Signed And Romantotal Edge Dominating Functuions Of Corana Product Graph Of A Cycle With A Star

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Abstract:

Graph Theory has been realized as one of the most useful branches of Mathematics of recent origin with wide applications to combinatorial problems and to classical algebraic problems. Graph theory has applications in diverse areas such as social sciences, linguistics, physical sciences, communication engineering etc.

The theory of domination in graphs is an emerging area of research in graph theory today. It has been studied extensively and finds applications to various branches of Science & Technology. An introduction and an extensive overview on domination in graphs and related topics is surveyed and detailed in the two books by Haynes et al [7, 8].

Frucht and Harary [6] introduced a new product on two graphs G_1 and G_2, called corona product denoted by $G_1 \odot G_2$. The object is to construct a new and simple operation on two graphs G_1 and G_2 called their corona, with the property that the group of the new graph is in general isomorphic with the wreath product of the groups of G_1 and of G_2 .

Products are often viewed as a convenient language with which one can describe structures, but they are increasingly being applied in more substantial ways. Every branch of mathematics employs some notion of product that enables the combination or decomposition of its elemental structures.

In this paper, some results on minimal signed and Roman total edge dominating functions of corona product graph of a cycle with a star are presented.

Keywords - *Corona Product, Total edge dominating set, Total edge domination number, Signed Total edge dominating function, Roman Total edge dominating function*

I. INTRODUCTION

Domination Theory has a wide range of applications to many fields like Engineering, Communication Networks, Social sciences, linguistics, physical sciences and many others. Allan, R.B. and Laskar, R.[1], Cockayne, E.J. and Hedetniemi, S.T. [4] have studied various domination parameters of graphs.

Products are often viewed as a convenient language with which one can describe structures, but they are increasingly being applied in more substantial ways. Every branch of mathematics employs some notion of product that enables the combination or decomposition of its elemental structures.

The concept of edge domination was introduced by Mitchell and Hedetniemi [16] and it is explored by many researchers. Arumugam and Velammal [3] have discussed the edge domination in graphs while the fractional edge domination in graphs is discussed in Arumugam and Jerry [2]. The complementary edge domination in graphs is studied by Kulli and Soner [15] while Jayaram [14] has studied the line dominating sets and obtained bounds for the line domination number. The bipartite graphs with equal edge domination number and maximum matching cardinality are characterized by Dutton and Klostermeyer [7] while Yannakakis and Gavril [22] have shown that edge dominating set problem is NP-complete even when restricted to planar or bipartite graphs of maximum degree. The edge domination in graphs of cubes is studied by Zelinka [23].

II. CORONA PRODUCT GRAPH $C_n \odot K_{1,m}$

The **corona product** of a cycle C_n wirh a star graph $K_{1,m}$ for m ≥ 2, is a graph obtained by taking one copy of a n-vertex graph C_n and n copies of $K_{1,m}$ and then joining the ith vertex of C_n to all vertices of ith copy of $K_{1,m}$. This graph is denoted by $C_n \odot K_{1,m}$.

The vertices in C_n are denoted by $v_1, v_2, \ldots\ldots\ldots, v_n$ and the edges in C_n by $e_1, e_2, \ldots\ldots, e_n$ where e_i is the edge joining the vertices v_i and v_{i+1}, i≠n. For i = n, e_n is the edge joining the vertices $v_n$and v_1.

The vertex in the first partition of $i^{th} copy of K_{1,m}$ is denoted by u_i and the vertices

in the second partition of $i^{th} copy of K_{1,m}$ are denoted by $w_{i1}, w_{i2}, \dots \dots \dots, w_{im}$. The edges in the $i^{th} copy of K_{1,m}$ are denoted by l_{ij} where l_{ij} is the edge joining the vertex u_i to the vertex w_{ij}. There are another type of edges, denoted by h_i, h_{ij}. Here h_i is the edge joining the vertex v_i in C_n to the vertex u_i in the $i^{th} copy of K_{1,m}$. The edge h_{ij} is the edge joining the vertex v_i in C_n to the vertex w_{ij} in the $i^{th} copy of K_{1,m}$.
The edge induced sub graph on the set of edges
$E_i = \{h_i, h_{ij}, l_{ij} : j = 1,2, \dots \dots .., m\}$is denoted by H_i, for i = 1,2,.......,n.
Some graph theoretic properties of corona product graph $C_n \odot K_{1,m}$ and edge dominating sets, edge domination number of this graph are studied by Sreedevi, J [19]. Some results on edge dominating functions of $C_n \odot K_{1,m}$ are presented in Sreedevi, J [18]. Further the total edge dominating functions of $C_n \odot K_{1,m}$ are discussed in Sreedevi, J [21]. Also some results on signed and Roman edge dominating functions of $C_n \odot K_{1,m}$ are studied by Sreedevi, J [20]

III. TOTAL SIGNED EDGE DOMINATING FUNCTIONS

Zelinka, B.[23] introduced the concept of total signed dominating function. Signed dominating function of a graph is a certain variant of Y - dominating function. This section contains the study of total signed edge dominating functions and minimal total signed edge dominating functions of the graph $G = C_n \odot K_{1,m}$. First we recall the definitions of total signed edge dominating function of a graph.

Definition: Let $G(V, E)$ be a graph. A function $f : E \to \{-1, 1\}$ is called a **total signed edge dominating function of G** if

$$f(N(e)) = \sum_{e' \in E(G)} f(e') \geq 1 \text{ for each } e \epsilon E.$$

A total signed edge dominating function f of G is called a **minimal total signed edge dominating function** (MTSEDF) if for all $g < f$, g is not a total signed edge dominating function.

Theorem 3.1: A function f : E→ $\{-1,1\}$ defined by

$$f(e) = \begin{cases} -1, & \text{for } \frac{m}{2} \text{ edges in each copy of } K_{1,m} \text{ in } G \text{ if m is even, for } \frac{m-1}{2} \\ & \text{edges in each copy of } K_{1,m} \text{ in G if m is odd,} \\ 1, & \text{otherwise,} \end{cases}$$

is a minimal total signed edge dominating function (MTSEDF) of $G = C_n \odot K_{1,m}$.

Proof: Let f be a function defined as in the hypothesis.

Case I: Suppose that m is even.

By the definition of the function, -1 is assigned to $\frac{m}{2}$ edges in each copy of $K_{1,m}$ and 1 is assigned to the remaining $\frac{m}{2}$ edges in each copy of $K_{1,m}$ in G.Also 1 is assigned to the remaining edges $e_i \in C_n$; $h_i, h_{ij} \in H_i$, i = 1,2,.....,n, j = 1,2,......,m.

The summation value taken over N(e) of $e \in E$ is as follows:

Case 1: Let $e_i \in C_n$ where i=1,2,........,n.

Then $adj(e_i) = 2m + 4$.

So $$\sum_{e \in N(e_i)} f(e) = (1+1) + \underbrace{(1+1+\cdots \dots \dots + 1)}_{(m+1)-times} + \underbrace{(1+1+\cdots \dots \dots \dots + 1)}_{(m+1)-times} = 2m + 4 > 1.$$

Case 2: Let $l_{ij} \in H_i$ where i=1,2,......n;j=1,2,.......,m.

Then $adj(l_{ij}) = m + 1$.

If $f(l_{ij}) = 1$, then $$\sum_{e \in N(l_{ij})} f(e) = \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2} - 1\right)(1)\right] + (1+1) = 1.$$

If $f(l_{ij}) = -1$, then $$\sum_{e \in N(l_{ij})} f(e) = \left[\left(\frac{m}{2}\right)(1) + \left(\frac{m}{2} - 1\right)(-1)\right] + (1+1) = 3 > 1.$$

Case 3: Let $h_i \in H_i$ where i=1,2,........n.

Then $adj(h_i) = 2m + 2$.

So $$\sum_{e \in N(h_i)} f(e) = (1+1) + \underbrace{(1+1+\cdots \dots \dots + 1)}_{m\ -times} + \left[\left(\frac{m}{2}\right)(1) + \left(\frac{m}{2}\right)(-1)\right] = m + 2 > 1.$$

Case 4: Let $h_{ij} \in H_i$ where i=1,2,........n;j=1,2,.....,m.

Then $adj(h_{ij}) = m + 3$.

So $\sum_{e\in N(h_{ij})} f(e) = (1+1) + \underbrace{(1+1+\cdots\ldots\ldots\ldots+1)}_{m\ -times} + f(l_{ij})$

$= m + 2 + f(l_{ij})$

$= m + 1, \text{if } f(l_{ij}) = -1$

$= m + 3, \text{if } f(l_{ij}) = 1.$

Therefore for all possibilities of e ∈ E, we get

$\sum_{e\in N(h)} f(e) > 1, \forall h \in E.$

This implies that f is a TSEDF.

Now we check for the minimality of f.

Define g: E → {−1,1} by

$$g(e) = \begin{cases} -1, & \text{for } \frac{m}{2} \text{ edges in each copy of } K_{1,m} \text{ in } G, \\ -1, & \text{if } e = h_k \in E \text{ for some k}, \\ 1, & \text{otherwise.} \end{cases}$$

Since strict inequality holds at the edge $h_k \in E$, it follows that g< f.

Case (i): Let $e_i \in C_n$ where i=1,2,.......,n.

Sub case 1: Let $h_k \in N[e_i]$. Then k = i or i + 1, if i ≠ 1 and k = 1 or n, if i = n.

So $\sum_{e\in N(e_i)} g(e) = (1+1) + (-1 + \underbrace{1+1+\cdots\ldots+1}_{m\ -times}) + \left(\underbrace{1+1+\cdots\ldots+1}_{(m+1)-times}\right)$

$= 2m + 2 > 1.$

Sub case 2: Let $h_k \notin N(e_i)$.

Then $\sum_{e\in N(e_i)} g(e) = (1+1) + (\underbrace{1+1+\cdots\ldots+1}_{(m+1)-times}) + (\underbrace{1+1+\cdots\ldots+1}_{(m+1)-times}) = 2m + 4 > 1.$

Case(ii): Let $l_{ij} \in H_i$ where i=1,2,......n; j=1,2,.......,m.

Sub case 1: Let $h_k \in N(l_{ij})$. Then k = i.

If $g(l_{ij}) = 1$, then $\sum_{e\in N(l_{ij})} g(e) = \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2} - 1\right)(1)\right] + (-1+1) = -1 < 1.$

If $g(l_{ij}) = -1$, then $\sum_{e\in N(l_{ij})} g(e) = \left[\left(\frac{m}{2}\right)(1) + \left(\frac{m}{2} - 1\right)(-1)\right] + (-1+1) = 1.$

Sub case 2: Let $h_k \notin N(l_{ij})$.

If $g(l_{ij}) = 1$, then $\sum_{e\in N(l_{ij})} g(e) = \left[\left(\frac{m}{2}\right)(-1) + \left(\frac{m}{2} - 1\right)(1)\right] + (1+1) = 1.$

If $g(l_{ij}) = -1$, then $\sum_{e\in N(l_{ij})} g(e) = \left[\left(\frac{m}{2}\right)(1) + \left(\frac{m}{2} - 1\right)(-1)\right] + (1+1) = 3.$

Case(iii): Let$h_i \in H_i$ where i=1,2,........n.

Then $\sum_{e\in N(h_i)} g(e) = (1+1) + \underbrace{(1+1+\cdots\ldots+1)}_{m\ -times} + \left[\left(\frac{m}{2}\right)(1) + \left(\frac{m}{2}\right)(-1)\right] = m + 2 > 1.$

Case(iv): Let $h_{ij} \in H_i$ where i=1,2,........n; j=1,2,.....,m.

Sub case 1: Let $h_k \in N(h_{ij})$. Then k=i.

Now $\sum_{e\in N(h_{ij})} g(e) = (1+1) + \left(-1 + \underbrace{1+1+\cdots\ldots\ldots\ldots+1}_{(m-1)-times}\right) + g(l_{ij}) = m + g(l_{ij})$

$= m - 1, \text{if } g(l_{ij}) = -1$

$= m + 1, \text{if } g(l_{ij}) = 1.$

Sub case 2: Let $h_k \notin N(h_{ij})$.

$\sum_{e\in N(h_{ij})} g(e) = (1+1) + \left(\underbrace{1+1+\cdots\ldots\ldots\ldots+1}_{m\ -times}\right) + g(l_{ij}) = m + 2 + g(l_{ij})$

$= m + 1, \text{if } g(l_{ij}) = -1$

$= m + 3, \text{if } g(l_{ij}) = 1.$

Thus we have seen that $\sum_{e\in N(h)} g(e) < 1$ for some h $\in$ E.
So, g is not a TSEDF.
Since g is defined arbitrarily, it follows that there exists no g< f such that g is a TSEDF.
Thus f is a MTSEDF.
Case II: Suppose that m is odd.
By the definition of the function, -1 is assigned to $\frac{m-1}{2}$ edges in each copy of $K_{1,m}$ and 1 is assigned to the remaining $\frac{m+1}{2}$ edges in each copy of $K_{1,m}$ in G. Also 1 is assigned to the edges $e_i \in C_n$; $h_i, h_{ij} \in H_i$ for i = 1,2,.....,n, j = 1,2,......,m.
The summation value taken over N(e) of e$\in$ E is as follows:
As in Case I, for Case 1, Case 4 we get the same functional values for
$\sum_{e\in N(e_i)} f(e)$, $\sum_{e\in N(h_{ij})} f(e)$ respectively.
For Case 2, the value is as follows.

$$\sum_{e\in N(l_{ij})} f(e) = \left[\left(\frac{m-1}{2}\right)(-1) + \left(\frac{m+1}{2} - 1\right)(1)\right] + (1+1) = 2 > 1, \text{if} f(l_{ij}) = 1$$

$$\text{and} \sum_{e\in N(l_{ij})} f(e) = \left[\left(\frac{m+1}{2}\right)(1) + \left(\frac{m-1}{2} - 1\right)(-1)\right] + (1+1) = 4 > 1, \text{if} f(l_{ij}) = -1.$$

For Case 3, we have

$$\sum_{e\in N(h_i)} f(e) = (1+1) + \underbrace{(1+1+\cdots\ldots\ldots+1)}_{m\ -times} + \left[\left(\frac{m+1}{2}\right)(1) + \left(\frac{m-1}{2}\right)(-1)\right]$$

$= m + 3 > 1.$
Therefore for all possibilities of e$\in$ E, we get

$$\sum_{e\in N(h)} f(e) > 1, \forall h \in E.$$

This implies that f is a TSEDF.
Now we check for the minimality of f.
Define g: E→ $\{-1,1\}$ by

$$g(e) = \begin{cases} -1, & \text{for} \frac{m-1}{2} \text{ edges in each copy of } K_{1,m} \text{ in } G, \\ -1, & \text{if } e = h_k \in \text{ E for some k}, \\ 1, & \text{otherwise.} \end{cases}$$

Since strict inequality holds at the edge $h_k \in E$, it follows that g < f.
As in Case I, for Case (i), Case (iv), we get the same functional values for
$\sum_{e\in N(e_i)} g(e)$, $\sum_{e\in N(h_{ij})} g(e)$ respectively.
Now in Case (ii), we have
if $h_k \in N(l_{ij})$, then

$$\sum_{e\in N(l_{ij})} g(e) = \left[\left(\frac{m+1}{2} - 1\right)(1) + \left(\frac{m-1}{2}\right)(-1)\right] + (-1+1) = 0, \text{if} g(l_{ij}) = 1$$

$$\text{and} \sum_{e\in N(l_{ij})} g(e) = \left[\left(\frac{m+1}{2}\right)(1) + \left(\frac{m-1}{2} - 1\right)(-1)\right] + (-1+1) = 2\ , \text{if} g(l_{ij}) = -1.$$

If $h_k \notin N(l_{ij})$, then

$$\sum_{e\in N(l_{ij})} g(e) = \left[\left(\frac{m+1}{2} - 1\right)(1) + \left(\frac{m-1}{2}\right)(-1)\right] + (1+1) = 2, \text{if} g(l_{ij}) = 1$$

$$\text{and} \sum_{e\in N(l_{ij})} g(e) = \left[\left(\frac{m+1}{2}\right)(1) + \left(\frac{m-1}{2} - 1\right)(-1)\right] + (1+1) = 4, \text{if} g(l_{ij}) = -1.$$

Again in Case (iii), we have

$$\sum_{e\in N(h_i)} g(e) = (1+1) + \underbrace{(1+1+\cdots\ldots\ldots+1)}_{mtimes} + \left[\left(\frac{m+1}{2}\right)(1) + \left(\frac{m-1}{2}\right)(-1)\right]$$
$$= m+3 > 1.$$

Thus we have seen that $\sum_{e\in N(h)} g(e) < 1$ for some h $\in$ E.

So, g is not a TSEDF.

Since g is defined arbitrarily, it follows that there exists no g< f such that g is a TSEDF.

Thus f is a MTSEDF. ■

IV. TOTAL ROMAN EDGE DOMINATING FUNCTIONS

Roman domination is suggested originally in the article Scientific American by Ian Stewart [24]. In this section the concept of total Roman edge dominating function of the graph $G = C_n \odot K_{1,m}$ is studied. Also some results on minimal total Roman edge dominating function of $G = C_n \odot K_{1,m}$ are obtained.

First we define total Roman edge dominating function of a graph.

Definition: Let $G(V, E)$ be a graph. A function $f : E \to \{0, 1, 2\}$ is called a total **Roman edge dominating function** (TREDF) of G if

$$f(N(e)) = \sum_{e'\in N(e)} f(e') \geq 1, \text{for each } e \in E$$

and satisfying the condition that every edge e' for which $f(e') = 0$ is adjacent to at least one edge e for which $f(e) = 2$.

A total Roman edge dominating function f of G is called a **minimal total Roman edge dominating function** (MTREDF) if for all $g < f$, g is not a total Roman edge dominating function.

Theorem 4.1: A function $f : E \to \{0,1,2\}$ defined by

$$f(e) = \begin{cases} 2, & \text{for } e = h_i, i = 1,2,\ldots\ldots\ldots,n, \\ & \text{and for any one edge in each copy of } K_{1,m} \text{ in G,} \\ 0, & \text{otherwise,} \end{cases}$$

is a minimal total Roman Edge dominating function (MTREDF) of $G = C_n \odot K_{1,m}$.

Proof: Let f be a function defined as in the hypothesis.

The summation value taken over N(e) of $e \in E$ is as follows:

Case 1: Let $e_i \in C_n$ where i = 1,2,........,n.

Then $$\sum_{e\in N(e_i)} f(e) = (0+0) + \left(2 + \underbrace{0+0+\cdots\ldots\ldots+0}_{m-times}\right) + \left(2 + \underbrace{0+0+\cdots\ldots\ldots+0}_{m-times}\right)$$
$$= 4 > 1.$$

Case 2: Let $l_{ij} \in H_i$ where i=1,2,......n;j=1,2,.......,m.

Then $$\sum_{e\in N(l_{ij})} f(e) = \left(\underbrace{0+0+\cdots\ldots\ldots+0}_{(m-1)-times}\right) + (2+0) = 2 > 1, \text{if } f(l_{ij}) = 2$$
$$= \left(2 + \underbrace{0+0+\cdots\ldots\ldots+0}_{(m-2)times}\right) + (2+0) = 4 > 1, \quad \text{if } f(l_{ij}) = 0.$$

Case 3: Let $h_i \in H_i$ where i=1,2,........n.

Then $$\sum_{e\in N(h_i)} f(e) = (0+0) + (\underbrace{0+0+\cdots\ldots\ldots+0}_{m-times}) + (2 + \underbrace{0+0+\cdots\ldots\ldots+0}_{(m-1)-times}) = 2 > 1.$$

Case 4: Let $h_{ij} \in H_i$ where i=1,2,........n;j=1,2,.....,m.

Then $$\sum_{e\in N(h_{ij})} f(e) = (0+0) + \left(2 + \underbrace{0+0+\cdots\ldots\ldots+0}_{(m-1)-times}\right) + f(l_{ij}) = 2 + f(l_{ij})$$
$$= 4, \quad \text{if } f(l_{ij}) = 2$$
$$= 2, \text{ if } f(l_{ij}) = 0.$$

Therefore for all possibilities of e$\in$ E, we get

$$\sum_{e\in N(h)} f(e) > 1, \forall h \in E.$$

That f is a TEDF.

We now verify that f is a TREDF.

Let e$\in$ E be such that $f(e) = 0$. Then $e = e_i$ for all i or $e = h_{ij}$ for all i and j or $e = l_{ij}$ for

all i and j, j≠k.
Suppose $e' \in$ E be such that $f(e') = 2$. Then $e' = h_i$ for all i or $e' = l_{ik}$ ineach copy .
Let $e = e_i$. Then e_i is adjacent to h_i, $\forall i$.
Let $e = l_{ij}$. Since all l_{ij} are adjacent, it follows that l_{ij} is adjacent to l_{ik} for which $f(l_{ik}) = 2$ in each copy.
Let $e = h_{ij}$. Then e is adjacent to h_i for which $f(h_i) = 2$.
Therefore f is a total REDF.
Now we check for the minimality of f.
Define g : E→ {0,1,2} by

$$g(e) = \begin{cases} 2, & \text{for} e = h_i,\ i = 1,2,\dots\dots\dots,n, i \neq k, \text{for some k} \\ & \text{and for any one edge in each copy of } K_{1,m} \text{ in G,} \\ 1, & \text{for} e = h_k, \\ 0, & \text{otherwise.} \end{cases}$$

Since strict inequality holds at the edge $h_k \in E$, it follows that g<f.
Case (i): Let $e_i \in C_n$ where i=1,2,.......,n.
Sub case 1: Let $h_k \in N(e_i)$. Then k = i or i + 1, if i ≠ 1 and k = 1 or n, if i = n.

$$\text{So} \sum_{e\in N(e_i)} g(e) = (0+0) + \Big(1 + \underbrace{0+0+\cdots\dots+0}_{m-\text{ times}}\Big) + \Big(2 + \underbrace{0+0+\cdots\dots+0}_{m\ -\text{ times}}\Big) = 3 > 1.$$

Sub case 2: Let $h_k \notin N(e_i)$.

$$\text{Then} \sum_{e\in N(e_i)} g(e) = (0+0) + \Big(2 + \underbrace{0+0+\cdots\dots+0}_{m\ -\text{times}}\Big) + \Big(2 + \underbrace{0+0+\cdots\dots+0}_{m\ -\text{times}}\Big)$$

$$= 4 > 1.$$

Case(ii): Let $l_{ij} \in H_i$ where i=1,2,......n; j=1,2,.......,m.
Then $g(l_{ij}) = 2$ or 0.
Sub case 1: Let $h_k \in N(l_{ij})$. Then k= i.

$$\text{If } g(l_{ij}) = 2, \text{then} \sum_{e\in N(l_{ij})} g(e) = \Big(\underbrace{0+0+\cdots\dots\dots+0}_{(m-1)-\text{ times}}\Big) + (1+0) = 1.$$

$$\text{If } g(l_{ij}) = 0, \text{then} \sum_{e\in N(l_{ij})} g(e) = \Big(2 + \underbrace{0+0+\cdots\dots\dots+0}_{(m-2)-\text{ times}}\Big) + (1+0) = 3 > 1.$$

Sub case 2: Let $h_k \notin N(l_{ij})$.

$$\text{If } g(l_{ij}) = 2, \text{then} \sum_{e\in N(l_{ij})} g(e) = \Big(\underbrace{0+0+\cdots\dots\dots+0}_{(m-1)-\text{ times}}\Big) + (2+0) = 2 > 1.$$

$$\text{If } g(l_{ij}) = 0, \text{then} \sum_{e\in N(l_{ij})} g(e) = \Big(2 + \underbrace{0+0+\cdots\dots\dots+0}_{(m-2)-\text{ times}}\Big) + (2+0) = 4 > 1.$$

Case(iii): Let $h_i \in H_i$ where i=1,2,........n.
Then

$$\sum_{e\in N(h_i)} g(e) = (0+0) + \underbrace{(0+0+\cdots\dots\dots+0)}_{m\ -\text{times}} + \Big(2 + \underbrace{0+0+\cdots\dots\dots+0}_{(m-1)-\text{ times}}\Big) = 2 > 1.$$

Case(iv): Let $h_{ij} \in H_i$ where i=1,2,........n; j=1,2,.....,m.
Then $g(l_{ij}) = 2$ or 0.
Sub case 1: Let $h_k \in N(h_{ij})$. Then k= i.

$$\text{So} \sum_{e\in N(h_{ij})} g(e) = (0+0) + \Big(1 + \underbrace{0+0+\cdots\dots\dots\dots+0}_{(m-1)-\text{ times}}\Big) + g(l_{ij})$$

$$= 1 + g(l_{ij})$$

$$\begin{aligned} &= 3, && \text{if } g(l_{ij}) = 2 \\ &= 1, && \text{if } g(l_{ij}) = 0. \end{aligned}$$

Sub case 2: Let $h_k \notin N(h_{ij})$.

$$\text{Then} \sum_{e\in N(h_{ij})} g(e) = (0+0) + \Big(2 + \underbrace{0+0+\cdots\dots\dots\dots+0}_{(m-1)-\text{ times}}\Big) + g(l_{ij})$$

$= 2 + g(l_{ij})$
$\quad = 4, \quad$ if $g(l_{ij}) = 2$
$\quad = 2, \quad$ if $g(l_{ij}) = 0.$
Therefore for all possibilities of e ∈ E, we get

$$\sum_{e \in N(h)} g(e) \geq 1, \forall h \in E.$$

i.e., g is a TEDF. But g is not a TREDF since the TREDF definition fails in the sub graph H_k of G.
Consider the edge h_{kj} in H_k which is adjacent to l_{kj} for which $g(l_{kj}) = 0$.
We know that $g(h_{kj}) = 0$.
Then there is no edge e in H_k such that g(e) = 2 and h_{kj} and e are adjacent.
So, g is not a TREDF.
Since g is defined arbitrarily, it follows that there exists no g< f such that g is a TREDF.
Therefore f is a MTREDF.■

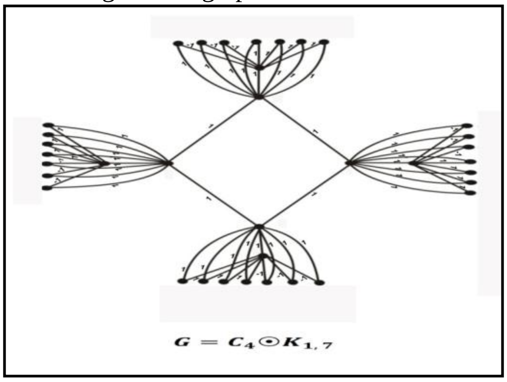
CONCLUSION

It is interesting to study various graph theoretic properties and domination parameters of corona product graph of a cycle with a star. Edge dominating functions and total edge dominating functions of this graph are studied by the authors. In recent years the study of signed and Roman domination and its variations is attracting the researchers. In this paper a study of these concepts for corona product graph of a cycle with a star is discussed. Study of these graphs enhances further research and throws light on further developments.

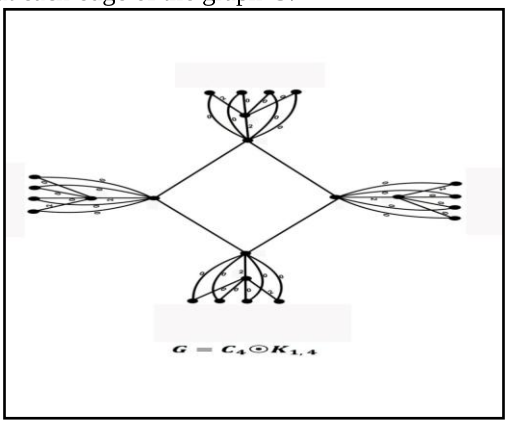
V. GRAPHS

MINIMAL TOTAL SIGNED EDGE DOMINATING FUNCTION

Theorem 3.1
Case II
The functional values are given at each edge of the graph G.



MINIMAL TOTAL ROMAN EDGE DOMINATING FUNCTION
Theorem 4.1
The functional values are given at each edge of the graph G.



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