 First order partial diferential equations

The working procedure of solving the given first order partial differential equations $f(x^m z^n p,\ y^l z^n q) = 0$.
**Step -1 :** Then the following are suitable substitutions.
Put $x = \int x^{-m}$, $y = \int y^{-l}$ and $z = \int z^n$.
Then $x^m z^n p = \frac{\partial Z}{\partial X} = P$.
Similarly $y^l z^n q = \frac{\partial Z}{\partial Y} = Q$.
**Step- 2:** On substitution, the given first partial differential equation reduces to on $e$ of the standard forms in new partial differential coefficients and new variables.
**Step- 3:** Solve the reduced partial differential equation which is in standard form.
**Step- 4:** Replace $X, Y, Z$ in terms of x,y,z to get the required solution.
**Problem 2.1** Solve $q^2 y^2 = z(z - xp)$
**Solution :** The given equation can be written in the form
$\frac{xp}{z} + \left(\frac{yq}{z}\right)^2 = 1$, which is a partial differential equation and it is not in any one of the standard form.
Here $m = 1$. Put

$$X = \int x^{-1} dx = logx$$
$$Y = \int y^{-1} dx = logy$$
$$Z = \int z^{-1} dx = logz.$$

Then $\frac{xp}{z} = \frac{\partial Z}{\partial X} = P$ and $\frac{\partial Z}{\partial Y} = Q$. (Say)
Therefore $P + Q^2 = 1 \quad \cdots \quad (1)$
Which is in the form $f(p, q) = 0$.
Let the solution be $Z = aX + bY + c$ then $P = a$ and $Q = b$.
Now put $P = a$ and $Q = b$ in equation (1). We get $b = \sqrt{1 - a}$.
Therefore the required solution is $z = aX + \sqrt{1 - a}Y + c$.
$logz = alogX + \sqrt{1 - a}logY + logk$, where $c = logk$.
Hence $z = XY\sqrt{1 - a}k$, where $a$ and $k$ are arbitrary constants.

**Problem 2.2** Solve $\frac{x^2}{p} + \frac{y^2}{q} = z$
**Solution :** The given equation can be written in the form
$x^2 z^{-1} p^{-1} + y^2 z^{-1} q^{-1} = 1$

$\Rightarrow\ (x^{-2}zp)^{-1} + (y^{-2}zp)^{-1} = 1$. Put

$$X = \int x^2 dx = \frac{x^3}{3}$$
$$Y = \int y^2 dx = \frac{y^3}{3}$$
$$Z = \int z dx = \frac{z^2}{2}.$$

Therefore $x^2z^{-1}p^{-1} = \frac{1}{P}$ and $y^2z^{-1}q^{-1} = \frac{1}{Q}$ then $\frac{1}{P} + \frac{1}{Q} = 1$.
Let the solution be $Z = aX + bY + c$.
Therefore the required solution $\frac{z^2}{2} = a\frac{x^3}{3} + b\frac{y^3}{3} + c$, where $b = \frac{a}{a-1}$.
**Problem 2.3** Solve $\frac{p}{x^2} + \frac{q}{y^2} = z$
**Solution :** $x^{-2}z^{-1}p + y^{-2}z^{-1}q = 1$. Put

$$X = \int x^2 dx = \frac{x^3}{3}$$
$$Y = \int y^2 dx = \frac{y^3}{3}$$
$$Z = \int z dx = \frac{z^2}{2}.$$

Then $x^{-2}z^{-1}p = P,\ y^{-2}z^{-1}q = Q$.
Then the equation becomes $P + Q = 1$. Therefore the solution $Z = aX + bY + c$, where $b = 1 - a$.
$logz + a\frac{x^3}{3} + (1-\alpha)\frac{y^3}{3} + c$.
**Problem 2.4** Solve $p^2 + pq = z^2$
**Solution :** The given equation can be written in the form $(z^{-1}p)^2 + (z^{-1}p)(z^{-1}q) = 1$.
Put $Z = \int zdx = \int \frac{1}{z}dz = logz$. Then $z^{-1}p = P$ and $z^{-1}q = Q$.
Therefore $P^2 + PQ = 1$.
Let the solution be $Z = aX + bY + c$, where $b = \frac{1-a^2}{a}$.
Therefore $logz = ax + \frac{1-a^2}{a}y + c$.

**Definition 6.1.** A partial differential equation of the form $Pp + Qq = R$, where $P, Q$ and $R$ are functions of $x, y$ and $z$ is called a Lagrange's linear partial differential equation.

**Problem 2.5** Solve $yzp + zxq = xy$
**Solution :** The given equation is Lagrange's first order linear partial differential equation.

The given equation can be written in the form $x^{-1}zp + y^{-1}zq = 1$. Put

$$X = \int xdx = \frac{x^2}{2}$$
$$Y = \int dx = \frac{y^2}{2}$$
$$Z = \int zdx = \frac{z^2}{2}.$$

Then $zx^{-1}p = P = \frac{\partial Z}{\partial X},\ y^{-1}zq = Q = \frac{\partial Z}{\partial Y}$.
Therefore $P + Q = 1$. Hence $Z = aX + bY + c$ and $a + b = 1$.
Thus $\frac{z^2}{2} = a\frac{x^2}{2} + b\frac{y^2}{2} + c$, where $b = 1 - a$. **Problem 2.6** Solve $\frac{y^2 z}{x}p + xq = y^2$
**Solution :** This equation can be written in the form $zx^{-1}p + x(y^{-2}zq) = 1$. Put

$$X = \int xdx = \frac{x^2}{2}$$
$$Y = \int y^2 dx = \frac{y^3}{3}$$
$$Z = \int zdx = \frac{z^2}{2}.$$

Then $zx^{-1}p = P = \frac{\partial z}{\partial x},\ x(y^{-2}zq) = Q = \frac{\partial z}{\partial y}$.
Therefore

$$P + \sqrt{2}\sqrt{X}Q = 1$$
$$\Rightarrow \sqrt{2}Q = \frac{1-P}{\sqrt{X}} = a$$
$$\Rightarrow Q = \frac{a}{\sqrt{2}} \text{ and } P = 1 - a\sqrt{X}.$$
$$dZ = PdX + QdY$$
$$dZ = (1 - a\sqrt{X})dX + \frac{a}{\sqrt{2}}dY$$
$$\Rightarrow z = X - a\frac{2}{3}X^{\frac{3}{2}} + \frac{a}{\sqrt{2}}Y + c$$
$$\Rightarrow \frac{z^2}{2} = \frac{x^2}{2} - \frac{2a}{3}\frac{x^3}{2^{\frac{3}{2}}} + \frac{a}{\sqrt{2}}\frac{y^3}{3} + c$$
$$\Rightarrow \frac{z^2 - x^2}{2} = \frac{a}{3\sqrt{2}}(y^3 - x^3) + c.$$

# References

[1] M. Murali Krishna Rao, Formation of differential equation in short method ,
DOI: 10.13140/RG.2.2.34597.35046.

[2] M. Murali Krishna Rao,Murali Krishna's method for Non-Homogeneous First Order Differential Equations,
DOI: 10.13140/RG.2.2.32592.71683/1

[3] M. Murali Krishna Rao,Solving the homogenous first order linear differential equation in short method.
DOI: 10.13140/RG.2.2.26721.40806

[4] M. Murali Krishna Rao, Murali Krishna Raos method for solving the first order partial dierential equations in short method DOI: 10.13140/RG.2.2.14884.94080

[6] M. Murali Krishna Rao, ON SECOND ORDER DIFFERENTIAL EQUATIONS DOI: 10.13140/RG.2.2.14464.69129