Weakly Generalized Star Pre Regular Closed Sets In Topology

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ABSTRACT

In this paper we introduce a new class of sets namely, wg*pr closed sets in topological spaces. For these sets we investigate its properties .

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1. INTRODUCTION

Levine [7] generalized the concept of closed set to generalized closed sets. Bhattacharya and Lahiri [2] generalized the concept of closed sets to semi-generalized closed sets. In this paper we generalize the concept of closed set to w closed set via g pr open set and study some of their relationship and properties. Furthermore the notion of wg*pr neighbourhoodwg*pr limit points,wg*pr derived sets,wg*pr closure,wg*pr interior and wg*pr R0 as well as weakly wg*pr R0 spaces are presented.

2. PRELIMINARIES:

Throughout this paper (X, □) denote a topological space on which no separation axioms is assumed. It is simply denoted by X. For a subset A of a space (X, □) closure of A and interior of A with respect to □ are denoted by Cl(A) and Int(A) respectively. The complement of A is denoted by X-A.

Definition 2.1:

A subset A of a topological space X is called

i. pre open [10] if A □ □ □ intcl (A) and pre closed [ ] ifclint (A) □ □ □ .
ii. regular open [13] if A= intcl (A) and regular closed [ ] if A= clint (A).
iii. semi open [6] if A □ □ □ clint (A) and semi closed [ ] if intcl (A) □ □ □ □

Definition 2.2:

A subset A of a topological space X is called

i. weakly closed [14] (briefly w closed) if cl A □ □ U whenever A□ □ U and U is semi open.
ii. generalized pre regular closed [4] (briefly g pr closed) if p cl (A) □ □ U whenever A □ □ U and U is regular open

iii. weakly generalized star pre regular closed (briefly wg*pr closed) if w cl A □ □ U whenever A □ □ U and U is gpr open.

Hereafter a topological space is simply written as TS.

3.wg*pr Neighbourhoods:

Definition 3.1:A subset P of a TS X is called as semigeneralized star pre regular-neighbourhood (in short wg*pr -nhd) of a point k of X if there arises a wg*pr -open set U so that k □ U □ P. The collection of entire wg*pr -nhds of x ∈ X is termed wg*pr -nhd system of x and is labeled as wg*pr -N(x).

Theorem3.2: Let p be any arbitrary point of a TS X. Then wg*pr -N(x) satisfies succeeding properties

- □ wg*pr -N(p) □ □
- □ Whenever N ∈wg*pr -N(p) then p ∈ N.
- □ Whenever N ∈wg*pr -N(p) and N ⊂ M at that time M ∈wg*pr -N(p).

Proof: (i) By the reason of each p∈X, X is a wg*pr -open set.Therefore x ∈ X ⊂ X, implies X is wg*pr -nhd of p, hence X ∈wg*pr-N(p). Accordingly, wg*pr -N(p)□ □.

- □ Given N ∈wg*pr -N(p), implies N is a wg*pr -nhd of p, which indicates there is a wg*pr -open set G so as p ∈ G ⊂ N. This implies p ∈ N.
- □ Given N ∈wg*pr-N(p) implies there is a wg*pr -open set G in such a manner p ∈ G ⊂ N. And N ⊂ M, which implies p ∈ G ⊂ M. This shows that M ∈wg*pr -N(p).

Assume, arbitrary intersection of wg*pr closed sets is wg*pr closed -----------------(R)

Theorem 3.3: Let A be a member of a TS X. Thereupon A is wg*pr-open iff A contains a wg*pr -nhd of each of its points when R holds

Proof:

Allow A be a wg*pr -open set in X. Make x□A, whichimplies x □A □ A. So A is wg*pr -nhd of x. Hence A contains a wg*pr -nhd of each of its points. Contrarily, A contains a wg*pr -nhd of each of its points. For each x □ A there arises a wg*pr neighbourhood N_x of x such that x □N_x□ A. By the definition of wg*pr -nhd of x, there is a wg*pr -open set G_x such that x □G_x□N_x□ A. Now we shall prove that A = □ {G_x: x □ A}. Let x □ A. Then there is wg*pr -open set G_x such that x □G_x. Therefore, x □ □{G_x: x □ A} which implies A □ □{G_x: x□ A}. Now let y□{G_x: x □ A} so that y □ some G_x for some x□ A and hence y□ A. Hence, □{G_x: x □ A} □ A. Hence A = □{G_x: x □ A}. Also each G_x is a wg*pr -open set. And hence A is a wg*pr-open set.

Theorem 3.4: Whenever A is a wg*pr -closed subset of X and x□X – A, accordingly there is a wg*pr -nhd N of x so that N □ A =□.

Proof: Assuming that A is a wg*pr -closed set in X, then X – Ais a wg*pr -open set. By the Theorem 4.3, X – A contains a wg*pr -nhd of each of its points. This implies that, there is an wg*pr -nhd N of x so as N □ X-A. That is, no point of N belongs to A and hence N □ A = □.

Definition 3.5: A point x□X is termed as wg*pr -limit point ofA iff each wg*pr -nhd of x contains a point of A different from x. That is (N – {x}) □ A □ □, for each wg*pr -nhd N of x. Also equivalently iff each wg*pr -open set G comprising x contains a point of A other than x. The collection of entire wg*pr -limit points of A is named as wg*pr -derived set of A and is labeled as wg*pr -d(A).

Theorem3.6 : Let A, B be subsets of X then A□B implieswg*pr -d(A) □wg*pr -d(B).

Proof: Enable x□wg*pr -d(A) implies x is a wg*pr -limit point of A. That is each wg*pr -nhd of x contains a point of A other than x. As A □ B, each wg*pr -nhd of x contains a point of B other than x. Consequently x is a wg*pr -limit point of B. That is x□wg*pr -d(B). Hence wg*pr -d(A) □wg*pr -d(B).

Theorem 3.7:A subset P of X is wg*pr closed iffwg*pr -d(P)□P assuming R

Proof: Let P be wg*pr -closed set. That is X-P is wg*pr -open. Now we prove that wg*pr -d(P) □ A. Allow x□wg*pr -d(P) which intend x is a wg*pr -limit point of P, that is each wg*pr -nhd of x contains a point of P different from x. Now think x□P so that x□ X-P, which is wg*pr -open and by definition of wg*pr -open sets, there is a wg*pr -nhd N of x in such a manner N □ X-P. From this we conclude that N contains no point of P, which is a contradiction. Therefore x□P and hence wg*pr -d(P) □ P. conversely assume that wg*pr -d(P)□ □P and we will prove that P is a wg*pr -closed set in X or X-P is wg*pr -open set. Ensure x be an arbitrary point of X-P, so that x□ P which implies that x□wg*pr -d(A). That is there exists a wg*pr -nbd N of x which consists of only points of X – P. This means that X-P is wg*pr -open. And hence P is wg*pr -closed set in X.

Theorem3.8: Each wg*pr -derived set in X is wg*pr closed,when R holds

Proof: Permit A be a member of X andwg*pr -d(A) is wg*pr -derived set of A. By Theorem 4.7, wg*pr -d(A) is wg*pr -closed iff wg*pr -d(wg*pr-d(A)) □wg*pr -d(A) . That is each wg*pr -limit point of wg*pr -d(A) belongs to wg*pr -d(A).

Now allow x be a wg*pr -limit point of wg*pr -d(A). That is x□wg*pr -d(wg*pr -d(A)). So that there is a wg*pr -open set G containing x such that {G – {x}} □wg*pr -d(A) □ □, which implies {G – {x}} □ A □ □,, as each wg*pr -nhd of an element of wg*pr -d(A) has at least one point of A. Hence x is a wg*pr -limit point of A. That is x belongs to wg*pr -d(A). So x□wg*pr -d(wg*pr -d(A)) implicit x□wg*pr -d(A). Accordingly wg*pr -d(A) is wg*pr -closed set in X.

Theorem 3.9: The following properties are true for A, B⊂X

i)wg*pr -d(□) = □
ii)Whenever A ⊂ B then wg*pr -d(A) ⊂wg*pr -d(B).
iii)Whenever q ∈wg*pr -d(A) then q ∈wg*pr -d(A−{q}).
iv)wg*pr -d(A)∪wg*pr -d(B) ⊂wg*pr -d(A∪B).
v)wg*pr -d(A∩B) ⊂wg*pr -d(A)∩ wg*pr -d(B).

Proof: (i) Let q∈X and G be a wg*pr -open involving q.Then (G −{q}) ∩ □ = □. This suggest q□wg*pr -d(□).Accordingly for any q ∈ X, q is not wg*pr -limit point of □.Hence wg*pr -d(□) = □.

ii) Allow q ∈wg*pr -d(A). Afterwards G∩(A−{q}) □ □,, for each wg*pr -open set G involving q. As A ⊂ B, implies G∩(B−{q}) □ □,. This impart q ∈wg*pr -d(B). Thereupon, q ∈wg*pr -d(A) implies q ∈wg*pr -d(B). Therefore, wg*pr -d(A) ⊂wg*pr -d(B).

iii)Let q ∈wg*pr -d(A). Then G∩(A−{q})□ □,, for each wg*pr -open set G containing q. This implies that each wg*pr -open set G including q, contains at least one point different from q of A−{q}. Therefore q ∈wg*pr -d(A−{q}).

iv)Since A ⊂ A∪B and B ⊂ A∪B and by (ii), wg*pr -d(A) ⊂wg*pr -d(A ∪ B) and wg*pr -d(B) ⊂wg*pr -d(A ∪ B). Hence, wg*pr -d(A) ∪wg*pr -d(B) ⊂wg*pr -d(A∪B).

v)Since A∩B ⊂ A and A∩B ⊂ B and by (ii), wg*pr -d(A∩B) ⊂wg*pr -d(A) and wg*pr -d(A∩B) ⊂wg*pr -d(B). Therefore wg*pr -d(A∩B) ⊂wg*pr -d(A) ∩ wg*pr -d(B).

Theorem 3.10: Whenever A is a member of X, then A∪wg*pr -d(A) is wg*pr -closed set, assuming R.

Proof: To prove A∪wg*prd(A) is wg*pr -closed set, it issufficient to prove X − (A ∪wg*pr -d(A)) is wg*pr open. Whenever X − (A ∪wg*pr -d(A))= □, then it is clearly wg*pr -open set. Enable X − (A ∪wg*pr -d(A)) □ □and x ∈ X − (A ∪wg*pr -d(A)), implies x□A∪wg*pr- d(A). This implies x□Aand x □wg*pr -d(A). Now x □wg*pr -d(A), which indicates x is not wg*pr -limit point of A. Therefore, there is a wg*pr -open set G containing x so that G ∩ (A −{x}) = □. As x □A, implies G ∩ A = □. This suggest x ∈ G ⊂ X − A——(1). Again G is wg*pr -open set and G ∩ A =□. implies no point of G can be wg*pr-limit point of A. This follows G∩ wg*pr -d(A) = □, implies x ∈ G ⊂ X− wg*pr-d(A)——(2). From (1) and (2), x ∈ G ⊂ (X −A)∩(X − wg*pr -d(A)) = X −(A∪wg*pr -d(A)). That is x ∈ G ⊂ X−(A∪wg*pr -d(A)). This impart X−(A∪wg*pr -d(A)) contains wg*pr-nhd of each of its points. By theorem 3.4, X−(A∪wg*pr -d(A)) is wg*pr open as well as A∪wg*pr -wg-d(A) is wg*pr closed set.

4. On wg*pr □ closure and wg*pr-interior operators

Definition 5.1: Consider X be a TS and Q□X. The set of intersection of entire wg*pr-closed sets including Q is named wg*pr-closure of Q and is labeled as wg*prCl(Q).

Theorem 4.2: For members A, B of X, the listed properties hold:

i)wg*pr Cl(X)=X and wg*pr Cl(□)=□□ .

ii)Whenever A □ B, then wg*pr Cl(A) □wg*pr Cl(B)

iii)wg*pr Cl(P)∪ wg*pr Cl(Q) □wg*pr Cl(P∪Q)

iv)wg*pr Cl(A∩B) □wg*pr Cl(A)∩ wg*pr Cl(B)

v)wg*pr Cl(wg*pr Cl(A)) = wg*pr Cl(A)

vi) A is wg*pr -closed iff wg*pr Cl(A)=A, when R holds

Theorem 4.3: For A□X, then x□wg*prCl(A) iff G∩A□□□for each wg*pr-open set G containing x.

Proof: Necessity: Enable x□wg*prCl(A) for any x□X.Expect there is a wg*pr-open set G comprising x so that G∩A=□□ . Then A□□ X-G. As X-G is wg*pr-closed set comprising A, we have wg*prCl(A) □ X-G, which indicates x□□ wg*prCl(A). This is contradiction to hypothesis. Hence G A□ □ □. Conversely, assume x□wg*prCl(A). There exist a wg*pr-closed set F containing A so that x□F. Then x□X-F and X-F is wg* pr□-open. Also (X-F) A=□. This is contradiction to the hypothesis. Therefore x□wg*prCl(A).

Definition 4.4: For a TS X and S□X the union of entire wg*pr open sets included in S is termed as wg*pr-interior of S and is labeled as wg*prInt(A).

Theorem 4.5: A and B be members of TS X. Then the listed results hold:

i)wg*pr Int(X)=X and wg*pr Int□□□ □ □□ .

ii)Whenever A □□ B, then wg*pr Int(A)□wg*pr Int(B)

iii)wg*pr Int(A) wg*pr Int(B)□□wg*pr Int(A B)

iv)wg*pr Int(A B) □wg*pr Int(A) wg*pr Int(B)

v)wg*pr Int(wg*pr Int(A))= wg*pr Int(A)

vi)A is wg*pr -open iff wg*pr Int(A)=A, assuming R

Theorem 4.6: For a member A of X, the listed results hold:

i)wg*prCl(X-A)= X- wg*prInt(A)

ii)wg*prInt(X-A) = X- wg*prCl(A)

iii)wg*prInt(A)= X- wg*prCl(X-A)

iv)wg*prCl(A)=X- wg*prInt(X-A)

Proof: (i) Allow x□X-wg*prInt(A). So x□wg*prInt(A),implies for each wg*pr-open set U comprising x we have U (X-A) □□□ . Thus, x□wg*prCl(X-A). Hence X-wg*prInt(A) □□ wg*prCl(X-A). Conservely, allow x□wg*prCl(X-A). So U □ A for each wg*pr-open set U comprising x. Hence x□wg*prInt(A), implies x□X-wg*prInt(A). This indicates wg*prCl(X-A) □□ X- wg*prInt(A).Therefore, X- wg*pr-sgInt(A) = wg*prClX-A).

(ii)Enable x□X- wg*prCl(A). So x□wg*prCl(A), implies for each wg*pr open set U including x we have U A=□ . This impart x□□ □□ A^c, so x□wg*prInt(A^c) or x□wg*prInt(X-A). Therefore we have X- wg*prCl(A) □□ wg*prInt(X-A) Conversely, make x□wg*prInt(X-A). Then there is a wg*pr-open set U including x so that x□U□ X-A. Hence U⋂A=□ , x□wg*prCl(A), implies x□X- wg*prCl(A). This indicates wg*prInt(X-A) □□ X-wg*prCl(A).Therefore, X- wg*prCl(A) = wg*prInt(X-A)

(iii)Replacing A by X-A in (ii) we result (iii)
(iv)Replacing A by X-A in (i) we result (iv)

5. wg*pr -R_oSPACES

Definition 5.1: Let A be a subset of a TS X. The wg*pr-kernel of A , labeled as wg*pr-ker(A) is defined to be the set wg*pr-ker (A) = □{U: A □□ U andU is wg*pr-open in X}

Definition 5.2: Let x be a point of a TS X. The wg*pr-kernel of x , labeled as wg*pr-ker ({x}) is defined to be the set wg*pr-ker ({x}) = □ {U: x □ U and U is wg*pr-open in (X,□)}

Lemma 5.3: Let X be a TS and x□X. Then wg*pr-ker (A) ={x □ X: wg*prCl ({x}) □ A ≠ □ }.

Proof: Let x□wg*pr-ker (A) and suppose wg*prCl ({x})□A =□. Hence x □X – wg*prCl ({x}) which is a wg*pr-open set including A. This is absurd, as x □wg*pr-ker (A).Hence wg*prCl ({x}) □ A ≠ □. Contrarily, let wg*prCl ({x}) □ A ≠ □ and assume that x □wg*pr-ker (A). Then there is a wg*pr open set U including A and x□□ U. Let y ϵwg*prCl ({x}) □ A. Hence , U is a wg*pr nhd of y in which x □ U. By this contradiction , x □wg*pr-ker (A) and the claim.

Definition 5.4: ATS X is named as w generalized star pre **regular**□-R_o(in short ,wg*prR_o) space iff for each wg*pr open set G and x □ G implies wg*prCl ({x}) □ G.

Lemma 5.5: Let X be a TS and x□X. Then y□ □wg*pr-ker({x}) iff x □wg*prCl ({y}).

Proof: Suppose that y□wg*pr-ker ({x}). Then there exists a wg*pr open set V comprising x such that y □ V. Therefore we have x □wg*prCl ({y}). The proof of converse can be done similarly.

Lemma 5.6: The following results are similar for any points x and y in a TS X:

i)wg*pr-ker ({x}) ≠ wg*pr-ker({y})

ii)wg*prCl ({x}) ≠ wg*prCl ({y}).

Proof: (i)□(ii).Suppose that wg*pr-ker ({x}) ≠ wg*pr-ker ({y})then there exists a point z in X such that z □wg*pr-ker ({x}) and z □wg*pr-ker ({y}). From z □wg*pr-ker ({x}) it follows that {x} □wg*prCl ({z}) ≠ □which implies x □wg*prCl ({z}).By z □wg*pr-ker ({y}) , we have {y}⋂wg*pr Cl ({z}) = □. Since x □wg*prCl ({z}) ,wg*prCl ({x}) □wg*prCl ({z}) and {y} □wg*prCl ({x}) =□. Therefore it follows that wg*prCl ({x}) ≠ wg*prCl ({y}). (ii) □ (i). Suppose that wg*prCl ({x}) ≠ wg*prCl ({y}).There exists a point z in X such that z □wg*prCl ({x}) and z □wg*prCl ({y}).Then there exists a wg*pr-open set containing z and therefore x but not y , namely , y □wg*pr-ker ({x}). Hence wg*pr-ker ({x}) ≠ wg*pr-ker ({y}).

Theorem5.7: A TS X is wg*pr-$R_o$space iff for any x , y in X ,wg*prCl ({x}) ≠ wg*prCl ({y}) implies wg*prCl {x}) □wg*prCl ({y}) = □.

Proof: Consider X is wg*pr-$R_o$space and x , y□X in that casewg*prCl ({x}) ≠ wg*prCl ({y}).Then there exists a point z □wg*pr-ker ({x}) so that z □wg*prCl ({y}) (or z □wg*pr-ker({y}) such that zϵwg*prCl ({x})).There exists a wg*pr-open set V such that y □ V and z □ V ; hence x □ V. Therefore , we have x □wg*prCl ({y}). Thus x □X – wg*prCl ({y}) a wg*pr-open set, which implies wg*prCl ({x}) □ X –wg*prCl ({y}) and wg*prCl ({x}) □wg*prCl ({y}) =□.

Contrarily , let V be a wg*pr-open set in X and let x □ V. Now we have claim that wg*prCl ({x}) □ V. Make y □ V, that is , yϵX – V. Then x ≠ y as well as x □wg*prCl ({y}).This implies ,wg*prCl ({x}) ≠ wg*prCl ({y}). By assumption ,wg*prCl{x}) □wg*prgCl ({y}) = □. Hence y □wg*prCl ({x}) and therefore wg*prCl ({x})) □ V.

Theorem5.8: If a TS X is wg*pr-$R_o$space, then for any x , y in X wg*pr-ker ({x}) ≠ wg*pr-ker ({y}) implies wg*pr-ker ({x}) □wg*pr-ker ({y}) =□.

Proof: Suppose X is wg*pr-$R_o$space. Thus by Lemma 6.6 forany points x , y □ X whenever wg*pr-ker ({x}) ≠ wg*pr-ker ({y}) then wg*prCl ({x}) ≠ wg*prCl ({y}). Now we prove that wg*pr-ker ({x}) □wg*pr-ker ({y}) = □ . Suppose that z □wg*pr-ker ({x}) □wg*pr-ker ({y}). By Lemma 6.5 and z □wg*pr-ker ({x}) implies x □wg*pr-ker ({z}). Since x □wg*prCl ({x}) , by Theorem 6.7, wg*prCl ({x}) = wg*prCl ({z}). Similarly , we have wg*prCl ({y}) = wg*prCl ({x}) a contradiction. Hence wg*pr-ker ({x}) □wg*pr-ker ({y}) = □.

Theorem 5.9: For a TS X the following properties are equivalent:

i)X is a wg*pr-R_o space.

ii)x□wg*prCl ({y}) if and only if y □wg*prCl {x}) for any points x and y in X.

Proof: (i)□(ii). Assume that X is a wg*pr-$R_o$space. Let x□wg*prCl ({y}) and U be any wg*pr-open set such that y □ U. Now by hypothesis x □ U. Therefore, every wg*pr-open set containing y contains x. Hence y □wg*prCl ({x}).

(ii) □ (i). Let V be a wg*pr-open set and x □ V. If y □ V then x □wg*prCl ({y}) and hence y □wg*prCl ({x}). This implies that wg*prCl ({x}) □ V. Hence X is a wg*pr-R_o space.

Theorem 5.10: For a topological space X the following properties are equivalent, when R holds

i)X is a wg*pr-R_o space.

ii)Whenever A is a wg*pr-closed , then A = wg*pr-ker(A).
iii)Whenever A is a wg*pr-closed as well as x □ A , thereupon wg*pr-ker ({x}) □ A

iv)Whenever x □X, then wg*pr-ker ({x}) □wg*prCl ({x}).

Proof: (i)□(ii). Let A be wg*pr-closed and x□A .Thus X –A is a wg*pr-open and x □X –A . Since X is a wg*pr-$R_o$spacewg*prCl ({x}) □ X – A. Thus wg*prCl({x}) □ A =□ and by the Lemma 5.3 , x □wg*pr-ker (A). Therefore wg*pr-ker (A) = A.

(ii) □ (iii). In general U □ V implies wg*pr-ker (U) □wg*pr-ker(V)

. Therefore wg*pr-ker ({x}) □wg*pr-ker (A) = A by (ii).

(iii) □ (iv). Since x □wg*prCl ({x}) and wg*prCl ({x}) is wg*pr-closed by

(iii) wg*pr-ker ({x}) □wg*prCl ({x}).

(iv) □ (i). Let x □wg*prCl ({y}) then by the Lemma 5.5 , y □wg*pr-ker ({x}). Since x □wg*prCl ({x}) and wg*prCl ({x})iswg*pr-closed , by (iv) we obtain y □wg*pr-ker ({x}) □wg*prCl ({x}).Therefore x □wg*prCl ({y}) implies y □wg*prCl ({x}) .The converse is obvious and X is a wg*pr-R_o space.

Definition 5.11: A TS X is termed as

- wg*pr-C_o whenever for x , y □ X with x ≠ y , there exists a wg*pr-open set G such that wg*prCl (G) contains one of x and y but not other.
- wg*prC1 whenever x,yϵX with x≠y,there exist wg*pr open sets G and H such that xϵwg*prCl(G),
- wg*prC1 whenever for x , y □ X with x ≠ y , there exist wg*pr-open sets G and H such that x □wg*prCl(G)

  y □wg*prCl (H) but x □wg*prCl (H) ,y □wg*prCl (G).

- weakly wg*pr-C_o whenever □□wg*pr-ker ({x}/xϵX} = □.
- weakly wg*pr–R_0 whenever□ □{ wg*prCl ({x}) / x□X }= □

Theorem 5.12: A topological space X is weakly wg*pr-$R_o$if and only if wg*pr-ker ({x}) ≠ X for x □X

Proof: Necessity: Assume that there is a point $x_o$in X with wg*pr-ker ({x_o}) = X. Then X is the only wg*pr-open set containing x_o. This implies that x_o□wg*prCl ({x}) for every x □X. Hence x_o□ □{ wg*pr Cl ({x}) / x □ X} ≠ □ , a contradiction.

Sufficiency: If X is not weakly wg*pr-R_0, then choose some $x_o$in X such that x_o□ □{ wg*prCl ({x}) / x □ X}. This implies that every wg*pr-open set containing x_o must contain every point of X. Thus the space X is the unique wg*pr-open set containing x_o. Hence wg*pr-ker ({x_o}) = X, which is a contradiction. Therefore X is weakly wg*pr -$R_{o.}$

Theorem 5.13: A space X is weakly wg*pr-$C_o$if and only if for each x □X, there exists a proper wg*pr-closed set containing y.

Proof: Suppose there is some y □ X such that X is the onlywg*pr-closed set containing y .Let U be any proper wg*pr-open subset of X containing a point of x_0 of X. This implies that X – U ≠ X. Since X – U is wg*pr-closed set, we have y □ X– U. So, y □ U. Thus y □ □ { wg*pr-ker ({x}) / x □ X} for any point x of X , a contradiction. Conversely, suppose X is not weakly wg*pr-Co , then choose y □ □ { wg*pr-ker ({x}) / x □ X}. So y belongs to wg*pr-ker ({x}) for any x □X. This implies that X is the only wg*pr-open set which contains the point y, a contradiction.

Theorem 5.14: Every wg*pr-Co space is weakly wg*pr-Co

Proof: Whenever p , q□X such that p ≠ q , where X is a wg*pr-C_o space , then without loss of generality , we can assume that there exists a wg*pr-open set G such that p □wg*prCl(G) but q □wg*prCl(G) . This implies that G ≠ □ . Hence we can choose some z in G. Now wg*pr-ker (z) □wg*pr-ker (q) □ G □ (wg*prCl (G))^C □wg*prCl (G) □ (wg*prCl (G))^C =□. Therefore □{wg*pr-ker ({p}) / p □ X} = □. Hence the space X is weakly wg*pr-$C_{o.}$

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