Eccentric Adjacent Vertex Sum Polynomial of Wheel Related Graphs

Sowmya.S[#]

[#]Assistant Professor in Mathematics, Malankara Catholic College, Mariagiri, Tamil Nadu, India.
(Affiliated to Manonmaniam Sundaranar University, Tirunelveli)

Abstract

Let G = (V(G), E(G)) be a graph. The eccentric adjacent vertex sum polynomial of graph G is defined as $EA(G,x)=\sum_{i=1}^{k}\varepsilon_G(v_i)\,x^{\delta_G(v_i)}$ *where the eccentricity* $\varepsilon_G(v_i)$ *is the largest distance from* v_i *to any other vertices of G and* $\delta_G(v_i) = \sum_{v_j\in N(v_i)} d_G(v_j)$, *is the sum of degrees of the neighboring vertices of* $v_i \in G$. *In this paper, I discussed the eccentric adjacent vertex sum polynomial of some wheel related graphs.*

Keywords - *eccentric adjacent vertex sum polynomial, eccentricity, wheel.*

.

I. INTRODUCTION

Let G be a simple connected graph with vertex set $V(G)$ and edge set $E(G)$, so that $|V(G)| = k$ and $|E(G)| = e$. For graph terminologies, I follow [1]. Let the vertices of G be labeled as $v_1, v_2, \dots, v_k$. For any vertex $v_i \in V(G)$, the number of neighbors of v_i is defined as the degree of the vertex v_i and is denoted by $d_G(v_i)$. Let $N(v_i)$ denote the set of vertices which are the neighbors of the vertex v_i, so that $|N(v_i)| = d_G(v_i)$. Also let $\delta_G(v_i) = \sum_{v_j\in N(v_i)} d_G(v_j)$, that is, sum of degrees of the neighboring vertices of $v_i \in G$. The distance between the vertices v_i and v_j is equal to the length of the shortest path connecting v_i and v_j. Also for a given vertex $v_i \in V(G)$, the eccentricity $\varepsilon_G(v_i)$ is the largest distance from v_i to any other vertices of G. The eccentric adjacent vertex sum polynomial of graph G is defined as $EA(G,x)=\sum_{i=1}^{k}\varepsilon_G(v_i)\,x^{\delta_G(v_i)}$ where the eccentricity $\varepsilon_G(v_i)$ is the largest distance from v_i to any other vertices of G and $\delta_G(v_i) = \sum_{v_j\in N(v_i)} d_G(v_j)$, is the sum of degrees of the neighboring vertices of $v_i \in G$.

1.1. Definition The Wheel W_n is the graph obtained by adding a new vertex joining to each of the vertices of C_n. That is, $W_n = C_n + K_1$. The new vertex is called the apex vertex and the vertices corresponding to C_n are called rim vertices of W_n. The edges joining rim vertices are called rim edges.

1.2. Definition [2] The Sunflower graph SF_n is the graph obtained by taking a wheel with the apex vertex v_0 and the consecutive rim vertices $v_1, v_2, \dots, v_n$ and additional vertices $w_1, w_2, \dots, w_n$ where w_i is joined by edges to v_i and v_{i+1}(mod n).

1.3. Definition [3] Duplication of a vertex v_k of a graph G produces a new graph G' by adding a new vertex v_k' such that $N(v_k') = N(v_k)$. In other words, v_k' is said to be a duplication of v_k if all the vertices which are adjacent to v_k in G are now adjacent to v_k' in G'.

1.4. Definition [3] Duplication of a vertex v_k by a new edge e = uv in a graph G produces a new graph G' by adding an edge e′ = u′v′ such that N(e′) = N(e).

1.5. Definition The Helm graph H_n is the graph obtained from a wheel W_n by adjoining a pendant edge at each node of the cycle.

1.6. Definition The Flower graph Fl_n is the graph obtained from a helm H_n by joining each pendant vertex to the apex of the helm H_n.

II. MAIN RESULTS

2.1. Theorem The eccentric adjacent vertex sum polynomial of sunflower graph SF_n is $EA(SF_n, x) = 2x^{5n} + 3nx^{n+14} + 4nx^{10}$ for $n \geq 6$.

Proof. The sunflower graph SF_n, is a graph that always has an odd number of vertices 2n + 1, say $v_0, v_1, v_2, \ldots, v_n$; $u_1, u_2, \ldots, u_n$, and a number of edges m = 4n. The central vertex v_0 has degree n, the vertices $v_1, v_2, \ldots, v_n$ have degree 5, and the vertices $u_1, u_2, \ldots, u_n$ have degree 2. In SF_n, one vertex has eccentricity 2 with neighboring vertices whose degree sum is 5n, n vertices has eccentricity 3 with degree sum n + 14 and n vertices has eccentricity 4 with neighboring vertices whose degree sum is 10. Hence $EA(SF_n, x) = 2x^{5n} + 3nx^{n+14} + 4nx^{10}$. This is true for all $n \geq 6$.

2.2. Illustration

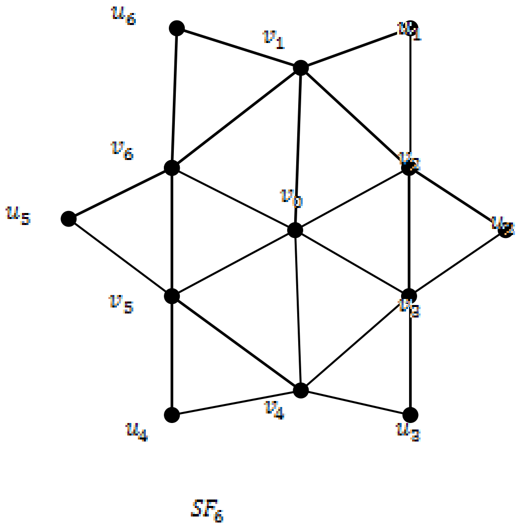

Fig. 1

2.3. Theorem The graph G obtained from duplication of each of the vertices w_i, i = 1, 2, …, n by a new vertex in the sunflower graph SF_n whose eccentric adjacent vertex sum polynomial is $EA(G, x) = 2x^{7n} + 3nx^{n+22} + 8nx^{14}$ for $n \geq 5$.

Proof. Let v_0 be the apex vertex of SF_n with $v_1, v_2, \ldots, v_n$ are the consecutive rim vertices of W_n and w_i, i = 1,2,…,n are the additional vertices where w_i is joined to v_i and v_{i+1}(mod n). Let the consecutive rim edges of the W_n be $e_1, e_2, \ldots, e_n$. e_i', i =1, 2,…,n are the corresponding edges joining apex vertex v_0 to the vertices $v_1, v_2, \ldots, v_n$ of C_n. For each $1 \leq i \leq n$, e_i^l are the edges joining w_i to v_i and e_i^r are the edges joining w_i to v_{i+1}(mod n). Let the graph obtained from SF_n by duplication of the vertices $w_1, w_2, \ldots, w_n$ by new vertices $u_1, u_2, \ldots, u_n$ respectively be denoted as G with 3n + 1 vertices and 6n edges. Let for each $1 \leq i \leq n$, f_i^l be the edges joining u_i to v_i and f_i^r be the edges joining u_i to v_{i+1}(mod n). In G, one vertex has eccentricity 2 with neighboring vertices whose degree sum is 7n, n vertices has eccentricity 3 with degree sum n + 22 and 2n vertices has eccentricity 4 with neighboring vertices whose degree sum is 14. Hence $EA(G, x) = 2x^{7n} + 3nx^{n+22} + 8nx^{14}$. This is true for all $n \geq 5$.

2.4. Illustration

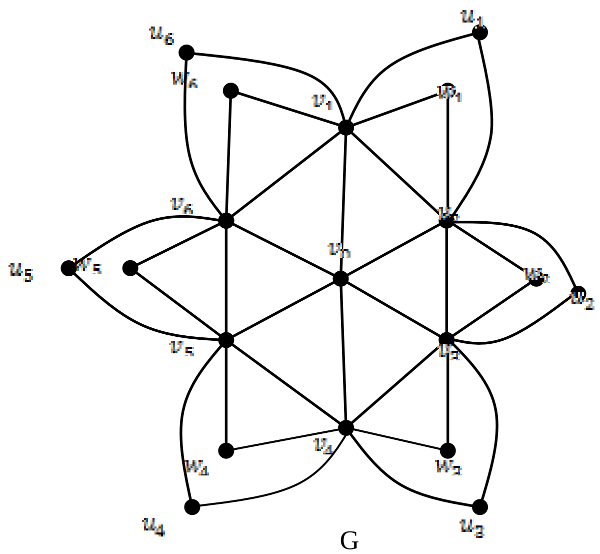

Fig. 2

2.5. Theorem The graph G obtained from duplication of each of the vertices w_i for i = 1, 2, …, n by a new edges f_i in the sunflower graph SF_n whose eccentric adjacent vertex sum polynomial is EA(G, x) = $3x^{5n} + 4nx^{n+18} + n\left\lfloor \frac{n+4}{2} \right\rfloor x^{14} + 2n\left(\left\lfloor \frac{n+4}{2} \right\rfloor + 1\right)x^6$, $n \geq 4$.

Proof. Let v_0 be the apex vertex of SF_n with v_i, i = 1, 2, …, n be the consecutive rim vertices of W_n and w_i, i = 1, 2, …, n be the additional vertices where w_i is joined to v_i and v_{i+1}(mod n). Let the consecutive rim edges of W_n be e_i, i = 1, 2, …, n. e_i', i = 1, 2, …, n are the corresponding edges joining apex vertex v_0 to the vertices v_i, i = 1, 2, …, n of C_n. Let for each $1 \leq i \leq n$, e_i^l be the edges joining w_i to v_i and e_i^r be the edges joining w_i to v_{i+1}(mod n). Let G be the graph obtained from SF_n by duplication of the vertices w_i, i = 1, 2, …, n by corresponding new edges f_i, i = 1, 2, …, n with new vertices u_i^l and u_i^r such that u_i^l and u_i^r join to the vertex w_i with 4n + 1 vertices and 7n edges. Let for each $1 \leq i \leq n$, f_i be the edges joining the vertices u_i^l and u_i^r and for each $1 \leq i \leq n$, f_i^l be the edges joining u_i^l to w_i and f_i^r be the edges joining u_i^r to w_i. In G, one vertex has eccentricity 3 with neighboring vertices whose degree sum is 5n, n vertices has eccentricity 4 with degree sum n + 18 and n vertices has eccentricity $\left\lfloor \frac{n+4}{2} \right\rfloor$ with neighboring vertices whose degree sum is 14, 2n vertices has eccentricity $\left(\left\lfloor \frac{n+4}{2} \right\rfloor + 1\right)$ with neighboring vertices whose degree sum is 6. Hence EA(G, x) = $3x^{5n} + 4nx^{n+18} + n\left\lfloor \frac{n+4}{2} \right\rfloor x^{14} + 2n\left(\left\lfloor \frac{n+4}{2} \right\rfloor + 1\right)x^6$. This is true for all $n \geq 4$.

2.6. Illustration

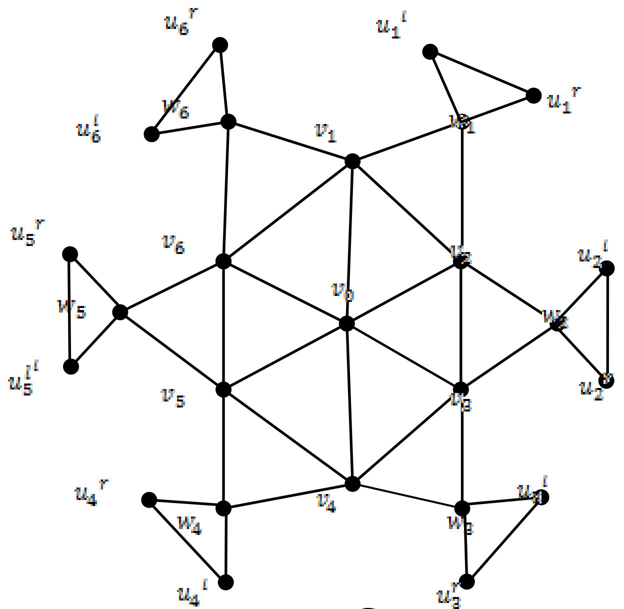

Fig. 3

2.7. Theorem The graph G is obtained by subdividing the edges w_iv_i and w_iv_{i+1} (subscripts are mod n) for all i = 1, 2, …, n by a vertex in the sunflower graph SF_n whose eccentric adjacent vertex sum polynomial is EA(G, x) = $3x^{5n} + 4nx^{n+14} + 10nx^7 + 6nx^4$, $n \geq 5$.

Proof. Let SF_n be the sunflower graph, where v_0 is the apex vertex, v_i, i = 1, 2, …, n be the consecutive rim vertices of W_n and w_i, i = 1, 2, …, n be the additional vertices where w_i is joined to v_i and v_{i+1}(mod n). Let e_i, i = 1, 2, …, n be the consecutive rim edges of the W_n. e_i', i = 1, 2, …, n are the corresponding edges joining apex vertex v_0 to the vertices $v_1, v_2, …, v_n$ of the cycle. Let G be the graph obtained from SF_n by subdividing the edges w_iv_i and w_iv_{i+1}(mod n) for $1 \leq i \leq n$ by a vertex u_i^l and u_i^r respectively. Let for each $1 \leq i \leq n$, e_i^l be the edges joining u_i^l to v_i and e_i^r be the edges joining u_i^r to v_{i+1}(mod n). Let for each $1 \leq i \leq n$, f_i^l be the edges joining w_i to u_i^l and f_i^r be the edges joining w_i to u_i^r. Thus |V(G)| = 4n + 1 and |E(G)| = 6n. In G, one vertex has eccentricity 3 with neighboring vertices whose degree sum is 5n, n vertices has eccentricity 4 with degree sum n + 14 and 2n vertices has eccentricity 5 with neighboring vertices whose degree sum is 7, n vertices has eccentricity 6 with neighboring vertices whose degree sum is 4. Hence $EA(G, x) = 3x^{5n} + 4nx^{n+14} + 10nx^{7} + 6nx^{4}$. This is true for all $n \geq 5$.

2.8. Illustration

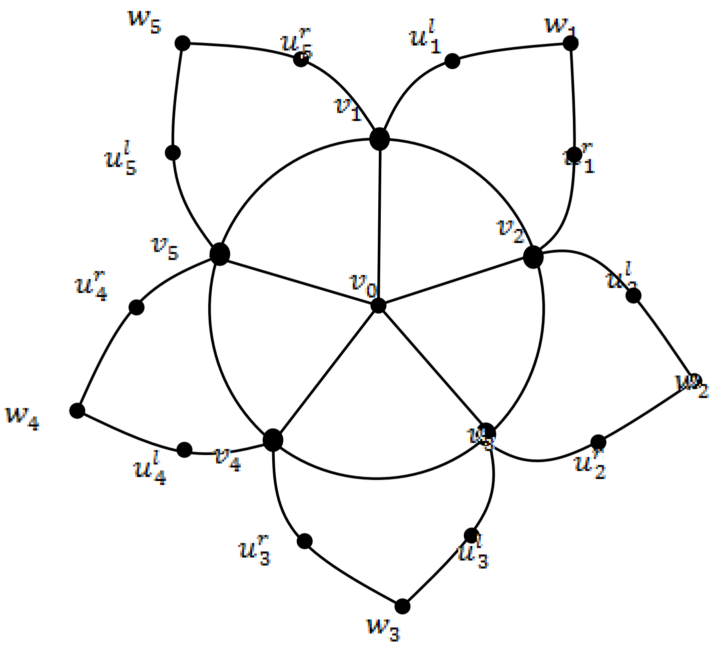

Fig. 4

2.9. Theorem The graph G obtained from flower graph Fl_n by adding n pendant vertices to the apex vertex v_0 whose eccentric adjacent vertex sum polynomial is $EA(G, x) = x^{7n} + 2nx^{3n+10} + 2nx^{3n+4} + 2nx^{3n}$, $n \geq 5$.

Proof. The Flower graph Fl_n is the graph obtained from a helm H_n by joining each pendant vertex to the apex of the helm H_n. Let G be a graph obtained from the flower graph Fl_n by attaching n pendant edges to the apex vertex with 3n + 1 vertices, 5n edges and let v_0 be the apex vertex with 3n degrees, $v_1, v_2, v_3, \ldots, v_n$ be the rim vertices with four degrees, w_i for i = 1, 2, …,n be the extreme vertices with two degrees, u_i for i = 1, 2, …,n be the pendant vertices of G. Thus |E(G)| = 5n and |V(G)| = 3n + 1. In G, one vertex has eccentricity 1 with neighboring vertices whose degree sum is 7n, n vertices has eccentricity 2 with degree sum 3n + 10 and n vertices has eccentricity 2 with neighboring vertices whose degree sum is 3n + 4, n vertices has eccentricity 2 with neighboring vertices whose degree sum is 3n. Hence $EA(G, x) = x^{7n} + 2nx^{3n+10} + 2nx^{3n+4} + 2nx^{3n}$.This is true for all $n \geq 5$.

2.10. Illustration

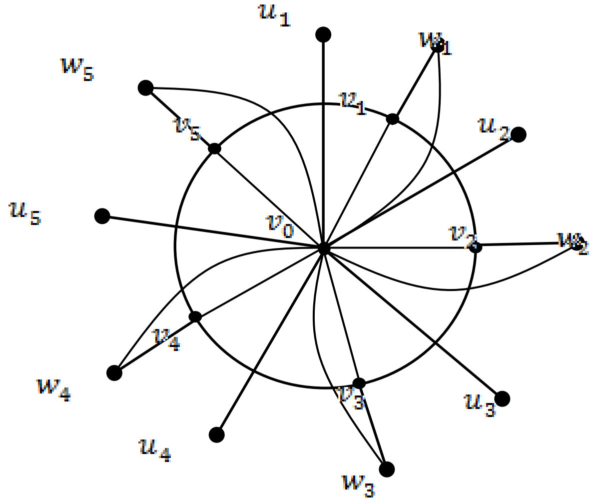

Fig. 5

ACKNOWLEDGEMENT

The author is highly thankful to the anonymous referee for valuable comments and constructive suggestions.

REFERENCES

[1] Gross, J.L. and Yellen, J., Handbook of Graph Theory. CRC Press, Boca Raton, 2004.
[2] Ponraj, R., Sathish Narayanan, S. and Kala, R., A Note on Difference Cordial Graphs. Palestine Journal of Mathematics, 4, 189-197, 2015.
[3] Udayan M. Prajapati, Nittal B. Patel, Edge Product Cordial Labeling of Some Cycle Related Graphs, Open Journal of Discrete Mathematics, 6, 268-278, 2016.
[4] Vaidya, S.K. and Barasara, C.M., Edge Product Cordial Labeling of Graphs, Journal of Mathematical and computational Science, 2, 1436-1450, 2012.
[5] Vaidya, S.K. and Prajapati, U.M., Prime Labeling in the Context of Duplication of Graph Elements, International Journal of Mathematics and Soft Computing , 3, 13-20, 2013.